Unobserved Heterogeneity in Price-Setting Behavior: a Duration Analysis Approach

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Abstract

There is strong empirical evidence that the degree of price stickiness is fairly different across commodity items and the nonparametric hazard function of price changes is downward sloping and has some spikes. We introduce the item-specific heterogeneity into the standard single-sector model of Calvo (1983) and estimate a hazard function of price adjustment applying the framework of duration analysis. This paper presents the appropriate form of heterogeneity for the data structure and shows that the decreasing (population) hazard function can be well-described. In the presence of item-specific heterogeneity, probability that prices remain unchanged is predicted higher than that of the single-sector model.

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1. Introduction

Previous studies (Bils and Klenow, 2004; Dhyne et al., 2005; Saita et al., 2006) showed that the degree of price stickiness is fairly different across commodity items. The time-dependent pricing model (Calvo, 1983) in which one single parameter represents the price stickiness cannot reproduce the strong empirical evidence such that the nonparametric hazard function of price changes is decreasing (Álvarez, Burriel and Hernando, 2005). Each item has specific factors which relate to its survival experience. These specific factors, whether observable or not, change the shape of the (individual) hazard function. If the variability in hazard is not fully captured by covariates, it is necessary to model unobserved heterogeneity.

In many empirical works of price-setting behavior, however, the unobserved heterogeneity has left unspecified. Therefore, we introduce the item-specific heterogeneity into the standard single-sector model of Calvo (1983) and estimate the hazard function of price adjustment applying the framework of duration analysis. This paper provides the appropriate form of heterogeneity for the data structure and shows that the decreasing (population) hazard can be well-described by modeling item-specific heterogeneity.

Recent papers analyzing monetary shock by using calibrated dynamic general equilibrium model shows that the degree of monetary non-neutrality implied by a multi-sector model is larger than that implied by its single-sector counterpart calibrated to the mean frequency of price change. The response to the monetary shock is much larger in sticky sector. These findings is consistent with our result that the existence of heterogeneity implies the speed of price adjustment slows down because as time elapses the effect of price spells with long duration gradually dominates.

The organization of the paper is as follows. In Section 2, we briefly summarize our approach to this problem. In section 3, we present the model with unobserved heterogeneity which is shared across price spells within an item. This model is called shared frailty model[1]. In Section 4, the results are discussed.

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[1] Unobserved heterogeneity is also referred to as frailty in the context of biostatistics. Shared frailty models can be understood as correlated heterogeneity models in the sense that observations within a group are correlated through the unobserved heterogeneity they share.

2. Duration Approach to the Problem of Price Stickiness

Following Aucremann and Dhyne (2004), methods used in this field of research can be classified into two approaches; frequency approach and duration approach. The method we use in this paper is the latter one.

In frequency approach, we first calculate the monthly frequency of price changes by items which is equal to the total number of price changes devided by the total number of observed prices. Then we aggregate the frequencies weighted by CPI weight to get the mean frequency. The expected value of waiting time is a reciprocal of the frequency of price change. Therefore the mean frequency implies the expected length of price spell as follows[2]:

$$\frac{1}{\delta} = -\frac{1}{\ln(1-\lambda)}, \tag{1}$$

where λ is monthly frequency and δ is implied instantaneous frequency. This follows from the derivation of the expected waiting time for the Poisson process. The median of price-change frequency is the middle value of weighted frequency which implies the median duration in the same way. Bils and Klenow (2004), for instance, use the "frequency" approach. They first compute the frequency of price changes and then infer the implied average duration of a price spell for each product category used in the US CPI.

In duration approach, we start off by specifying the functional form of hazard function. The price setting behavior described by the Calvo model corresponds to the exponential model with constant hazard rate. Calvo (1983) assumes the probability

[2]Monthly frequency and instantaneous frequency satisfy the equality $\lambda = 1 - e^{-\delta}$, or equivalently $\delta = -\ln(1-\lambda)$. If we divide 1 month into n equidistant intervals Δt and assume all firms change their price with probability $\delta\Delta t$ during the period $(t, t+\Delta t)$, the probability that price change does not occur for one month becomes $(1-\delta\,\Delta t)^n$. Therefore the monthly probability of price change is

$$\lambda = 1-(1-\delta\Delta t)^n = 1-\left(1-\delta\cdot\frac{1^{month}}{n}\right)^n.$$

Letting $\Delta t \to 0$, that is, $n \to \infty$, we obtain $\lambda = 1-e^{-\delta}$.

density function of a price spell with duration t as follows:

$$\delta \cdot e^{-\delta t}, \qquad \delta > 0. \tag{2}$$

This pdf is devided into two parts. The first one, δ, is equal to the price-change probability, i.e., the hazard rate in the Calvo model. The second one, $e^{-\delta t}$, is the survivor function which means the probability that a price has not been changed until t. Then we construct likelihood function according to this functional form and maximize it using all of the data to obtain the maximum likelihood estimators which summarize the shape of hazard function and survivor function. The median duration is the elapsed time which satisfies the condition that the survival probability is 0.5, that is, the proportion of the unchanged price spells is just 50 percent.

One advantage of this approach is that we can clearly evaluate the pattern of price adjustment. As shown in later, the nonparametric hazard rate is significantly higher at 12, 24, and 36 month, which suggests price changes tend to occur annually. Secondly, we can introduce the various types of heterogeneity. Álvarez, Burriel and Hernando (2005) use finite mixture models, which suppose that population consists of some homogeneous subpopulations. They specify the hazard function for each subpopulation and show that the mixture of hazard functions varies according to how they specify functional forms and how many subpopulations in the model. They document that it is optimal to estimate a model composed of 3 groups with a different but constant hazard rate, plus 1 group with a positive hazard rate at every 12 month[3]. In our shared frailty model, we assume *a priori* that population consists of heterogeneous items, which is supported by the result that the degree of price stickiness is fairly different across items as we mentioned earlier. Formally, the difference between the finite mixture model and our shared frailty model is that the former is a fixed effects model with random groups, whereas the latter is a random effects model with known groups[4].

[3] Ikeda and Nishioka (2007) slightly modified the model of Álvarez, Burriel and Hernando (2005)'s. They apply the Weibull hazard model to each component, which is monotonically increasing (or decreasing) according to the parameter value.

[4] See Mosler (2003) and Cameron and Trivedi (2005) for further discussion.

3. Shared frailty model for price-setting behaviors

In this analysis, we exploit a sample of retail prices underlying the computation of the Japanese CPI. These prices are collected on a monthly basis by the Statistic Bureau, Ministry of Internal Affairs and Communication and appeared in *Monthly Report on the Retail Price Survey.* Prices are reported at each of the cities with prefectural governments and the cities with a population of 150,000 or more. The period covered by this analysis starts in January 2000 and ends in December 2005. Our raw data set considers 498 items which covers 68 % of the Japanese CPI in 2000 and consists of 2,063,148 price records.

Our data is a Japanese counterpart to that of previous studies after Bils and Klenow (2004). Whereas these previous studies use outlet-level price data, we use the average of prices quoted at each outlet. Since we cannot access price data of each outlet where the price report has been carried out, our data set provides the best available information in Japan for measuring the degree of price stickiness. Saita et al. (2006) also uses the source of data. As they have pointed out, the frequency of price change may have upper bias when we use average prices. Because we count the number of price changes even if some of the outlet-level prices do not change.

We assume that price change does not occur more than twice because our data is monthly data, so that we can observe the price of each category only once a month. This is the limitation of our analysis as well as all of the previous studies using monthly data.

Table 1. Descriptive statistics

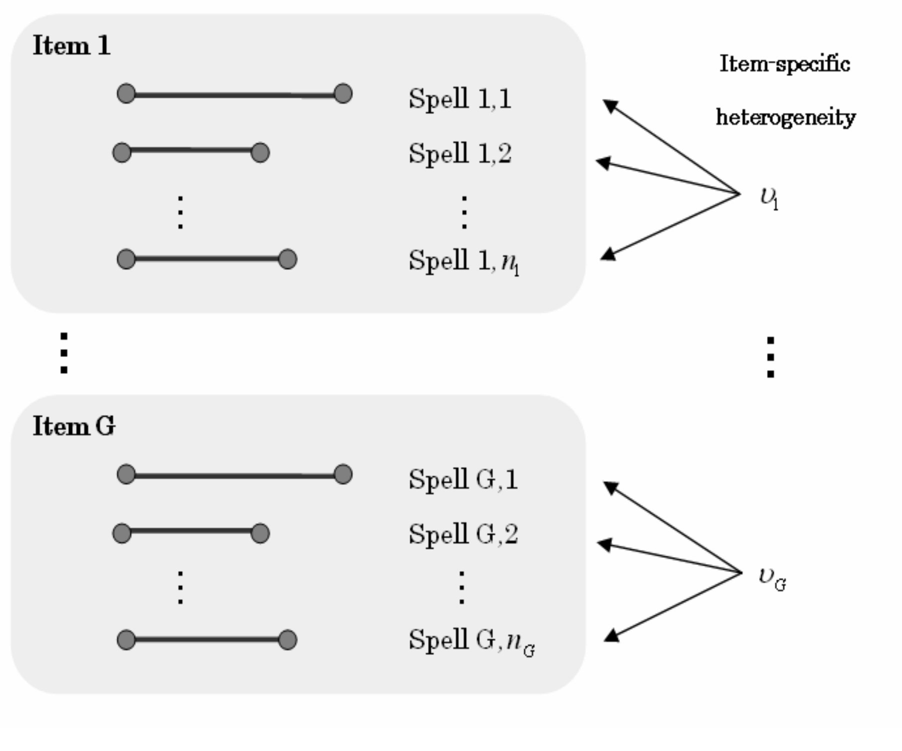
Category	total	Per price spell			
		mean	min	median	max
Number of items	498				
Number of price spells	677169				
(First) entry time		0	0	0	0
(Final) exit time		2.721	1	1	71
Time at risk	1798327	2.656	1	1	71
Number of uncensored spells	656581	0.970	0	1	1

Source.—Retail price data used for the calculation of the Japanese CPI (2000-2005).

Our data set is described in Table 1. In our retail price data, several price spells are observed per item and thus we call them multiple-spell data.

When we compute directly the duration of price spells, we need some trimming of the original dataset. First, we discard all left-censored spells. The duration of price spells quoted before the beginning of our observation period cannot be calculated because we do not know the starting time of price spells. This is called the problem of left-censoring (Amemiya, 1984). Since our data is multiple-spell data and the observation period is long enough, this exclusion does not lead to a severe problem. Second, we remove price spells which end with an item substitution. This is also negligible because the number of these spells is relatively small compared to the total number of price spells.

In order to analyze item-specific effects, we must assume the heterogeneities are not specific for price spell, but instead are shared within each items. Therefore, the multiple-spell data lead to the following hierarchical structure.



More precisely, let υ_i $(i = 1, \cdots, G)$ be independently and identically distributed random variables with a common distribution. And we assume the hazard function for the i th subject in the j th group given the j th heterogeneity is

$$\lambda_{ij}(t) = \upsilon_i \cdot \lambda(t \mid X_{ij}). \tag{3}$$

This implies that the cumulative hazard function for the same subject conditional on the j th heterogeneity is

$$\Lambda_{ij}(t) \equiv \int_0^t \lambda_{ij}(s)ds = \upsilon_i \int_o^t \lambda(s \mid X_{ij})ds = \upsilon_i \cdot \Lambda(t \mid X_{ij}). \tag{4}$$

Using the identity $\Lambda(t) = -\ln\{S(t)\}$, we obtain the conditional survivor function

$$S_{ij}(t) = \left[S(t \mid X_{ij})\right]^{\upsilon_i}. \tag{5}$$

For the i th subject in the j th group, the shared frailty model treats the hazard as equation (3). We assume that the shared frailties are i.i.d. sample from a Gamma distribution. Since the scale parameter of the distribution is unidentifiable, the mean and variance is normalized to set $E[\upsilon] = 1$ and $V[\upsilon] = \delta$, respectively[5]. The density becomes therefore

$$g(\upsilon) = \frac{\upsilon^{(1/\delta - 1)} \exp(-\upsilon/\delta)}{\Gamma(1/\delta)\delta^{1/\delta}}, \quad \upsilon > 0. \tag{6}$$

The joint survivor function for the i th group is given by

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[5] The gamma distribution $\Gamma(k, \delta)$ has $E[\upsilon] = k\delta$, $V[\upsilon] = k\delta^2$. Setting $k = 1/\delta$, we obtain the normalized parameter values.

$$
\begin{aligned}
S(t_{i1},\cdots,t_{in_i}) &= \Pr[T_{i1} > t_{i1},\cdots,T_{in_i} > t_{in_i}] \\
&= LP[\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})] \qquad (7) \\
&= [1+\delta\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})]^{-1/\delta}
\end{aligned}
$$

Here, $LP(x) = E_{\upsilon}[\exp(-\upsilon x)]$ is the Laplace transform of the frailty υ. See Appendix A for the derivation in detail. The log-likelihood contribution of the i th group is

$$
L_i(\delta,\lambda,\beta) = \left[\prod_{j=1}^{n_i}\lambda(t_{ij} \mid X_{ij})^{d_{ij}}\right]\frac{\Gamma(1/\delta + D_i)}{\Gamma(1/\delta)}\delta^{D_i}\left[1+\delta\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})\right]^{-(1/\delta+D_i)}, \qquad (8)
$$

where d_{ij} is the indicator of the price change and $D_i = \sum_{j=1}^{n_i} d_{ij}$ is the number of price changes in the i th group. We discuss in detail in Appendix B. We then obtain the full log-likelihood

$$
l(\delta,\lambda,\beta) = \log\prod_{i=1}^{G} L_i(\delta,\lambda,\beta). \qquad (9)
$$

We specify that the baseline hazard function is constant over time, that is, the baseline hazard function and the cumulative hazard function become $\lambda(t \mid X_{ij}) = \lambda$ and $\Lambda(t \mid X_{ij}) = \lambda t$, respectively. Referring to equation (3), even though the baseline hazard is constant and the same across the items, the individual hazard function $\lambda_{ij}(t)$ can be different to each other due to frailty υ_i. This exponential model with Gamma shared frailty is the natural extension of Calvo model, which allows the price stickiness to vary across the items.

4. The Estimation Results

Since we specify the baseline hazard function, the maximum likelihood estimators are obtained by maximizing equation (6) using Newton-Raphson method[6].

Figure 1 shows the predicted hazard from the parametric shared frailty model and nonparametric hazard function, i.e., the Kaplan-Meier product limit estimators of hazard rate. The shared frailty model reproduces the decreasing hazard function and describes the shape of the nonparametric hazard function fairly well except for the short-run prediction. If we ignore unobserved heterogeneity, the hazard rate is constant at 0.365 as shown in Table 2. This means that Calvo model overestimates the hazard rate for a long term spell.

Table 2. The Comparison of Calvo Model and Shared Frailty Model

	Hazard rate of price change	Duration of price spells			
		Mean	25%	Median	75%
Calvo (1983) model	0.37	2.74	3.60	1.90	0.72
Shared frailty model	-	-	9.36	2.87	0.72

Source.—Retail price data used for calculation of the Japanese CPI (2000-2005).
Note. — Hazard rate of price change is unweighted predicted hazard function. Price spell durations are reported in months.

One reason is that, in Calvo model, hazard rate of price changes is common to all items, because it assumes that economy consists of homogeneous firms: they adjust their price randomly but they share a common probability of price change. But this assumption of homogeneity cannot be supported empirically. The estimated frailty variance $\hat{\delta}$ is 1.149. Examining the likelihood-ratio test of $H_0 : \delta = 0$, the null hypothesis is soundly rejected at the 1% level of significance. Therefore, we conclude that the prediction errors are caused by item-specific heterogeneity.

6 If we fit a Cox model with shared frailty in which the baseline hazard function is not specified, the estimates are obtained by using an EM algorithm. See Klein and Moeschberger (1997) and Yu (2006) for further discussion. Nakamura and Steinsson (2006b) use Cox proportional hazard model with covariates (seasonal dummies) and Gamma frailty and analyze the baseline hazard function by sector. I tried to estimate Cox proportional hazard in full sample, but the speed of convergence was so slow that I could not obtain the likelihood estimator for the Cox model.

Figure 2 shows that survival probabilities of Calvo model are significantly smaller than those of the shared frailty model. The median duration from the shared frailty model is slightly longer than the one from Calvo model. The predicted median duration is 2.87 months, which is about 1.5 times longer than the median duration implied by the price-change frequency of single-sector model. But the significant difference lies in the predictions at longer duration. The smaller probabilities in Calvo model arise because it ignores the effect of sticky items which gradually dominates as time elapses.

The implication of our findings relates to a matter of great importance for monetary economics, since the dynamics of monetary economies is depend on to some extent how to deal with the sectors with lower frequency of price change.

Recent papers analyzing monetary shock by using dynamic stochastic general equilibrium model show that the degree of monetary shock implied by a multi-sector model is larger and more persistent than that implied by a single-sector model calibrated to the mean frequency of price change (Carvalho, 2006; Nakamura and Steinsson, 2006a). Carvalho (2006) introduces heterogeneity into Calvo (1983)'s model and concludes that in order to better approximate a single-sector model requires much lower frequency of price changes than that of multi-sector model. Our main finding can be restated as follows: the existence of heterogeneity implies the speed of price adjustment slows down as time elapses. In order to approximate the survivor function of the shared frailty model, it is necessary to use a lower value for the hazard rate in the Calvo model. This conclusion is consistent with that of Carvalho (2006)'s.

5. Conclusion

In this paper, we analyze the heterogeneity in price-setting behavior using the framework of duration analysis. We introduce the item-specific heterogeneity into the single-sector model and show that the shared frailty model reproduces the decreasing hazard function and describes the shape of the nonparametric hazard function fairly well.

In the Calvo (1983) model, the hazard rate of price changes is common to all items, because it assumes that economy consists of homogeneous firms. We present that this assumption of homogeneity cannot be supported empirically. We examine the likelihood-ratio test of the null hypothesis that the frailty variance is equal to zero. The hypothesis is soundly rejected at the 1% level of significance, which suggests that the CPI basket consists of highly heterogeneous components. Therefore, we conclude that the prediction errors are caused by item-specific heterogeneity.

We find that in the presence of item-specific heterogeneity, the probability that prices remain unchanged is higher than that of the single-sector model. This is because the Calvo model ignores the effect of sticky items which gradually dominates as time elapses. We document that in order to approximate the survivor function of the shared frailty model, it is necessary to use a lower value for the hazard rate in the Calvo model.

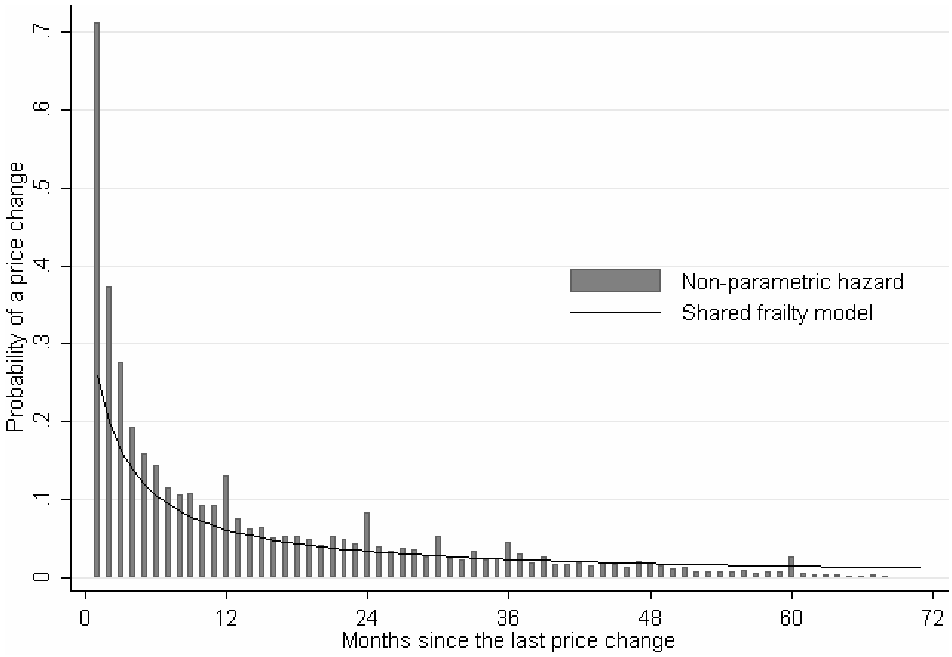

Figure 1: Population hazard from the exponential model with Gamma shared frailty vs. Kaplan-Meier estimator: Retail price data in Japan from 2000–2005: Unweighted sample (498 items).

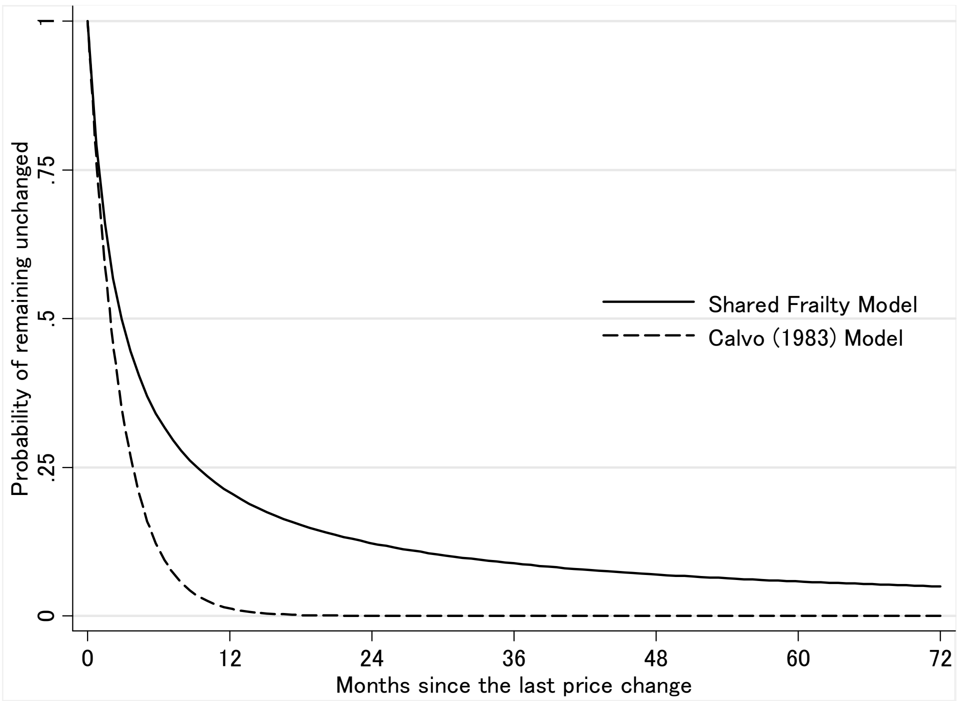

Figure 2: Survivor function from the shared frailty model and the Calvo (1983) model. Same data as Figure 1.

Appendix A. The derivation of the joint survivor function for the i th item.

From equation (7), we obtain

$$\begin{aligned} S(t_{i1},\cdots,t_{in_i}) &= E_\upsilon\{\exp[-\upsilon\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})]\} \\ &= \int_0^\infty \exp[-\upsilon\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})]\frac{\upsilon^{(1/\delta-1)}\exp(-\upsilon/\delta)}{\Gamma(1/\delta)\delta^{1/\delta}}d\upsilon \qquad \text{(A.1)} \\ &= \frac{1}{\Gamma(1/\delta)\delta^{1/\delta}}\int_0^\infty \exp\{-\upsilon[\frac{1}{\delta}+\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})]\}\upsilon^{(1/\delta-1)}d\upsilon. \end{aligned}$$

Letting $\frac{1}{\delta}+\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij}) = M$, we get

$$S(t_{i1},\cdots,t_{in_i}) = \frac{1}{\Gamma(1/\delta)\delta^{1/\delta}}\int_0^\infty \exp(-M\upsilon)\frac{\{M\upsilon\}^{(1/\delta-1)}}{M^{(1/\delta-1)}}d\upsilon. \qquad \text{(A.2)}$$

Define $y = M\upsilon$, so that $d\upsilon = M^{-1}dy$ and

$$\begin{aligned} S(t_{i1},\cdots,t_{in_i}) &= \frac{1}{\Gamma(1/\delta)\delta^{1/\delta}}\int_0^\infty \exp(-y)\cdot y^{1/\delta-1}\cdot M^{-1/\delta+1}\cdot M^{-1}dy \\ &= [\delta M]^{-1/\delta} \qquad \text{(By definition, } \Gamma(1/\delta) = \int_0^\infty \exp(-y)\cdot y^{1/\delta-1}dy) \\ &= [1+\delta\sum_{j=1}^{n_i}\Lambda(t_{ij} \mid X_{ij})]^{-1/\delta}. \end{aligned}$$

(A.3)

Appendix B. The likelihood function for the i th item

Given the i th heterogeneity υ_i, the contribution to the likelihood for the j th price spell in i th item is

$$\begin{aligned} L_{ij}(\upsilon_i) &= S_{ij}(t_{ij})\cdot\lambda_{ij}(t_{ij})^{d_{ij}} \\ &= S(t_{ij} \mid X_{ij})^{\upsilon_i}\cdot\left[\upsilon_i\lambda(t_{ij} \mid X_{ij})\right]^{d_{ij}} \qquad \text{(by equations (3) and (5))} \end{aligned} \qquad \text{(B.1)}$$

Using the expression of cumulative hazard function, equation (B.1) can be written as

$$L_{ij}(\upsilon_i) = \exp\left[-\sum_{j=1}^{n_i} \Lambda(t_{ij} \mid X_{ij}) \cdot \upsilon_i\right] \cdot \left[\upsilon_i \lambda(t_{ij} \mid X_{ij})\right]^{d_{ij}} \quad .$$

Consequently, the conditional likelihood function for the i th item is

$$\begin{aligned} L_i(\upsilon_i) &= \prod_{j=1}^{n_i} S(t_{ij} \mid X_{ij})^{\upsilon_i} \cdot \left[\upsilon_i \lambda(t_{ij} \mid X_{ij})\right]^{d_{ij}} \\ &= \upsilon_i^{D_i} \cdot \prod_{j=1}^{n_i} \exp\left[-\sum_{j=1}^{n_i} \Lambda(t_{ij} \mid X_{ij}) \cdot \upsilon_i\right] \lambda(t_{ij} \mid X_{ij})^{d_{ij}}, \end{aligned} \tag{B.2}$$

where $D_i = \sum_{j=1}^{n_i} d_{ij}$. Integrating out υ_i, we obtain the unconditional likelihood

$$L_i = \int_0^\infty L_i(\upsilon_i) g(\upsilon_i) d\upsilon_i, \tag{B.3}$$

where $g(\upsilon_i)$ is the density function given in equation (6). Therefore we have

$$\begin{aligned} L_i &= \int_0^\infty \upsilon_i^{D_i} \prod_{j=1}^{n_i} \exp\left[-\sum_{j=1}^{n_i} \Lambda(t_{ij} \mid X_{ij}) \cdot \upsilon_i\right] \cdot \lambda(t_{ij} \mid X_{ij})^{d_{ij}} \cdot \frac{\upsilon_i^{(1/\delta-1)} \exp(-\upsilon_i/\delta)}{\Gamma(1/\delta)\delta^{1/\delta}} d\upsilon_i \\ &= \left[\prod_{j=1}^{n_i} \lambda(t_{ij} \mid X_{ij})^{d_{ij}}\right] \frac{1}{\Gamma(1/\delta)\delta^{1/\delta}} \int_0^\infty \upsilon_i^{(D_i+1/\delta-1)} \prod_{j=1}^{n_i} \exp\left[-\upsilon_i\left(\frac{1}{\delta} + \sum_{j=1}^{n_i} \Lambda(t_{ij} \mid X_{ij})\right)\right] d\upsilon_i. \end{aligned} \tag{B.4}$$

Following from the same manner as in Appendix A, the integration in the last equality becomes

$$\Gamma(1/\delta + D_i)\left[\frac{1}{\delta} + \sum_{j=1}^{n_i} \Lambda(t_{ij} \mid X_{ij})\right]^{-(D_i+1)} . \tag{B.5}$$

Substituting (B.5) into (B.4), we recover equation (8).

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