A Theory of Deflation*

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Abstract

This paper explores the link between fiscal–monetary policy interaction and deflation. The key feature of this paper is that the central bank's balance sheet is separated from the government's budget constraint. As a result, government bonds are held entirely by households, and money is supplied via 'helicopter drops'. Interestingly, helicopter money works as a subsidy to purchasing government bonds. Thus, the central bank's commitment to a low nominal interest rate implies that it must inject money to keep the price of government bonds from falling when debt increases. The injection of helicopter money also increases the demand for real money, raising the value of money.

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1 Introduction

Academic research in recent years has witnessed the revival of theoretical studies into monetary policy and a surprising renewal of interest in liquidity traps.[1] Figure 1 presents the rate of change in the GDP deflator in Japan. This clearly indicates that Japan has been experiencing a decade of *very* low inflation.[2] The study of deflation is no longer a theoretical curiosity: rather, it is a real economic issue. Although there is a large and growing body of literature on liquidity traps, relatively little attention has been paid to deflation.[3] The purpose of this paper is to present a theory of deflation.

The dominant analytical framework for studying deflation is arguably the model of liquidity traps (Benhabib *et al.,* 2002; Taylor, 2001; Woodford, 2001, 2003). If an economy is in a liquidity trap, then the nominal interest rate is zero, and the Fisher relation implies that the rate of inflation *plus* the real interest rate must be zero. Thus, in terms of this literature, the liquidity trap implies deflation. The conventional view has two important implications. One is that deflation is chosen by the central bank. The liquidity trap is a situation where the nominal interest rate remains stuck at zero. However, if the central bank *raises* the nominal interest rate, then the rate of inflation can easily be positive. In this sense, the central bank creates a deflationary trap by following a Taylor rule. The other implication of the theory is that any country that adopts a Taylor rule faces the grave risk of a liquidity trap. Questions remain. Has the Bank of Japan been itself trapped by following a Taylor rule? Will other countries that follow Taylor rules also experience deflation?

A recent defining characteristic in the Japanese economy is its level of outstanding public debt. Figure 2 shows the evolution of the amount of bonds per GDP in Japan. This suggests that the ratio was relatively stable in the 1980s, but reached approximately 0.76 by 2003. Of greater significance is the speed of change: in the 1990s, the annual growth rate of the bonds–GDP ratio

[1] A partial listing of this literature includes Benhabib *et al.* (2002), Buiter (2003a, 2003b), Krugman (1998) and Woodford (2003).

[2] For empirical studies on Japan's deflation, see Ahearne *et al.* (2002), Baba (2006), Baba *et al.* (2005), and Taylor (2001).

[3] A notable exception is Buiter (2003a).

was roughly 9%. Thus, the government is borrowing money at a rate considerably faster than the growth rate of income.

Motivated by this observation, in this paper I present a theory of *dis*inflation and deflation from a new perspective—namely, fiscal–monetary policy interactions. The basic structure is not new. I follow the tradition of research on fiscal–monetary policy interaction and adopt a simple overlapping generations model with multiple assets (Aiyagari and Gertler, 1985; Bhattacharya *et al.,* 1998; Sargent, 1999; Sargent and Wallace, 1981). Although it is tempting to study recessions associated with deflation, I focus on an environment where output is exogenous.

Two new features are worth emphasizing: (i) the real interest rate on government bonds is endogenous, and (ii) the central bank's balance sheet is separated from the government's budget constraint. Some existing models assume that the real interest rate on bonds is constant in equilibrium. However, a constant real interest rate immediately implies that the rate of inflation is *perfectly* controlled by nominal interest rate policies, and thereby implies that the central bank chooses deflation. This paper does *not* argue whether or not economic policies can affect the real interest rate. Rather, it explores a scenario in which the rate of inflation cannot be controlled by the central bank alone. In this sense, this paper is intended to contribute to the literature on the fiscal side of price stability (Aiyagari and Gertler, 1985; Bhattacharya *et al.,* 1998; Cochrane, 2005; King, 1990; King and Plosser, 1985; Leeper, 1991; Sargent, 1999; Sargent and Wallace, 1981; and Woodford, 2001).

The other new feature of this paper is the notion of central bank independence. Existing models of fiscal–monetary policy interactions assume a subordinate central bank. This does not well match the recent central banking, because it implies that any shortfall in government revenue creates the need for revenue from an inflation tax. Motivated by this observation, this paper proposes a model in which the central bank's balance sheet is separated from the government budget constraint.[4] It thus helps us understand fiscal–monetary policy interactions in the era of central bank independence. To completely separate the central bank's balance sheet from the

[4] Jeanne and Svensson (2006) take up the issue of the central bank's balance sheet in the conduct of monetary policy.

government's budget constraint, open market operations are ruled out. Thus, the central bank injects newly printed money into the economy through "helicopter drops" (Buiter, 2003b). One important implication is that government bonds are held entirely by households.

Interestingly, the analysis suggests that, even though the central bank's balance sheet is separated from the government's budget constraint, monetary policy continues to be constrained by the actions of the fiscal authority. Suppose the government issues new bonds. The central bank does not purchase these bonds, because open market operations are ruled out. If the nominal interest rate were free to adjust, then the nominal interest rate would rise to induce households to purchase these extra bonds. However, if the central bank commits to a certain level of the nominal interest rate, then it must inject money into the economy to prevent the nominal interest rate from rising (prevent the bond price from falling). In short, a central bank committing to a low nominal interest rate is forced to inject new money. As money is a free good (Correia and Teles, 1999), helicopter money creates an income effect, through which the demand for bonds expands. This will also increase the demand for real money at the same level of output, implying a decrease in the price level.

This paper is organized as follows. Section 2 presents a review of the existing theories of deflation. Section 3 presents a model and its steady state when a fiscal–monetary policy regime is sustainable. Section 4 considers a regime in which the fiscal authority follows an unsustainable debt policy, and Section 5 concludes.

2 Related Literature

2.1 Taylor Rules and Zero Lower Bound

Consider a simple dynamic economy in which (i) output is exogenous, (ii) the representative household's behavior is expressed by the standard Euler equation, and (iii) the central bank follows a Taylor-type feedback rule. Now focus on the steady state. A Taylor rule is then expressed by $I = A\Pi^{\alpha}$, where I is the gross nominal interest rate, Π is the inflation factor, A is a scale parameter, and $\alpha > 0$. The standard Euler equation implies that, in a steady state, the gross real interest rate R equals the inverse of the household's discount factor. Then the Fisher equation is $I = R\Pi$, where R is constant. Substitute this into $I = A\Pi^{\alpha}$ to obtain $\Pi = (R/A)^{1/(\alpha-1)}$. According to this model, the central bank should be able to perfectly control the long-run rate of inflation by appropriately choosing A and α.

However, it is not clear whether this steady state is stable. Suppose a decrease in inflation occurs in this model. The central bank reduces the nominal interest rate according to the Taylor rule. However, in a frictionless pure-exchange economy, a reduction in the nominal interest rate reduces the inflation rate further, because of the Fisher equation with a predetermined real interest rate. Thus, in this simple economy, the Taylor rule accelerates disinflation (or deflation). Similarly, it accelerates inflation once it begins. In this frictionless economy with predetermined output and real interest rate, Taylor rules are destabilizing.

An easy way to model deflation in such an environment is to add a zero lower bound (ZLB) on the nominal interest rate to the analysis. When the rate of inflation starts falling, sooner or later the nominal interest rate will hit the ZLB. In this case, the Fisher equation implies $\Pi = 1/R$, which equals the household's discount factor, and is therefore less than 1. In this simple economy, deflation occurs whenever the nominal interest rate hits the ZLB.

As a theory of deflation, the above argument has an important drawback. It is implicitly assumed that the central bank *accelerates* the process of disinflation by blindly following a Taylor rule, even though it knows that cutting the nominal interest rate reduces the rate of inflation further. Thus, the central bank creates a deflationary trap. In such a model, deflation can easily be prevented by raising the nominal interest rate. In this paper, I search for an alternative explanation for deflation.

2.2 Monetary Policy Arithmetic

One approach to modeling a constrained central bank is to focus on the *consolidated* government budget constraint given by

$$G_t + T_t + I_t B_{t-1} = B_t + M_t - M_{t-1} \tag{1}$$

for $t \geq 2$ and $G_1 + T_1 = M_1 + B_1$ for $t = 1$, where G_t is the government expenditure, T_t is the tax revenue, B_t is the government bonds issued in period t, and M_t is the quantity of base money. Money is supplied through the channel of open market operations by the central bank. The fiscal authority determines the total government liability, and the central bank determines its composition. Divide (1) by $p_t y_t$ to obtain

$$g_t - \tau_t = b_t - \frac{1}{\Pi_t n} I_t b_{t-1} + m_t - \frac{1}{\Pi_t n} m_{t-1}, \tag{2}$$

where $b_t = B_t/p_t y_t$, $\tau_t = T_t/p_t y_t$, $m_t = M_t/p_t y_t$, and $\Pi_t \equiv p_t/p_{t-1}$. I assume that g_t and τ_t are constant over time. Thus, $g - \tau$ is the permanent primary deficit that must be financed by money and bonds. Throughout, it is assumed that $g \in (0, 1)$ and $\tau \in (0, 1)$.

Consider the steady state. Then, (2) implies

$$g - \tau = \left(1 - \frac{I}{\Pi n}\right) b + \left(1 - \frac{1}{\Pi n}\right) m, \tag{3}$$

Suppose that the real interest rate is exogenously given. For example, consider the Euler equation, implying that the steady-state real interest rate equals the inverse of the household's discount factor. Another possible interpretation is that there is a storage technology with a fixed gross real return. In any case, (3) is solved for Π as $\Pi = n^{-1}[1 - (g - \tau)/m + (1 - R)b/m]^{-1}$. Thus, $\Pi < 1$ holds if and only if

$$\frac{g - \tau}{m} < \frac{n - 1}{n} + \left(1 - \frac{R}{n}\right) \frac{b}{m}.$$

That is, the budget arithmetic implies deflation if (i) the primary deficit is sufficiently small, and (ii) the real interest rate is sufficiently low.

Kudoh (2005) considered the case where the real interest rate is endogenous. Then, solve (3) for Π as $\Pi = (m + Ib)/[b + m - (g - \tau)]n$, which implies that $\Pi < 1$ holds if and only if

$$\frac{g - \tau}{b} < \frac{n - I}{n} + \frac{m}{b}\frac{n - 1}{n}.$$

Thus, the budget arithmetic implies deflation if (i) the deficit-debt ratio is small, and (ii) the nominal interest rate is low.

Irrespective of whether the real interest rate is exogenous or not, the problem with this approach is that the central bank is forced to create deflation when the fiscal authority does not need to rely on the inflation tax. In other words, *the central bank is forced to implement a tax-cut through deflation.* Has the Japanese economy then also been deflationary because the government has been running a primary surplus? Apparently not.

2.3 Buiter's Model of Hyperdeflation

Following Buiter (1987), I assume that the real interest rate is fixed and that the real debt outstanding is held constant over time. Then, (2) is rewritten as

$$m_t = \frac{1}{\Pi_t n} m_{t-1} + (g - \tau) + \left(\frac{R}{n} - 1\right) \bar{b}. \tag{4}$$

Assume that the real money demand function is given by $m_t = S(\Pi_{t+1})$, with $S' < 0$. Thus, real money demand increases as the real return on money increases. Assume S is invertible and express $\Pi_{t+1} = s(m_t)$ with $s' < 0$. Substitute this into (4) to obtain

$$m_t = \frac{m_{t-1}}{s(m_{t-1})\, n} + (g - \tau) + \left(\frac{R}{n} - 1\right) \bar{b} \equiv \Psi(m_{t-1}). \tag{5}$$

From (5), it is easy to verify that if $(g - \tau) + (R/n - 1)\, \bar{b} > 0$, then the deficit is *unsustainable*, and as shown in Figure 3, m_t increases over time without bound. If the nominal stock of money is fixed, then $m_t \to \infty$ implies $p_t \to 0$. This is the case of Buiter's (1987) *hyperdeflation*. It is important to point out that since such an equilibrium path will eventually violate the economy's resource constraints, the path cannot be part of a rational expectations equilibrium. Even so, the argument is quite compelling, especially if we accept the possibility that the real world differs slightly to the world of rational expectations.

2.4 Fiscal Theory of the Price Level

The fiscal theory of the price level (FTPL) has been popularized by Buiter (2002), Cochrane (2005), Leeper (1991) and Woodford (2001, 2003). Its basic premise can be described as follows. Rewrite (2) as

$$b_t = \frac{R}{n} b_{t-1} + g - \tau - cs, \tag{6}$$

where cs is the currency seigniorage, which, for simplicity, is assumed to be constant over time. It is assumed that $\tau + cs > g$. Suppose that the real interest rate is constant and $R > n$ holds. That is, dynamic efficiency is assumed. The steady state of the system is given by

$$b = \frac{\tau - g + cs}{R/n - 1} \equiv \bar{b} > 0. \tag{7}$$

Consider Figure 4. As is easily demonstrated, the dynamic system (6) is unstable—the deficit is unsustainable unless the economy is forever at the steady state. Note that this economy can generate Buiter's (1987) hyperdeflation as $b_t \to \infty$. Unlike Buiter (1987), in the FTPL, the households are assumed to correctly anticipate that equilibrium conditions will be violated in the distant future and reject all candidate equilibrium paths that lead to fiscal insolvency. Then, the only possible equilibrium is obtained at $b_t = B_t/p_t y_t \equiv \bar{b}$, at which the price level is determined if the nominal stock of government bonds B_t and the output y_t are given.

Consider a decrease in the government surplus. Ceteris paribus, this reduces the value of $\bar{b}$. Thus, for any given B_t and y_t, a decrease in the government surplus *raises* the price level through "revaluation effect" (Leeper and Yun, 2006). The key difference between FTPL and Buiter (1987) is that while FTPL rules out all equilibrium paths that imply fiscal insolvency, Buiter (1987) describes what will happen when the economy is actually in a situation in which the government will be insolvent in the distant future. Under the FTPL equilibrium selection device, one price level that is consistent with government solvency is chosen. Thus, a shortfall in government revenue implies inflation in the FTPL and hyperdeflation in Buiter (1987).

As demonstrated above, the price level is *negatively* related with the government surplus. Thus, in FTPL, deflation is explained by an increase in the government surplus. The logic is similar to the one explained in the budget arithmetic, where an increase in the government surplus reduces the need of inflation tax. In FTPL, an increase in the surplus implies a higher real debt in (7). This *reverses* the revaluation effect, thereby reducing the price level for a given stock of nominal debt.

As a theory of deflation, FTPL has two unappealing features. One is that an increase in money growth raises cs, and this is potentially deflationary. The other is that, as in the budget arithmetic, deflation occurs by way of tax cut. To the government, deflation and a tax cut are perfect substitutes in these theories because fiscal and monetary policies are linked as a result of a single *consolidated budget constraint*. Has the Japanese economy been deflationary because the government has been using deflation as a means of providing a tax cut? If this is true, then it should have been easy to get out of deflation.

3 The Model

3.1 Environment

Consider an economy consisting of an infinite sequence of two-period-lived overlapping generations, an initial old generation, and an infinitely lived government. Let $t = 1, 2, ...$ index time. At each date t, a new generation is born. The population is normalized to 1 in each period. Each young agent is endowed with y_t units of the consumption good, and the endowment grows at the gross rate of n: $y_{t+1} = ny_t$. It is worth emphasizing that the purpose of this paper is to explain deflation alone, not a recession associated with deflation.[5]

3.2 Government

Since Sargent and Wallace's (1981) contribution, a consolidated government budget constraint such as (1) has become the building block of monetary policy analysis. However, such budget constraints with fiscal deficits imply that the monetary authority must raise revenue by printing money to maintain the government's solvency. King (1990) has argued, "The issue basically boils down to whether or not it is reasonable to view Congress and the executive branch as 'dominant,' requiring the monetary authority to deliver a given amount of revenue from money creation." Further, King and Plosser (1985) found little empirical support for the link between deficits and money creation.

The implicit assumption of a weak or subordinate central bank appears inconsistent with the recent state of central banking found in many developed economies, where inflation rates are

[5] In fact, Atkeson and Kehoe (2004) found no empirical link between deflation and recession.

generally low. In order to capture a tough, or independent, central bank, this paper uses a model which *separates* the central bank's budget from the fiscal authority's budget constraint. The two budget constraints are separated when (i) the fiscal authority does not receive any seigniorage revenue from the central bank, and (ii) the central bank never purchases government bonds.

In such an economy, the fiscal authority's budget constraint is written in nominal terms as

$$G_t + I_t B_{t-1} = T_t + B_t \tag{8}$$

for $t \geq 2$ and $G_1 = T_1 + B_1$ for $t = 1$. In addition, $B_0 = 0$. It is clear from (8) that the fiscal authority must finance its consumption by tax and borrowing; no seigniorage revenue is available. I assume that the government simply consumes G_t and that it does not affect the utility of any generation or production process at any date. Divide (8) by $p_t y_t$ to obtain

$$g_t - \tau_t = b_t - \frac{R_t}{n} b_{t-1}. \tag{9}$$

Throughout, it is assumed that the government simply consumes $g_t = g$ for all t and that it does not affect the utility of any generation or production process at any date.

Since open market operations are ruled out, money in this economy is injected directly into the economy by means of "helicopter drops". The amount of monetary transfer from the central bank satisfies

$$H_t = M_t - M_{t-1} \tag{10}$$

for $t \geq 1$, where the initial old is endowed with $M_0 \geq 0$. Thus, the stock of money evolves according to $M_t = M_{t-1} + H_t$. Divide (10) by $p_t y_t$ to obtain

$$h_t = m_t - \frac{1}{\Pi_t n} m_{t-1}, \tag{11}$$

which equates the real transfer and the currency seigniorage. Throughout this analysis, this particular policy regime is referred to as the CBI regime, and one with a subordinate central bank expressed by (1) is referred to as the SCB regime.

3.3 Consumers

In order to focus on agents' portfolio choice, I assume that agents care only about consumption in their old age. Thus, young individuals save all their income. As a means of saving, agents may hold money and government bonds. To motivate the demand for money as a liquid asset, divide each period into two subperiods. Bonds, denoted by B_t, are assumed to yield the gross nominal return of $I_{t+1} \geq 1$ in the next period. However, bonds cannot be liquidated until the second subperiod. Money, the nominal interest rate of which is zero, is assumed to be the only liquid asset. Thus, the only distinction between money and bonds is that bonds must be held a little longer (Antonio and Martins, 1980).

Suppose that each individual wishes to consume in both subperiods. Let c_{1t} and c_{2t} denote the consumption of the final good in the first and second subperiods by an old agent born at date t (*i.e.*, generation t). The consumer's objective function is

$$\phi u\left(c_{1t}\right) + \left(1 - \phi\right) u\left(c_{2t}\right), \tag{12}$$

where ϕ captures the relative weight of utility between the two subperiods. Throughout, I use the following specification: $u\left(c\right) = [1 - \rho]^{-1} c^{1-\rho}$ with $\rho \neq 1$ and $\rho > 0$. Since the individual cannot liquidate bonds in the first subperiod, the agent faces a cash-in-advance constraint:[6]

$$p_{t+1} c_{1t} \leq M_t, \tag{13}$$

which is binding for $I_t > 1$. The individual's budget constraint for the young is

$$M_t + B_t = p_t y_t - T_t + H_t, \tag{14}$$

where H_t is the money injection from the central bank. It is possible to interpret (14) as a private bank's balance sheet. The left-hand side is the use of funds, and the right-hand side is the source of funds. According to this interpretation, the central bank injects money into the private bank. Similarly, the budget constraint for the old is

$$p_{t+1} c_{1t} + p_{t+1} c_{2t} = M_t + I_{t+1} B_t. \tag{15}$$

[6] For an alternative environment in which a cash-in-advance constraint is used in an overlapping generations model, see Crettez *et al.* (1999).

Thus, each young individual maximizes (12) subject to (13)–(15). Since (15) becomes $p_{t+1}c_{2t} = I_{t+1}B_t$ under the binding cash-in-advance constraint, it is easy to transform the consumer's problem into

$$\max_{M_t} \left\{ \phi \frac{\left[M_t/p_{t+1}\right]^{1-\rho}}{1-\rho} + (1-\phi) \frac{\left[\left(p_t y_t - T_t - M_t + H_t\right) I_{t+1}/p_{t+1}\right]^{1-\rho}}{1-\rho} \right\}. \tag{16}$$

Use the first-order condition to derive the money demand function as

$$M_t = \gamma\left(I_{t+1}\right)\left[p_t y_t - T_t + H_t\right], \tag{17}$$

$$\gamma\left(I_{t+1}\right) \equiv \left[1 + \left(\frac{1-\phi}{\phi}\right)^{1/\rho} I_{t+1}^{1/\rho - 1}\right]^{-1}. \tag{18}$$

It is easy to verify that $\gamma'(I) < 0$ holds for $\rho \in (0,1)$; $\lim_{I\to\infty} \gamma(I) = 0$ for $\rho \in (0,1)$; and $\lim_{I\to 1} \gamma(I) = [1 + ((1-\phi)/\phi)^{1/\rho}]^{-1}$. The value of ρ captures the strength of the income effect associated with a change in I. Throughout, I focus on the case in which $\rho \in (0,1)$ so that the income effect is relatively weak.[7]

Divide (17) by $p_t y_t$ to obtain

$$m_t = \gamma\left(I_{t+1}\right)\left[1 - \tau_t + h_t\right], \tag{19}$$

from which the demand for real bonds is given by $b_t = [1 - \gamma(I_{t+1})][1 - \tau_t + h_t]$. Thus, $m_t/b_t = \gamma(I_{t+1})[1 - \gamma(I_{t+1})]^{-1} \equiv \Gamma(I_{t+1})$.

3.4 Steady State with Endogenous Debt

A monetary equilibrium in the CBI regime with endogenous debt is a set of sequences for real allocations $\{m_t, b_t\}$ and relative prices $\{I_t, \Pi_t\}$ such that (a) each generation maximizes utility,

[7] There are several other environments that induce this money demand function. In Schreft and Smith (2002), for example, markets are spatially separated and communication across markets is limited. Thus, only money is universally accepted as a means of payment. "Relocation shock" similar to the liquidity preference shock of Diamond and Dybvig (1983) induces agents to hold a mix of money and interest-bearing assets. Financial intermediation arises to provide perfect risk-sharing through demand deposit contracts, and the deposit demand function is of the form (17). I do not adopt Schreft and Smith's (2002) environment in this paper because the banking sector does not play any role in the theory presented.

(b) the asset market clears, (c) the government's budget constraint and the central bank's balance sheet are satisfied, (d) fiscal policy specifies g_t and τ_t, and (e) monetary policy specifies I_t or Π_t.

Consider the steady state of the economy in which $g_t = g$, $\tau_t = \tau$, and $I_t = I$ for all t. Steady-state equilibrium under this policy is given by $m = \gamma(I)[1 - \tau + h]$, $b = [1 - \gamma(I)][1 - \tau + h]$, and $h = (1 - 1/\Pi n)m$. Solve these equations for b as

$$b = \frac{[1 - \gamma(I)](1 - \tau)}{1 - \left(1 - \frac{1}{\Pi n}\right)\gamma(I)}. \tag{20}$$

Solve the government budget constraint (9) and (20) for Π to obtain

$$\Pi = \frac{1 - \tau}{1 - g}\frac{I}{n} + \frac{g - \tau}{1 - g}\frac{\gamma(I)}{1 - \gamma(I)}\frac{1}{n}.$$

Thus, the following is obtained.

Proposition 1 *In the CBI regime, the economy is deflationary in the steady state if and only if*

$$\frac{g - \tau}{1 - \tau} < \frac{n - I}{n + \Gamma(I)}.$$

The condition for deflation is expressed in terms of the deficit, nominal interest rate and the output growth rate. An interesting special case is when $\tau = g$. In this case, the economy is deflationary if and only if $n > I$. This result is easily understood. Without primary deficits, the government budget constraint is written as $b_t = (R_t/n)b_{t-1}$. A nontrivial steady state exists if and only if $R_t = I/\Pi = n$, so $\Pi < 1$ obtains if and only if $n > I$. In this case, the price level decreases over time because output grows faster than the growth of base money, which is governed by $M_t = IM_{t-1}$.[8] Interestingly, the nominal interest rate equals the growth rate of base money—lower nominal interest rates imply slower money growth.

[8] In the absence of the primary deficit, the government budget constraint implies $B_t = IB_{t-1}$. From the demand functions for money and bonds, it is evident that the bond–money ratio is constant over time under a nominal interest rate target. Thus, money and bonds grow at the same rate.

4 Public Debt Management and Deflation

4.1 Preliminaries

The previous section presented the steady state of the model in which the stock of debt is determined as part of equilibrium. However, studying the condition for *steady-state deflation* is more than necessary, because it implies that the price level will eventually be zero. The purpose of this section is to find the condition for deflation that persists for a limited duration. To do so, I examine the nonstationary equilibria of the model. In particular, I consider a scenario in which the fiscal authority accumulates debt at a rate faster than the output growth rate, which necessarily results in fiscal reform.[9]

For $t \leq k$, the tax rate τ is constant and the *real* stock of debt evolves according to $b_t = \theta b_{t-1}$, where $\theta > 1$. In period $t = k$, a fiscal reform takes place, so that for $t \geq k+1$, the fiscal authority maintains the real debt at $b_t = b_k \equiv \bar{b}$, and the tax rate τ_t is endogenous. Thus, generations up to $k-1$ experience the regime where the government plays an unsustainable debt policy and generation $t \geq k$ will experience the regime in which $b_t = \bar{b}$.

4.2 Economy after Fiscal Reform

Suppose that the fiscal authority maintains $b_t = \bar{b}$ for $t = k, ..., \infty$. In this case, the fiscal authority's budget constraint under nominal interest rate pegging satisfies

$$g - \tau_t = \left(1 - \frac{R_t}{n}\right)\bar{b} = \left(1 - \frac{I}{\Pi_t n}\right)\bar{b}. \tag{21}$$

From the household's budget constraint (14), $m_t + \bar{b} = 1 - \tau_t + h_t$. Use the central bank's budget constraint to eliminate h_t and obtain

$$\tau_t = 1 - \bar{b} - \frac{1}{\Pi_t n} m_{t-1}. \tag{22}$$

Note that $m_t = \Gamma(I)\bar{b}$. Eliminate τ_t from (21) and (22), and arrange the terms to obtain

$$\Pi_t = \frac{I + \Gamma(I)}{(1-g)\, n}\bar{b}, \tag{23}$$

[9] The sustainability of a policy regime has been considered by Burnside *et al.* (2001), Drazen and Helpman (1990), Paal (2000), Sargent and Wallace (1981) and Smith (1998), to name just a few.

which implies that the inflation rate after the fiscal reform is higher as $\bar{b}$ increases. In other words, a delay in the reform results in higher inflation, as observed in Sargent and Wallace's (1981) unpleasant monetarist arithmetic. Substitute $m_t = \Gamma(I)\bar{b}$ and (23) into (22) to finally obtain

$$\tau_t = g - \bar{b} + \frac{1-g}{1+\Phi(I)}, \tag{24}$$

where $\Phi(I) \equiv \Gamma(I)/I = \gamma(I)[1-\gamma(I)]^{-1}I^{-1}$. The equilibrium after fiscal reform is characterized as a set of m_t, τ_t, Π_t that solve $m_t = \Gamma(I)\bar{b}$, (23), and (24), given $\bar{b}$. As is clear, all variables are constant over time, and this regime is sustainable.

It is important to note that the economy is necessarily characterized as being dynamically inefficient if the primary deficit is positive ($g > \tau$). This result is straightforward from (9). As no currency seigniorage is available, a permanent primary deficit must imply dynamic inefficiency if there is any steady-state equilibrium to be considered.

Since nominal interest rate targeting is considered, the rate of inflation is determined as part of the equilibrium. From the government budget constraint (9), it is easy to verify that

$$\Pi = \frac{I}{n}\left[1 - \frac{g-\tau}{\bar{b}}\right]^{-1}. \tag{25}$$

It is then easy to establish the following.

Proposition 2 *(a) $\partial\Pi/\partial\bar{b} < 0$. (b) $\partial\Pi/\partial g > 0$. (c) The post reform economy is deflationary if and only if*

$$\frac{g-\tau}{\bar{b}} < \frac{n-I}{n}. \tag{26}$$

Thus, for deflation, the deficit–debt ratio must be small and the nominal interest rate must be sufficiently lower than the output growth rate. In the liquidity trap scenario, what matters is whether the nominal interest rate hits the ZLB. Here, the ZLB plays no role, as long as the nominal interest rate is below the output growth rate. In addition, the government's indebtedness matters. Proposition 2 suggests that whereas the deficit is inflationary, the debt is disinflationary.

4.3 Transition

Consider the date of fiscal reform, $t = k$. Generation k is young in this period, and belongs to the regime following the reform. Thus, $b_k = \bar{b}$ is given, $m_t = \Gamma(I)\bar{b}$, and (24) suggests that τ_k is determined. Given m_k, b_k, and τ_k, h_k is determined by the household's budget constraint: $m_t + \bar{b} = 1 - \tau_t + h_t$. Turn to Generation $k - 1$, which is old in period k. The key question here is whether the shift of fiscal regime influences the behavior of agents of this generation. Interestingly, their behavior is no different from generations $t \leq k - 2$. The reason is because there is no *intergenerational trade* in this economy. In the traditional overlapping generations model of a Samuelson-type (1958), the young generation exchanges their consumption goods with the old generation for fiat money. Thus, the young's demand for money and the old's demand for goods must be balanced. In contrast, agents in this model acquire fiat money directly from the central bank when young, and sell it to the central bank when old. For a recent discussion of this issue, see Bhattacharya *et al.* (2005).

4.4 Economy before Fiscal Reform

Suppose that the real debt evolves according to $b_t = \theta b_{t-1}$ for $t = 2, .., k$. It is then easy to obtain the sequence $\{b_t\}$ for $t = 2, ..., k$ as $b_t = \theta^{t-1} b_1$ given $b_1 = g - \tau > 0$. The government's budget constraint (9) and $b_t = \theta b_{t-1}$ determine the sequence for R_t:

$$R_t = \left[1 - \frac{g - \tau}{b_t}\right] \theta n = \left[1 - \frac{g - \tau}{\theta^{t-1} b_1}\right] \theta n \tag{27}$$

for $t = 2, ..., k$. It is then easy to establish that the real interest rate increases over time. Interestingly, the sequence for the real interest rate is *independent of how monetary policy is conducted.* The key to the theory is that, even though the real interest rate is endogenous, it is beyond the control of the central bank (Friedman, 1968).

Remember that (27) determines the sequence for the real interest rate. The role of monetary policy in such an economy is to determine a combination of the nominal interest and inflation rates; the value of one variable is implied by the other. Suppose that the central bank targets nominal interest rate at $I_t = I$ for all $t \leq k$. The Fisher equation and (27) imply that

$$\Pi_t = \frac{I/\theta n}{1 - (g - \tau)/b_t}. \tag{28}$$

The path of the inflation rate is completely written down by specifying fiscal and monetary policies. From (28), $\Pi_t > 0$ if and only if $b_t > g - \tau$.

Proposition 3 *(a) A necessary condition for* $\Pi_t < 1$ *for some* t *under permanent budget deficit is* $\theta > I/n$*. (b)* $\Pi_t < 1$ *holds if*

$$b_t > \frac{\theta}{\theta - I/n}(g - \tau). \tag{29}$$

According to Proposition 3, the economy is deflationary if the debt–GDP ratio exceeds the threshold level, and the deflationary region expands as θ increases and as the primary deficit falls. Under the regime considered, the evolution of the real interest rate is pinned down by the supply of the government's bonds. In equilibrium, the demand and supply of government bonds must be balanced. An increase in the supply of bonds must accompany an increase in the demand of the same quantity. If the nominal interest rate were allowed to adjust, then an increase in the demand for bonds would imply an increase in the nominal interest rate. Suppose the central bank commits to a (low) level of the nominal interest rate. The central bank can implement its commitment to a low nominal interest rate by injecting money into the economy. Since money creation requires no real resource cost, money is a free good (Correia and Teles, 1999). Thus, money injection creates an income effect, and expands the demand for bonds without raising the nominal interest rate. As a by-product, there is an increase in the real demand for money, and this raises the value of fiat money.

To illustrate the condition (29), some numerical examples are presented in Tables 1 and 2. I let $n = 1.02$. I choose 0.01 and 0.05 for the deficit–GDP ratio. For the value of θ, I use the relevant Japanese data. The debt–GDP ratio was about 0.2289 in 1991 and 0.5032 in 2000. Thus, θ must satisfy $0.5032 = 0.2289\theta^9$, which implies that θ is approximately 1.09. Similarly, θ is about 1.00 for the 1980s, which is consistent with existence of a steady state. The value of θ in Tables 1 and 2 varies from 1.01 to 1.09. The nominal interest rate is chosen to be 2% and 0% for Tables

1 and 2, respectively. From Figure 2, it is easily verified that the actual debt–GDP ratio ranges roughly from 0.2 to 0.7, and the nominal interest rate is near zero. Table 2 suggests that the economy is deflationary when $\theta \geq 1.05$. Clearly, the Japanese economy in early 2000 satisfies this condition. Tables 1 and 2 suggest that, as the speed of real debt expansion increases, the condition for deflation is significantly relaxed.

Consider another numerical example to illustrate how each variable evolves over time, before and after the fiscal reform under nominal interest rate pegging. I let $I = 1.01$ and $\theta = 1.09$. I choose $n = 1.015$ to match the average real output growth rate in Japan in the 1990s. Similarly, I allow $g = 0.16$ and $\tau = 0.11$. With regard to the initial bonds–GDP ratio, I choose $b_1 = 0.23 > g - \tau$, to be consistent with the 1991 value. With regard to the preference parameters, I select $\phi = 0.42$ and $\rho = 0.3$ to match the average bonds–money ratio, which is about 3.

Figure 5 illustrates the time paths of the bonds–GDP ratio and the gross inflation rate for $k = 13$, where $b_{13} = 0.65$. As shown in Figures 5(a) and 5(b), the debt–GDP ratio increases over time while inflation declines. The fiscal reform is assumed to take place in $t = 14$, and the inflation rate becomes constant at $\Pi = 1.02$.

4.5 Discussion

The literature repeatedly emphasizes the positive association between fiscal deficits and inflation rates. Thus, it may seem counterintuitive to obtain disinflation and deflation when the public debt is growing. The key here is to distinguish the public debt from the deficit. Mathematically, the key result is derived by exploiting the government's budget constraint: $g - \tau = (1 - R/n)b$, which is written as a static form for the purpose of exposition. It is easy to see that *ceteris paribus*, an increase in b raises R whereas an increase in $g - \tau$ reduces it. Suppose the nominal interest rate is constant, so that the real interest rate and the inflation rate are negatively related. Then, it is evident that greater fiscal deficits are inflationary, but greater outstanding government bonds are disinflationary.

The real question is, what is happening to the economy? For $t \leq k$, the stock of real debt evolves exogenously according to $b_t = \theta b_{t-1}$. Remember that, in this economy, the central bank does not purchase government bonds. Thus, the exogenous evolution of the debt implies that the fiscal authority *targets the household's bonds holdings*. The household's portfolio choice suggests that the nominal interest rate must increase in order to induce households to hold more government bonds. If the nominal interest rate were free to adjust, then there would be no downward pressure on the inflation rate. The key assumption here is the central bank's commitment to a (possibly low) nominal interest rate, which occasionally occurs when the economy is in a recession. The central bank targets a nominal interest rate by injecting money into the economy. As open market operations are ruled out, money supply takes a form of transfer to households. Households use this extra income to purchase both government bonds and money, leaving the nominal interest rate unchanged. A result is that the fiscal authority succeeds in selling government bonds without raising the nominal interest rate. However, as a by-product, the demand for money also increases and thereby raising the value of money. This is the key insight of this theory.

Money injection considered in this paper is similar to the episode of "quantitative monetary easing policy" implemented in Japan in 2001–2006 (Baba, 2006; Baba *et al.*, 2005). Baba (2006) documents that, in 2004, the target level of the current account balances held by financial institutions at the Bank of Japan was more than five times the required reserves. This indicates that the Bank of Japan was injecting a massive base money into the economy through the banking sector, and I argue that part of this money was used to purchase government bonds. Figure 6 indicates that financial institutions have increased government bonds holdings since the late 90s.[10] This is consistent with the argument of this paper—a massive base money injected into the economy acted as a subsidy to purchasing government bonds. Deflation is an inevitable by-product of this policy.

4.6 Welfare Implications

In order to make a relatively fair comparison of welfare across generations, let $y_t = 1$ for all t so that $n = 1$. Then, from (12), the welfare of generation t under nominal interest rate pegging is defined by $W_t = \phi[m_t p_t/p_{t+1}]^{1-\rho}/(1-\rho) + (1-\phi)[Ib_t p_t/p_{t+1}]^{1-\rho}/(1-\rho)$. For $t \leq k$, $\tau_t = \tau$,

[10] The data is taken from Keizai Tokei Nenkan (Economic Yearbook) published by TOYO KEIZAI INC.

$b_t = \theta b_{t-1}$, (28) and $m_t/b_t = \Gamma(I)$ hold. After some algebra, the welfare of generation t is given by

$$W_t = \frac{\phi[\Phi(I)]^{1-\rho} + (1-\phi)}{1-\rho} \{[b_t - (g-\tau)]\,\theta\}^{1-\rho},$$

where $\Phi(I) \equiv \Gamma(I)/I = \gamma(I)[1-\gamma(I)]^{-1}I^{-1}$. Note that $W_t > 0$ if and only if $b_t > g - \tau$. This is satisfied because $b_1 > g - \tau$ is assumed.

Proposition 4 *(a)* $\partial W_t/\partial I < 0$. *(b)* $\partial W_t/\partial b_t > 0$.

Proof. Since $\partial W_t/\partial \Phi > 0$, it is enough to verify the sign of $\Phi'(I)$, which is computed as $\Phi'(I) = \{\gamma'(I)I - [1-\gamma(I)]\gamma(I)\}[I - I\gamma(I)]^{-2} < 0$. ■

Proposition 4(a) asserts that, in an economy before fiscal reform, a lower nominal interest rate increases welfare. This provides a rationale for the central bank's commitment to a low nominal interest rate. Proposition 4(b) establishes that welfare increases with b_t. Thus, for $t \leq k$, welfare increases over time. This is because consumption grows faster than income for these generations.

Now consider welfare following fiscal reform. Welfare is given by

$$W_t = \frac{\phi[\Gamma(I)]^{1-\rho} + (1-\phi)}{1-\rho} \left\{\frac{1-g}{I - \Gamma(I)}\right\}^{1-\rho}.$$

Thus, welfare after the fiscal reform is constant over time. Interestingly, welfare is independent of the debt outstanding $\bar{b}$.

These results are interesting because welfare increases over time with explosive real debt, and the level of debt outstanding at the date of fiscal reform has no influence on the welfare of any generation. There seems to be no welfare loss associated with the explosive debt policy, with the exception that the policy is unsustainable. However, I doubt the robustness of this welfare implication, because the model adopts a pure-exchange economy so the conventional crowding-out channel—where a higher real interest rate reduces investment and output—is absent from the analysis.

4.7 Alternative Monetary Policy Rules

Suppose that the central bank follows strict inflation targeting: $\Pi_t = \Pi$ for all $t \leq k$. Then, from (27),

$$I_t = \left(1 - \frac{g - \tau}{\theta^{t-1} b_1}\right) \theta n \Pi.$$

Thus, the nominal interest rate increases over time. Because the real interest rate increases over time, the central bank must choose an ever-declining inflation or an ever-increasing nominal interest rate.

Suppose that the central bank conducts monetary policy according to $I_t = \Pi_t^{\alpha}, \alpha > 0$. According to Leeper (1991), monetary policy is said to be "active" if $\alpha > 1$ and "passive" if $\alpha < 1$. Then, the Fisher equation implies $R_t = I_t / \Pi_t = \Pi_t^{\alpha-1}$. From (27),

$$\Pi_t = R_t^{\frac{1}{\alpha-1}} = \left[\left(1 - \frac{g - \tau}{b_t}\right) \theta n\right]^{\frac{1}{\alpha-1}}.$$

Since R_t grows over time, the inflation rate increases over time if and only if $\alpha > 1$. Thus, the inflation rate increases over time if monetary policy is active, and decreases over time if it is passive. A related result is found in the literature on liquidity traps, in which two steady-state equilibria exist, and the paths leading to the passive equilibrium are deflationary.

5 Conclusion

This paper has presented a model of deflation based on fiscal–monetary policy interaction. The key intuition obtained is that the central bank's commitment to a (low) nominal interest rate implies that the central bank must inject money into the economy when the fiscal authority tries to sell more government bonds. Under a subordinate central bank, this creates the inflationary pressure suggested by Friedman (1968). This paper has demonstrated that, if monetary policy is conducted via helicopter drops, the money injection increases the demand for bonds without changing the nominal interest rate. At the same time, it increases the demand for money, thereby raising the value of money.

An important limitation of the analysis is that it assumes output to be exogenous. Although the real interest rate increases over time along the divergent path, there is no crowding-out because investment is absent from the analysis. Thus, the analysis fails to capture the loss associated with the explosive debt policy. It is important to consider a model with productive capital to integrate recessions into the theory of deflation.

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Figure 1: Inflation rate in Japan

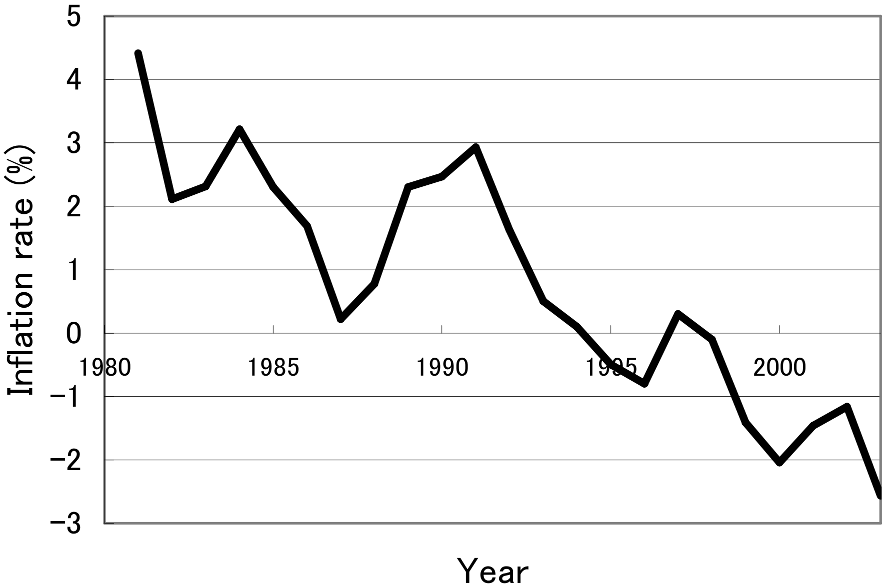


Figure 2: Debt to GDP ratio

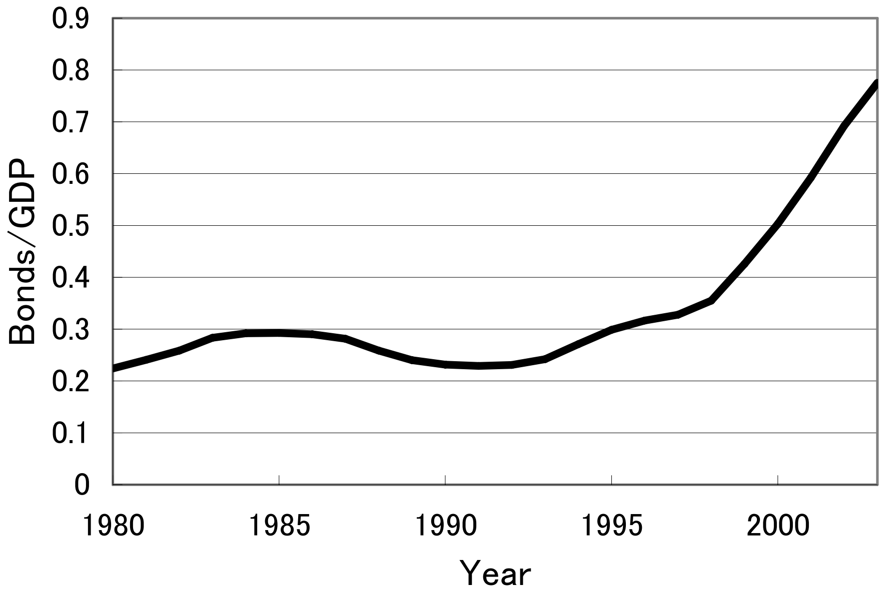


Figure 3: Buiter's (1987) example

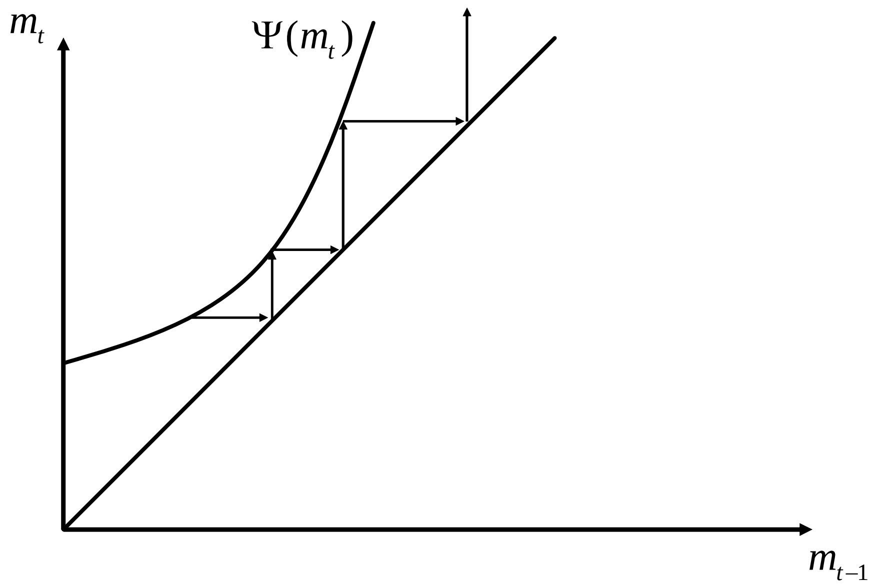


Figure 4: FTPL

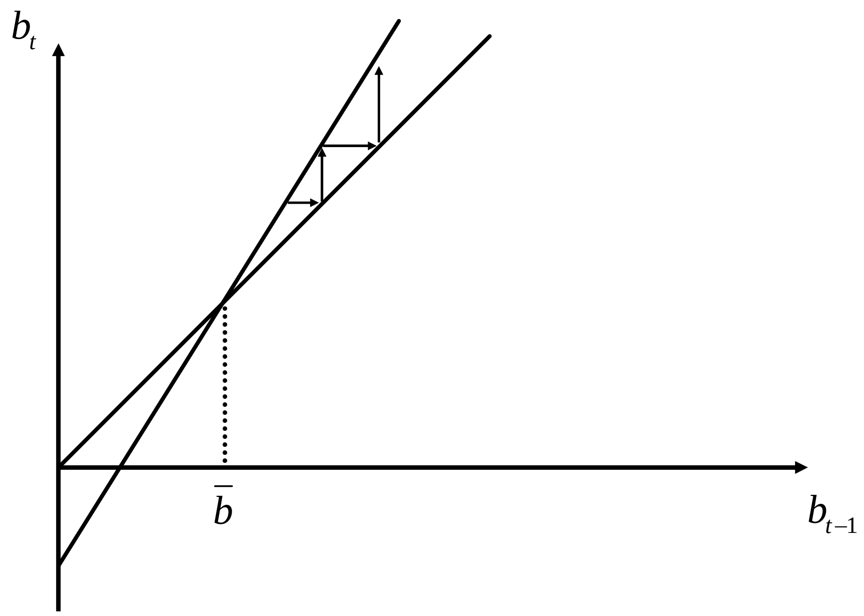
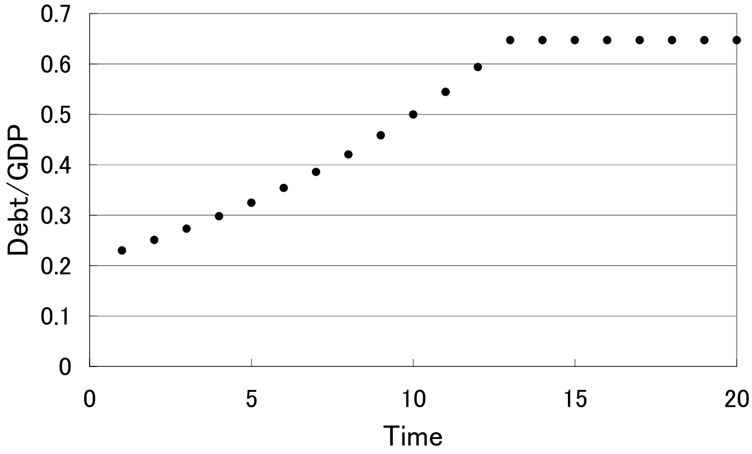


Figure 5: Numerical Example

(a)



(b)

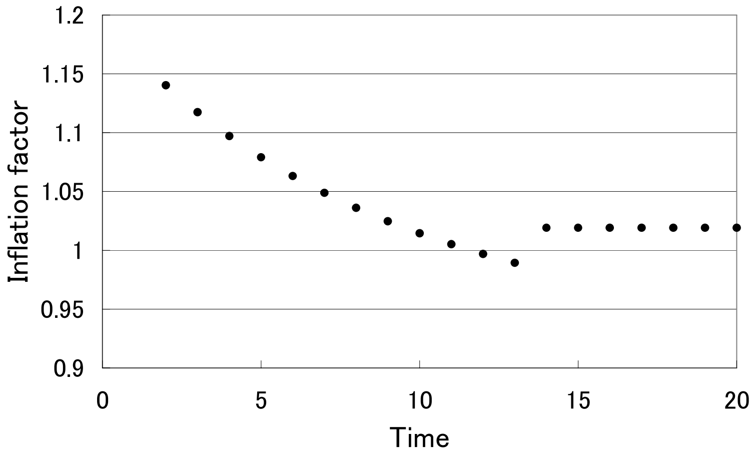


Figure 6: Government bonds held by domestic banks (in billion yen)

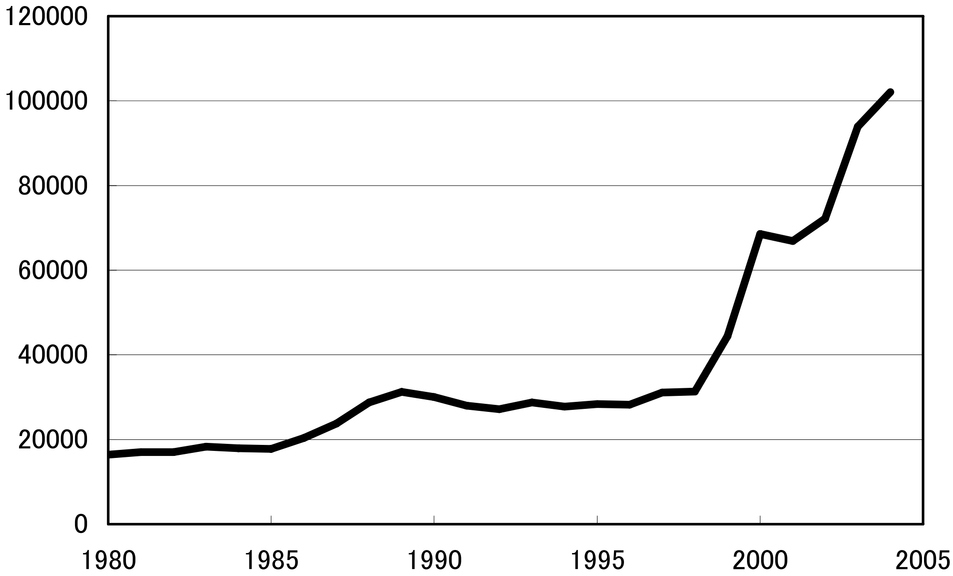


Table 1: Threshold levels of the debt-GDP ratio (n = 1.02, I = 1.02)

	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09
0.01	1.01	0.51	0.34	0.26	0.21	0.18	0.15	0.14	0.12
0.05	5.05	2.55	1.72	1.30	1.05	0.88	0.76	0.68	0.61

Table 2: Threshold levels of the debt-GDP ratio (n = 1.02, I = 1.00)

	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09
0.01	0.34	0.26	0.21	0.17	0.15	0.13	0.12	0.11	0.09
0.05	1.71	1.29	1.04	0.87	0.75	0.67	0.60	0.54	0.50