Stabilizing Expectations under Monetary and Fiscal Policy Coordination*

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Abstract

This paper analyzes the constraints imposed on monetary and fiscal policy design by expectations formation. Households and firms are uncertain about the statistical properties of aggregate variables, and, in particular, the policy regime characterized by a nominal interest rate rule and tax rule, and must learn about their dynamics using historical data. The presence of regime uncertainty substantially narrows, relative to a rational expectations analysis of the model, the menu of policies consistent with expectations stabilization. Moreover, there is greater need for policy coordination — the specific choice of monetary policy limits the set of fiscal policies consistent with macroeconomic stability. Resolving uncertainty about the prevailing policy regime improves stabilization policy, enlarging the menu of policy options consistent with stability. However, there are limits to the benefits of communicating precise details of the policy regime: the more heavily indebted the economy, the greater is the likelihood of expectations driven instability.

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1 Introduction

In a broad class of monetary models it is well understood that unique bounded rational expectations equilibria obtain under two distinct configurations of fiscal and monetary policy: i) monetary policy is active and fiscal policy is passive and ii) monetary policy is passive and fiscal policy active.[1] Alternative configurations of passive-passive and active-active give rise to either non-unique or unbounded dynamics. These fundamental insights, drawn together by Leeper (1991), underscore the importance of appropriately coordinating the choice of monetary and fiscal policy to achieve macroeconomic stability. Subsequent analyses under the rubric *fiscal theory of the price level* explore further the interaction of monetary and fiscal policy — see, inter alia, Cochrane (1998), Sims (1994) and Woodford (1996, 2001) for early contributions.

Underpinning such analyses is the assumption that agents hold rational expectations and correctly understand that any given policy regime will be adhered to with certainty into the indefinite future. Yet there are clearly historical examples which question the accuracy of this assumption. The existence of non-recurring regimes, such as the bond price support regime in the U.S. in the late 1940s discussed by Woodford (2001), and recent empirical evidence of on-going shifts in the configuration of monetary and fiscal policy in the post war era — see Davig and Leeper (2005a) — raises the possibility that agents may not be able to accurately assess the likelihood of any given policy regime. And given this possibility, it is natural to ask whether this constrains the set of monetary and fiscal policies consistent with expectations stabilization.

To this end, this paper explores the constraints imposed on policy design by expectations formation, and specifically uncertainty about the policy regime. Motivated by Friedman (1947, 1968), a model of output gap and inflation determination is developed — see, for instance, Clarida, Gali, and Gertler (1999) and Woodford (2003) — in which stabilization policy is conducted in the presence of two informational frictions. First, the central bank has imperfect information about the current state of the economy and must forecast the current inflation rate when setting nominal interest rates. Because of this observation lag,

[1] The terms active and passive are formally defined in the sequel.

the central bank responds to information about the state of the economy with a delay: policy is implementable in the sense of McCallum (1999) and Orphanides (2003). Fiscal policy is described by two objects. Taxes are determined by a one parameter family of rules, and adjusted in response to the outstanding level of real debt. The fiscal authority also chooses a desired steady state structural surplus-to-output ratio. Because only one period debt is issued, in contrast to the monetary authority, the fiscal authority faces no prediction problem: outstanding liabilities need not be forecasted to implement current tax policy.

Second, households and firms have an incomplete model of the macroeconomy, knowing only their own objectives, constraints and beliefs. Consequently, they do not have a model of how aggregate state variables are determined. They forecast exogenous variables relevant to their decision problems by extrapolating from historical patterns in observed data. In not knowing how nominal interest rates and taxes are determined such beliefs capture uncertainty about the prevailing policy regime. In a rational expectations analysis of the model, the policy regime is known, and agents' subjective beliefs coincide with the objective probability laws that describe the evolution of state variables. Here, instead, it is assumed that beliefs need not necessarily coincide with objective probabilities, as households and firms learn about the nature of policy from observed data. Expectations need not be consistent with the implemented monetary and fiscal policy rules.

An implication of this modeling assumption is that variations in taxes have traditional Keynesian expenditure effects. Because households imperfectly forecast future tax obligations, holdings of government debt are treated as net wealth, even if fiscal policy is Ricardian. Hence, Ricardian equivalence fails when agents make small expectational errors relative to rational expectations — compare the seminal analysis Barro (1974). The existence of wealth effects out of rational expectations equilibrium has consequences for the design of stabilization policy.

The central task is to discern whether uncertainty about the precise nature of the policy regime — that is the specification of monetary and fiscal policy — serves to restrict the menu of policy options consistent with stabilizing expectations. And, in particular, whether tighter coordination of monetary and fiscal policy is desirable in such an environment relative to a rational expectations equilibrium analysis of the model in which agents know the policy regime.

Two core results are distinguished by the absence or presence of knowledge of the policy regime. Across these cases, stability hinges on the relative magnitudes of two channels: i) traditional aggregate demand management through manipulation of real interest rates, and ii) wealth effects originating either from operation of the government's intertemporal budget constraint when fiscal policy is active or from departures from Ricardian equivalence. The magnitude of the latter depends on the average indebtedness of the economy. A rational expectations analysis of the model reveals determinacy of equilibrium is independent of this quantity.

In our benchmark analysis, agents have no knowledge of the monetary and fiscal policy rules. Stabilization policy is demonstrated to be more difficult than in a rational expectations analysis of the model: the menu of policies consistent with expectations stabilization is considerably narrowed. Indeed, for a large class of active monetary policies that satisfy the Taylor principle, there is no choice of fiscal policy consistent with expectations stabilization. In contrast, for passive monetary policies that do not satisfy the Taylor principle, there is always a choice of fiscal policy consistent with macroeconomic stability — though admissible choices depend on the precise choice of monetary policy, underscoring the need for coordination in policy design. Instability arises due to a failure of traditional aggregate demand management. Because agents are uncertain about the policy regime, their expectations need not be consistent with the implemented monetary and fiscal policies. This can be destabilizing as real interest rates are not accurately projected. Stability arises when uncertainty about real interest rates is small and countervailing, stabilizing, wealth effects are strong enough.

To further source instability, the analysis considers a model where agents have full knowledge of the prevailing policy regime. Hence households and firms know the adopted monetary and fiscal policy rules. This knowledge serves to simplify agents' forecasting problems, as a smaller set of state variables need to be forecasted to make current spending and price setting decisions. Eliminating uncertainty about the policy regime unambiguously improves stabilization policy: a larger menu of policies is consistent with macroeconomic stability. Under active monetary policy and passive fiscal policy, the improvement in macroeconomic stability stems from effective demand management under communication: in response to a shock to inflation expectations, agents correctly predict higher future real interest rates when monetary policy is active. Under passive monetary policy and active fiscal policy, despite the fact that the Taylor principle is not satisfied, there is less uncertainty about real interest rates. This gives greater force to the wealth effects generated by active fiscal policy.

In general, however, the full set of policies consistent with expectations stabilization under rational expectations remains unavailable, and depends on the average structural surplus-to-output ratio in the economy (equivalently, the debt-to-output ratio). The more heavily indebted an economy, the smaller the menu of policies consistent with stability. That the structural surplus-to-output ratio mitigates the efficacy of policy is because: i) the elasticity of demand with respect to changes in current and future real interest rates is reduced by precisely this quantity under non-rational expectations, as increases in real interest rates imply increases in the current value of holdings of the public debt, and ii) this quantity indexes the magnitude of departures from Ricardian equivalence. Because households incorrectly forecast future tax changes, variations in current taxes lead to wealth effects, and the magnitude of these wealth effects are proportional to the average debt-to-output ratio. These wealth effects are destabilizing. As a special case, in economies with a structural surplus-to-output ratio of zero, all policies consistent with determinacy of rational expectations equilibrium deliver stability of expectations under learning dynamics.

Related Literature: The analysis presented here owes much to Leeper (1991) and the subsequent literature on the fiscal theory of the price level. It also contributes to a growing literature on policy design under learning dynamics — see, inter alia, Howitt (1992), Bullard and Mitra (2002, 2006), Eusepi (2007), Evans and Honkapohja (2003, 2005, 2006), Preston (2004, 2005, 2006) — but is most directly related to Evans and Honkapohja (2007) and Eusepi and Preston (2007a). The former paper considers the interaction of monetary and fiscal policy in the context of Leeper's model under learning dynamics rather than rational expectations. The analysis here advances their findings by considering a model in which agents are optimizing conditional on their beliefs. This has the advantage that intertemporal budget constraints and transversality conditions are accounted for — a property pertinent to analyzing the fiscal theory of the price level, since this theory is explicitly grounded on shifting expectations of various macroeconomic objects appearing in households' intertemporal budget constraints.

The latter paper analyzes the role of communication in stabilizing expectations. The presence or absence of knowledge about the policy regime is adapted from the notions of full communication and no communication developed in that paper. The results here differ in non-trivial ways as a broader class of fiscal policy is considered. Rather than assuming a zero debt Ricardian fiscal policy, which is understood by households, the analysis here considers a class of locally Ricardian and non-Ricardian fiscal policies determined by the dual specification of a tax rule, which is unknown to agents, and choice of debt-to-output ratio. This engenders significantly richer model predictions regarding policy interactions and expectations stabilization, as agents must forecast future taxes to make current spending decisions and holdings of the public debt are treated as net wealth.

Our analysis also makes contact with various papers exploring economic environments that question the desirability of the Taylor principle as a foundation of monetary policy design. In particular, Benhabib, Schmitt-Grohe, and Uribe (2001) show that incorporating money in household and firm decisions leads to indeterminacy in the Ricardian regime even if the Taylor principle is satisfied. Building on Edge and Rudd (2002), Leith and von Thadden (2006) show in a Leeper (1991) style model with capital that conditions for determinacy of rational expectations equilibrium depend on the debt-to-output ratio as in results presented here. Bilbiie (2005) and Gali, Lopez-Salido, and Valles (2006) develop models of limited asset market participation, and adduce evidence that the Taylor principle may be neither sufficient nor necessary for determinacy of rational expectations equilibrium. Our paper builds on this literature by showing that uncertainty about the true statistical laws characterizing the evolution of prices can similarly compromise the effectiveness of standard policy advice — despite being a minimal departure from the standard New Keynesian framework.

The paper proceeds as follows. Section 2 lays out the microfoundations of a simple model of output gap and inflation determination under an arbitrary assumption on expectations formation. Section 3 specifies the adopted belief structure and learning dynamics. Section

4 revisits the analysis of Leeper (1991) in the context of our model, describing the model properties under the rational expectations assumption. Section 5 gives the core results under regime uncertainty. Section 6 discusses means to improving stabilization policy, with particular focus on resolving uncertainty about the prevailing policy regime. Section 7 compares the findings of this paper to those of Evans and Honkapohja (2007). Section 8 concludes.

2 A Simple Model

The following section details a model similar in spirit to Clarida, Gali, and Gertler (1999) and Woodford (2003). A continuum of households faces a canonical consumption allocation problem and decides how much to consume of available differentiated goods and how much labor to supply to firms for the production of such goods. A continuum of monopolistically competitive firms produces differentiated goods using labor as the only input and faces a price setting problem of the kind proposed by Calvo (1983) and implemented by Yun (1996). The major difference is the incorporation of non-rational beliefs. The analysis follows Marcet and Sargent (1989a) and Preston (2005b), solving for optimal decisions conditional on current beliefs.

2.1 Microfoundations

Households: The economy is populated by a continuum of households which seeks to maximize future expected discounted utility

$$\hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[\ln\left(C_T^i + g\right) - h_T^i \right] \tag{1}$$

where utility depends on a consumption index, C_T^i, the amount of labor supplied for the production of each good j, h_T^i, and the quantity of government expenditures $g > 0$.[2] The consumption index, C_t^i, is the Dixit-Stiglitz constant-elasticity-of-substitution aggregator of the economy's available goods and has an associated price index written, respectively, as

$$C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \quad \text{and} \quad P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \tag{2}$$

[2] The adopted functional form facilitates analytical results.

where $\theta > 1$ is the elasticity of substitution between any two goods and $c_t^i(j)$ and $p_t(j)$ denote household i's consumption and the price of good j. The discount factor is assumed to satisfy $0 < \beta < 1$.

$\hat{E}_t^i$ denotes the beliefs at time t held by each household i, which satisfy standard probability laws. Section 3 describes the precise form of these beliefs and the information set available to agents in forming expectations. Households and firms observe only their own objectives, constraints and realizations of aggregate variables that are exogenous to their decision problems and beyond their control. They have no knowledge of the beliefs, constraints and objectives of other agents in the economy: in consequence agents are heterogeneous in their information sets in the sense that even though their decision problems are identical, they do not know this to be true.

Asset markets are assumed to be incomplete. The only asset in non-zero net supply is government debt to be discussed below. The household's flow budget constraint is

$$B_{t+1}^i \leq R_t \left(B_t^i + W_t h_t^i + P_t \Pi_t - T_t - P_t C_t^i \right) \tag{3}$$

where B_t^i is household i's holdings of the public debt, with $B_0^i > 0$ given, R_t the gross nominal interest rate, W_t the nominal wage and T_t lump-sum taxes. Π_t denotes profits from holding shares in an equal part of each firm. Period nominal income is therefore determined as

$$P_t Y_t^i = W_t h_t^i + \int_0^1 \Pi_t(j)\, dj$$

for each household i. Finally, there is a No-Ponzi constraint

$$\lim_{T \to \infty} \hat{E}_t^i R_{t,T} B_T^i \geq 0$$

where $R_{t,T} = \prod_{s=t}^{T-1} R_s^{-1}$ for $T \geq 1$ and $R_{t,t} = 1$.[3]

A log-linear approximation to the first order conditions of the household problem provides the Euler equation

$$\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \left(\hat{\imath}_t - \hat{E}_t^i \hat{\pi}_{t+1} \right)$$

[3] In general, No Ponzi does not ensure satisfaction of the intertemporal budget constraint under incomplete markets. However, given the assumption of identical preferences and beliefs, a symmetric equilibrium will have the property that all households have non-negative wealth.

and intertemporal budget constraint

$$s_C \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \hat{C}_T^i = \frac{\bar{b}}{\bar{Y}} \hat{b}_t^i + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\hat{Y}_T^i - \frac{\bar{\tau}}{\bar{Y}} \hat{\tau}_T + \frac{\bar{b}}{\bar{Y}} \left(\beta \hat{\imath}_T - \hat{\pi}_T \right) \right] \tag{4}$$

where

$$\begin{aligned} \hat{Y}_t &\equiv \ln(Y_t/\bar{Y});\ \hat{C}_t \equiv \ln(C_t/\bar{C});\ \hat{\imath}_t \equiv \ln(R_t/\bar{R});\ \hat{\pi}_t = \ln\left(P_t/P_{t-1}\right); \\ \hat{\tau}_t &\equiv \ln(\tau_t/\bar{\tau});\ \tau_t = T_t/P_t;\ \hat{b}_t^i = \ln\left(\tilde{B}_t^i/\bar{B}\right) \text{ and } \tilde{B}_t^i = B_t^i/P_{t-1} \end{aligned}$$

and $\bar{z}$ denotes the steady state value of any variable z_t.

The appendix shows that solving the Euler equation recursively backwards, taking expectations at time t and substituting into the intertemporal budget constraint gives

$$\begin{aligned} \hat{C}_t^i &= s_C^{-1} \delta \left(\hat{b}_t^i - \hat{\pi}_t \right) + \\ & \quad s_C^{-1} \hat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta) \left(\hat{Y}_T - \delta \hat{s}_T \right) - (1-\delta) \beta \left(i_T - \pi_{T+1} \right) \right] \end{aligned}$$

where

$$\hat{s}_t = \bar{\tau} \hat{\tau}_t / \bar{s};\ \ s_C = \bar{C}/\bar{Y} \ \text{ and } \ \delta = \bar{s}/\bar{Y}$$

are the structural surplus (defined below), the steady state consumption-to-income ratio and the steady state structural surplus-to-income ratio. Optimal consumption decisions depend on current wealth and on the expected future path of after tax income and the real interest rate.[4] The optimal allocation rule is analogous to permanent income theory, with differences emerging from allowing variations in the real rate of interest, which can occur due to variations in either the nominal interest rate or inflation. Note also, that as households become more patient, current consumption demand is more sensitive to expectations about future macroeconomic conditions. Furthermore, the steady state structural surplus-to-income ratio, δ, affects consumption decisions in two ways. First, it regulates wealth effects on consumption spending that result from variations in the real value of government debt holdings. Second,

[4] Using the fact that total household income is the sum of dividend and wage income, combined with the first order conditions for labor supply and consumption, delivers a decision rule for consumption that depends only on forecasts of prices: that is, goods prices, nominal interest rates, wages and dividends. However, we make the simplifying assumption that households forecast total income, the sum of dividend payments and wages received.

it mitigates the elasticity of consumption spending with respect to changes in current and future expected real interest rates. Both these influences have consequences for stabilization policy.

Firms. There is a continuum of monopolistically competitive firms. Each differentiated consumption good is produced according to the linear production function $y_t(j) = Z_t h_t(j)$ where $Z_t > 0$ denotes an aggregate technology shock. Each firm faces a demand curve $Y_t(j) = (P_t(j)/P_t)^{-\theta_t} Y_t$ where Y_t denotes aggregate output, and solves a Calvo-style price setting problem. A price p is chosen to maximize the expected discounted value of profits

$$\hat{E}_t^j \sum_{T=t}^{\infty} Q_{t,T} \Pi_T^j (p)$$

where

$$\Pi_T^j (p) = p^{1-\theta} P_T^{\theta} Y_T - p^{-\theta} P_T^{\theta} Y_T W_T / Z_T$$

denotes period profits. Given the incomplete markets assumption it is assumed that firms value future profits according to the marginal rate of substitution evaluated at aggregate income $Q_{t,T} = \beta^{T-t} P_t Y_T / (P_T Y_t)$ for $T \geq t$.[5]

Denote the optimal price p_t^*. Since all firms changing prices in period t face identical decision problems, the aggregate price index evolves according to

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) p_t^{*1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Log-linearizing the first order condition for the optimal price we obtain

$$\hat{p}_t = \hat{E}_t^i \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[(1-\alpha\beta)\hat{\chi}_T + \alpha\beta\pi_{T+1}\right]$$

where $\hat{p}_t = \log(p_t^*/P_t)$ and $\hat{\chi}_t \equiv \ln(\chi_t/\bar{\chi})$ is average marginal costs defined below. Each firm's current price depends on the expected future path of real marginal costs and inflation. The higher the degree of nominal rigidity, the greater the weight on future inflation in determining current prices. The average real marginal cost function is $\chi_t = W_t/(P_t Z_t) = Y_t/Z_t$, where the second equality comes from the household's labor supply decision. Log-linearizing we obtain $\hat{\chi}_t = \hat{Y}_t - z_t$ so that current prices depend on expected future demand, inflation and technology.

[5] The precise details of this assumption are not important to the ensuing analysis so long as in the log linear approximation future profits are discounted at the rate β^{T-t}.

2.2 Monetary and Fiscal Authorities

Monetary Policy: The central bank is assumed to implement monetary policy according to a one parameter family of interest rate rules

$$R_t = \bar{R}\left(E_{t-1}^{cb}\pi_t\right)^{\phi_\pi}$$

where $E_{t-1}^{cb}\pi_t$ is a measure of current inflation and $\phi_\pi \geq 0$. The nominal interest rate rule satisfies the approximation

$$\hat{\imath}_t = \phi_\pi E_{t-1}^{cb}\hat{\pi}_t. \tag{5}$$

For simplicity, it is assumed the central bank has the same forecasting model for inflation as private agents. This is easily generalized.

This class of rule has had considerable popularity in the recent literature on monetary policy. It ensures determinacy of rational expectations equilibrium if the Taylor principle is satisfied under certain assumptions about fiscal policy and exhibits other robustness properties noted by Batini and Haldane (1999) and Levin, Wieland, and Williams (2003). This has led to advocacy of forecast-based instrument rules for the implementation of monetary policy. Indeed, such policy rules appear in a number of central bank forecasting models — see, for instance, the Bank of Canada. Furthermore, Clarida, Gali and Gertler (1998, 2000) adduce empirical evidence for such interest rate reaction functions.

The study of optimal policy is not pursued on two grounds. If appropriately chosen, simple rules of the postulated form deliver much of the welfare gains inherent in more complex optimal policy rules — see Schmitt-Grohe and Uribe (2005). Second, optimal policy in the context of learning dynamics is not trivial. Assumptions have to be made about the precise information a central bank has about the structure of the economy. While households and firms need only know their own objectives and constraints to make decisions, for a central bank to design optimal policy, it needs accurate information on all agents in the economy including the nature of beliefs. This is informationally demanding — and left to future work.

Fiscal Policy: The fiscal authority finances government purchases of g per period by issuing public debt and levying lump-sum taxes. Denoting B_t as the outstanding government debt at the beginning of any period t, and assuming for simplicity that the public debt is comprised entirely of one period riskless nominal Treasury bills, government liabilities evolve according to

$$B_{t+1} = (1 + i_t) [B_t + gP_t - T_t] .$$

For later purpose it is convenient to rewrite this constraint as

$$b_{t+1} = (1 + i_t) \left(b_t \pi_t^{-1} - s_t \right)$$

where $s_t = T_t/P_t - g$ denotes the primary surplus and $b_t = B_t/P_{t-1}$ a measure of the real value of the public debt. Observe that b_t is a predetermined variable since B_t is determined a period in advance.[6] The government's flow budget constraint satisfies the log-linear approximation

$$\hat{b}_{t+1} = \beta^{-1} \left(\hat{b}_t - \hat{\pi}_t - (1 - \beta) \hat{s}_t \right) + \hat{\imath}_t. \tag{6}$$

The model is closed with an assumption on the path of primary surpluses $\{s_t\}$.[7] Analogous to the monetary authority, it is assumed that the fiscal authority adjusts the primary surplus according to the one parameter family of rules

$$s_t = \bar{s} \left(\frac{b_t}{\bar{b}} \right)^{\phi_\tau}$$

where $\bar{s}, \bar{b} > 0$ are constants coinciding with the steady state level of the primary surplus and the public debt respectively. $\phi_\tau \geq 0$ is a policy parameter. The fiscal authority faces no uncertainty about outstanding liabilities as they are determined a period in advance. The tax rule satisfies the log-linear approximation

$$\hat{s}_t = \phi_\tau \hat{b}_t. \tag{7}$$

Similar remarks on the matter of optimal policy apply here.

2.3 Market clearing and aggregate dynamics

General equilibrium requires goods market clearing,

$$\int_0^1 C_t^i di + g = C_t + g = Y_t. \tag{8}$$

[6] See Eusepi and Preston (2007b) for a more general analysis with multiple debt maturities.

[7] This is without loss of generality. It would be straightforward to specify separate policies for the revenues and expenditures of the government accounts without altering the substantive implications of the model.

This relation satisfies the log-linear approximation

$$s_C \int_0^1 \hat{C}_t^i di = s_C \hat{C}_t = \hat{Y}_t.$$

It is useful to characterize the natural rate of output — the level of output that would prevail absent nominal rigidities under rational expectations. Under these assumptions, optimal price setting implies the log-linear approximation $\hat{Y}_t^n = a_t$. Hence movements in the natural rate of output are determined by variations in aggregate technology shocks. Using this definition, aggregate dynamics of the economy can be characterized in terms of deviations from the flexible price equilibrium. Finally, asset market clearing requires

$$\int_0^1 B_t^i di = B_t,$$

implying the sum of individual holdings of the public debt equals the supply of one period bonds.

Aggregating household and firm decisions provides

$$\begin{aligned} \hat{x}_t &= \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\delta\hat{s}_t + \\ & \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\left(\hat{x}_{T+1} - \delta\hat{s}_{T+1}\right) - (1-\delta)\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + r_T\right] \end{aligned} \quad (9)$$

and

$$\hat{\pi}_t = \kappa\hat{x}_t + \hat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\left[\kappa\alpha\beta\hat{x}_{T+1} + (1-\alpha)\beta\hat{\pi}_{T+1}\right] \quad (10)$$

where $\int_0^1 \hat{E}_t^i di = \hat{E}_t$ gives average expectations; $x_t = \hat{Y}_t - \hat{Y}_t^n$ denotes the log-deviation of output from its natural rate; $r_t^n = \hat{Y}_{t+1}^n - \hat{Y}_t^n$ the corresponding natural rate of interest — assumed to be an identically independently distributed process; and $\kappa = (1-\alpha)(1-\alpha\beta)\alpha^{-1} > 0$.

The average expectations operator does not satisfy the law of iterated expectations due to the assumption of completely imperfect common knowledge on the part of all households and firms. Because agents do not know the beliefs, objectives and constraints of other households and firms in the economy, they cannot infer aggregate probability laws. This is the property of the irreducibility of long horizon forecasts noted by Preston (2005).

To summarize, the model comprises the structural relations (5), (6), (7), (9) and (10).

3 Belief Formation

This section describes agents' learning behavior and the criterion to assess convergence of beliefs. Agents do not know the true structure of the economic model determining aggregate variables. To forecast state variables relevant to their decision problems, though beyond their control, agents make use of atheoretical regression models. The regression model is assumed to contain the set of variables that appears in the minimum state variable rational expectations solution to the model. Each period, as additional data become available, agents re-estimate the coefficients of their parametric model.

An immediate implication is that model dynamics are self-referential: the evolution of firm and household beliefs influence the realizations of observed macroeconomic variables. In turn, changes in observed data affect agents' belief formation. Learning induces time variation in the data generating process describing inflation, output, nominal interest rates, taxes and real debt. The central technical question concerns the conditions under which beliefs converge to those that would obtain in the model under rational expectations, in which case the data generating process characterizing the evolution of macroeconomic variables is time invariant. Convergence is assessed using the notion of expectational stability outlined in Evans and Honkapohja (2001).

These assumptions on the structure of beliefs have the advantage that agents learn about the current policy regime only by observing historical data. Indeed, in periods of significant change in the policy regime it seems hardly reasonable to suppose that households and firms are able to assign probabilities — to the various objects that they must forecast in order to make decisions — that necessarily coincide with the objective probabilities implied by the true economic model. And given that constraint, it is equally plausible that agents make use of historical data to form inferences about the future evolution of the economy. If there is a change in regime and, therefore, the underlying data generating process, agents only learn about it through observing new data. Such an approach to modeling belief formation obviates the requirement of specifying what beliefs agents hold about future possible policy regimes, as would be the case in a rational expectations equilibrium analysis. As has been highlighted in recent discussion of determinacy of rational expectations equilibrium in regime switching models, analysis of this kind is difficult — see Davig and Leeper (2005a, 2005b) and Farmer, Waggoner and Zha (2006a, 2006b).

3.1 Forecasting

This section outlines the beliefs of agents in our benchmark analysis. Each agent's estimated model at date t can be expressed as

$$X_t = \begin{bmatrix} x_t \\ \pi_t \\ b_{t+1} \\ i_t \\ s_t \end{bmatrix} = \omega_{0,t} + \omega_{1,t} X_{t-1} + \bar{e}_t \tag{11}$$

where ω_0 denotes the constant, ω_1 is defined as

$$\omega_1 = \begin{bmatrix} 0 & 0 & b_x^b & 0 & 0 \\ 0 & 0 & b_\pi^b & 0 & 0 \\ 0 & 0 & b_b^b & 0 & 0 \\ 0 & 0 & b_i^b & 0 & 0 \\ 0 & 0 & b_s^b & 0 & 0 \end{bmatrix}$$

and $\bar{e}_t$ represents an i.i.d. estimation error. The fact that only one period debt is issued is exploited in the belief structure — agents know that debt is predetermined. This assumption can be relaxed without consequence, though at the price of considerably more algebra. Agents are further assumed to know the coefficients on the lags of output, inflation, nominal interest rates and taxes, but estimate remaining parameters (with time subscripts being dropped for convenience) on real debt.[8]

In period t agents form their forecast about the future evolution of the macroeconomic variables given their current beliefs about reduced form dynamics. Expectations $T+1$ periods

[8] To the extent that these beliefs constrain policy, requiring agents to learn more about the underlying dynamics can only render the stabilization problem more difficult.

ahead are calculated as

$$\hat{E}_t X_{T+1} = \left(I_5 - \omega_{1,t-1}\right)^{-1} \left(I_5 - \omega_{1,t-1}^{T-t+1}\right) \omega_{0,t-1} + \omega_{1,t-1}^{T-t+1} X_t$$

for each $T \geq t$, where I_5 is a (5×5) identity matrix. To evaluate expectations in the optimal decision rules of households and firms, note that the discounted infinite-horizon forecasts are

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} X_{T+1} = \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\left(I_5 - \omega_{1,t-1}\right)^{-1} \left(I_5 - \omega_{1,t-1}^{T-t+1}\right) \omega_{0,t-1}\right]$$
$$+\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[\omega_{1,t-1}^{T-t+1} X_t\right].$$

This expression can be compactly written as

$$\hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} X_{T+1} = F_0\left(\omega_{0,t-1}, \omega_{1,t-1}\right) + F_1\left(\omega_{1,t-1}\right) X_t,$$

where

$$\begin{aligned} F_0\left(\omega_{0,t-1}, \omega_1\right) &= \left(I_5 - \omega_{1,t-1}\right)^{-1} \left[(1-\beta)^{-1} I_5 - \omega_{1,t-1}\left(I_5 - \beta\omega_{1,t-1}\right)^{-1}\right] \omega_{0,t-1} \\ F_1\left(\omega_1\right) &= \omega_{1,t-1}\left(I_5 - \beta\omega_{1,t-1}\right)^{-1} \end{aligned}$$

are, respectively, a (5×1) vector and (5×5) matrix.

3.2 Expectational Stability

Substituting for expectations in the equations for the output gap, inflation and the nominal interest rate, permits writing aggregate dynamics of the economy as

$$X_t = \Gamma_0\left(\omega_{0,t-1}, \omega_{1,t-1}\right) + \Gamma_1\left(\omega_{1,t-1}\right) X_{t-1} + \Gamma_2 r_t^n \tag{12}$$

with obvious notation. This expression captures the dependency of observed dynamics on agents' beliefs about the future evolution of the economy. Moreover, it implicitly defines the mapping between agents' beliefs and the actual coefficients describing observed dynamics as

$$T\left(\omega_{0,t-1},\ \omega_{1,t-1}\right) = \left(\Gamma_0\left(\omega_{0,t-1}, \omega_{1,t-1}\right),\ \Gamma_1\left(\omega_{1,t-1}\right)\right).$$

A rational expectations equilibrium is a fixed point of this mapping. For such rational expectations equilibria we are interested in asking under what conditions does an economy with learning dynamics converge to each equilibrium. Using stochastic approximation methods, Marcet and Sargent (1989b) and Evans and Honkapohja (2001) show that conditions for convergence are characterized by the local stability properties of the associated ordinary differential equation

$$\frac{d(\omega_0, \omega_1)}{d\tau} = T(\omega_0, \omega_1) - (\omega_0, \omega_1), \tag{13}$$

where τ denotes notional time. The rational expectations equilibrium is said to be expectationally stable, or E-Stable, when agents use recursive least squares if and only if this differential equation is locally stable in the neighborhood of the rational expectations equilibrium.[9]

4 Foundations: Leeper Revisited

In the standard account of monetary policy design, nominal interest rates are determined to actively stabilize inflation and output. Less emphasized, but no less important, is the accompanying assumption that fiscal policy is Ricardian in nature — taxes are assumed to adjust in such a way as to ensure intertemporal solvency of the government budget. Under these assumptions, a central recommendation is that monetary policy should satisfy the Taylor principle: nominal interest rates should be adjusted more than one for one with variations in inflation. As shown by Leeper (1991), however, other configurations of policy are consistent with determinacy of rational expectations equilibrium. They involve a more active role for fiscal policy in which it is non-Ricardian in nature and has monetary consequences. The following section describes the fiscal theory of the price level and studies the determinacy properties of our model under rational expectations. Ricardian and non-Ricardian fiscal policies are formally defined.

[9] Standard results for ordinary differential equations imply that a fixed point is locally asymptotically stable if all eigenvalues of the Jacobian matrix $D[T(\omega_0, \omega_1) - (\omega_0, \omega_1)]$ have negative real parts (where D denotes the differentiation operator and the Jacobian is understood to be evaluated at the relevant rational expectations equilibrium).

4.1 The Fiscal Theory of the Price Level

Household optimization implies

$$E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c\left(C_T^i + g\right)}{U_c\left(C_t^i + g\right)} C_T^i = \frac{B_t^i}{P_t} + E_t^i \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c\left(C_T^i + g\right)}{U_c\left(C_t^i + g\right)} \left[Y_T - \frac{T_T}{P_T}\right].$$

In a symmetric rational expectations equilibrium, goods and asset markets clearing imply $C_T^i = C_T^j$, $Y_T^i = Y_T^j = Y_T = C_T$ and $B_t^i = B_t^j = B_t$ for all $i \neq j$ and in all periods $T \geq t$. Substituting these conditions into the above relation yields

$$\frac{B_t}{P_t} = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c\left(Y_T\right)}{U_c\left(Y_t\right)} \left[\frac{T_T}{P_T} - g\right] = E_t \sum_{T=t}^{\infty} \beta^{T-t} \frac{U_c\left(Y_T\right)}{U_c\left(Y_t\right)} s_T.$$

Under the rational expectations assumption, and making use of the Euler equation, this relation satisfies the log-linear approximation

$$\begin{aligned} \hat{b}_t - \hat{\pi}_t &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\,\hat{s}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right] \\ &= E_t \sum_{T=t}^{\infty} \beta^{T-t} \left[(1-\beta)\,\hat{s}_T - \beta\left(\hat{Y}_T - \hat{Y}_{T+1}\right)\right]. \qquad (14) \end{aligned}$$

The real value of outstanding government liabilities is equal to the present discounted value of future primary surpluses adjusted for variation in real returns.

As emphasized by Woodford (2001) and Leeper and Yun (2005), this intertemporal solvency condition is imposed on the government by household optimization. To understand the fiscal theory of the price level consider (14). Suppose for the sake of simplicity that the path of primary surpluses $\{s_t\}$ is exogenously determined. Under the assumption of flexible price setting, which implies output is equal to the natural rate of output, the right hand side of the intertemporal solvency condition is exogenously determined. Because the model assumes the government to issue only one period public debt, which is a predetermined variable, this intertemporal solvency condition imposes a restriction on the path of equilibrium goods prices and therefore inflation. This is the heart of the fiscal theory of the price level.

As an example, consider a government choosing to increase expenditures by some constant amount each period (or equivalently a reduction in the level of taxes levied each period). This leads to a fall in the present discounted value of primary surpluses. Because outstanding public debt is predetermined, equilibrium is guaranteed by an increase in the price level. This is a wealth effect. Households expect to pay a smaller present discounted value of taxes over their lifetime, implying a rise in permanent income and, concomitantly, in expenditure in the current period.

4.2 Rational Expectations

The following characterizes the set of unique equilibria under the rational expectations assumption. The analysis is analogous to Leeper (1991), though in the context of the model of section 2. All proofs are collected in the appendix.

Proposition 1 *There exist unique bounded rational expectations equilibria of the indicated form if and only if the following conditions are satisfied: either*

1. *Monetary policy is active and fiscal policy is locally Ricardian such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > 1$$

with inflation dynamics determined as

$$\hat{\pi}_t = \phi_0 r_t^n; \quad or$$

2. *Monetary policy is passive and fiscal policy is locally non-Ricardian such that*

$$0 \leq \phi_\pi < 1 \text{ and } 0 \leq \phi_\tau < 1 \text{ or } \phi_\tau > \frac{1+\beta}{1-\beta}$$

with inflation dynamics determined as

$$\hat{\pi}_t = \phi_1 \hat{b}_t + \phi_2 r_t^n.$$

All coefficients are reported in the Appendix.

The descriptors locally Ricardian and non-Ricardian refer to the combined implications of the government's flow budget constraint and tax policy. When $1 < \phi_\tau < (1+\beta)/(1-\beta)$ the eigenvalue of the difference equation (6) is inside the unit circle, and, for all bounded sequences $\{\pi_t,\ i_t\}$, real debt converges to its steady state value. Because taxes are adjusted to ensure intertemporal solvency of the government accounts for all possible paths of the price level, this configuration of fiscal policy is termed locally Ricardian, where locally refers to the use of a log-linear approximation. In the language of Leeper this is passive fiscal policy. In contrast, if either $0 \leq \phi_\tau < 1$ or $\phi_\tau > (1+\beta)/(1-\beta)$, then the eigenvalue is outside the unit circle and real debt dynamics are inherently explosive. It is this property that requires a specific path of the price level to ensure solvency of the intertemporal accounts. Hence, locally non-Ricardian, or in the language of Leeper, active fiscal policy.

Whether fiscal policy is locally Ricardian or non-Ricardian has implications for macroeconomic dynamics. In the former case, inflation dynamics are independent of the public debt. In the latter case, the path of real debt has consequences for the determination of inflation dynamics. Moreover, and in further contrast to the case of a locally Ricardian fiscal policy, current inflation also depends on the previous period's inflation rate — a richer set of macroeconomic dynamics obtain. The conditions for determinacy of rational expectations equilibrium in each regime are referred to as the Leeper conditions.[10]

5 Regime Uncertainty and Expectations Stabilization

Having laid out preparatory foundations, the analysis turns to the consequences of regime uncertainty for stabilization policy. One final assumption is required to facilitate analytical results: the economy is assumed to have only a small degree of nominal rigidity. Formally, the conditions for expectational stability are studied in the neighborhood of the limit, $\alpha \to 0$. It is important to note that this is not equivalent to analyzing a flexible price economy. For an arbitrary degree of nominal friction, $0 < \alpha < 1$, analytical results are unavailable except in two special cases. For a numerical treatment with locally Ricardian fiscal policy, see Eusepi and Preston (2007b), which explores related issues and the consequences of the debt maturity structure for stabilization policy.

[10] Two other classes of equilibria are possible. One concerns the case of Ricardian fiscal policy combined with a passive monetary policy satisfying $0 < \phi_\pi < 1$. In this case, there is indeterminacy of rational expectations equilibrium for all parameter values. It is easily demonstrated that none of these equilibria is stable under the alternative non-rational expectations assumption being considered. The second concerns the case of non-Ricardian fiscal policy and monetary policy satisfying the Taylor principle. Under rational expectations it can be shown that there exist a class of unbounded equilibria that have explosive debt and inflation dynamics.

5.1 Constraints on Stabilization Policy

In the model described by section 2 under regime uncertainty, the following results obtain.

Proposition 2 *Stabilization policy ensures expectational stability if and only if*

1. *Monetary policy is active and fiscal policy is locally Ricardian such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > \frac{1}{1-\beta}; \text{ or}$$

2. *Monetary policy is passive and fiscal policy is locally non-Ricardian such that* $0 \leq \phi_\pi < 1$, *and either*

   (a)

$$0 \leq \phi_\tau < \min(\phi_\tau^*, 1) \text{ where } \phi_\tau^* = \frac{2}{[(1-\beta\phi_\pi)^{-1} + (1-\beta)]}; \text{ or}$$

   (b)

$$\phi_\tau > \frac{1+\beta}{1-\beta}.$$

This proposition demonstrates that regime uncertainty constrains the menu of policies consistent with expectations stabilization relative to the class of policies given by the Leeper conditions. If fiscal policy is locally Ricardian then monetary policy must be highly aggressive to prevent self-fulfilling expectations. For many monetary policies satisfying the Taylor principle there is no choice of fiscal policy that can guarantee stability. The restriction on the choice of monetary policy depends on the household's discount factor, β, since this parameter regulates the impact of revisions about future macroeconomic conditions on current spending and pricing decisions. The more patient are households the larger will be the impact of revisions to expectations on current macroeconomic conditions.

If fiscal policy is non-Ricardian there are greater incentives to coordinate monetary and fiscal policy relative to a rational expectations analysis of the model. Indeed, under rational expectations, conditional on monetary policy being passive, any choice of locally non-Ricardian fiscal policy delivers a unique bounded rational expectations equilibrium. Under regime uncertainty this is no longer true. The precise choice of monetary policy constrains the set of fiscal policies consistent with macroeconomic stability. However, for a given choice of monetary policy there always exists a choice of fiscal policy that prevents expectations driven instability. Part 2(b) of the proposition shows that a fiscal policy characterized by an exogenous surplus or an extremely aggressive fiscal rule is conducive to macroeconomic stability for all parameter configurations. Thus, perhaps surprisingly, non-Ricardian regimes appear to be more robust to learning dynamics.

What are the sources of instability and stability under learning dynamics? The next section considers a simple example to provide intuition for the robustness of the non-Ricardian regime. The general case is then discussed.

5.2 Learning to Believe in the Fiscal Theory: An Example

Consider a deterministic economy with fully flexible prices; fiscal policy characterized by zero steady state debt, $\delta = 0$, and an exogenous constant surplus, $\phi_\tau = 0$; and a central bank with perfect information about inflation so that $i_t = \phi_\pi \pi_t$. Under these assumptions, aggregate supply equals the natural rate of output, and the model is given by the aggregate demand and debt equations

$$\phi_\pi \hat{\pi}_t = (1 - \beta\phi_\pi)\, \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\pi}_{T+1} \tag{15}$$

$$\hat{b}_{t+1} = \beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right). \tag{16}$$

Beliefs are specified by the regressions

$$\hat{\pi}_t = \omega_\pi \hat{b}_t \quad \text{and } \hat{b}_{t+1} = \omega_b \hat{b}_t$$

where for simplicity assume that the intercept is not estimated.[11] The belief structure implies

$$\begin{aligned} \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \hat{\pi}_{T+1} &= \omega_\pi \frac{\omega_b}{1 - \beta\omega_b} \hat{b}_{t+1} \\ &= \omega_\pi \frac{\omega_b}{1 - \beta\omega_b} \left[\beta^{-1}\hat{b}_t - (\beta^{-1} - \phi_\pi)\hat{\pi}_t\right] \end{aligned} \tag{17}$$

[11] This is formally what Evans and Honkapohja (2001) call a restricted perceptions equilibrium, since beliefs about debt dynamics do not nest the minum state variable form. This is irrelevant to the established point.

where the second inequality uses the definition of the flow budget constraint. Inserting (17) in (15) and rearranging provides

$$\begin{aligned}\hat{\pi}_t &= \left[\phi_\pi \frac{1-\beta\omega_b}{\omega_\pi\omega_b} + (1-\beta\phi_\pi)(\beta^{-1}-\phi_\pi)\right]^{-1}(1-\beta\phi_\pi)\beta^{-1}\hat{b}_t \\ &= T(\omega_\pi,\omega_b)\,\hat{b}_t\end{aligned}$$

which denotes the actual evolution of inflation as a function of real debt and agents' beliefs.

In the special case $\phi_\pi = 0$, where monetary policy is a nominal interest rate peg, the expression simplifies to

$$\hat{\pi}_t = \hat{b}_t \qquad (18)$$

and observed dynamics are independent of agents' beliefs. Indeed, relation (18) corresponds to the restriction between inflation and debt that obtains in a rational expectations equilibrium under maintained assumptions. Given $T(\omega_\pi, \omega_b) = 1$, the associated ordinary differential equation characterizing learning dynamics is

$$\begin{aligned}\dot{\omega}_\pi &= 1-\omega_\pi \\ \dot{\omega}_b &= -\omega_b,\end{aligned}$$

implying stability for all parameter values.

More generally, stability under learning depends crucially on the relation between inflation and government debt. Suppose agents' inflation expectations increase — formally $\omega_\pi > 1$. The increase in inflation expectations leads to an increase in current inflation, with the increase being larger the lower is ϕ_π. Simultaneously, higher inflation decreases the real value of next period holdings of the public debt, which in turn lowers expectations. In the limiting case, $\phi_\pi \to 0$, inflation remains unchanged — the two effects on inflation are equal and opposite. Regardless, the initial rise in inflation expectations is not validated by subsequent inflation data and the agents' estimate of ω_π converges back to its rational expectations equilibrium value. As long as agents' beliefs permit a possible relation between inflation and real debt, as assumed in this paper, their learning process converges to rational expectations equilibrium.

5.3 Aggregate Demand Management and Instability

Now consider the general case. The mechanism generating instability is the same in both Ricardian and non-Ricardian regimes and depends fundamentally on monetary policy. Consider an increase in inflation expectations in the locally Ricardian regime.[12] Aggregate demand rises immediately, as does inflation. The initial monetary policy response is weak because the nominal interest rate is set before observing current prices. As inflation increases, the central bank revises its expectations of current inflation and starts increasing the nominal interest rate more than proportionally, as dictated by the Taylor Principle. Because private agents do not know the policy rule their *expected path* for the interest rate is flatter than under full knowledge of the policy rule: as a consequence, the gradual increase in the nominal interest rate has little initial effect on inflation expectations. As inflation continues to rise, the central bank adjusts policy until inflation expectations and actual inflation start declining. Eventually interest rates are too high and the economy contracts. A process of recessions followed by expansions ensues, leading to instability. Uncertainty about both the policy rule and the delay in the monetary policy response drive instability. Failure to manage expectations through effective restraint of aggregate demand generates destabilizing dynamics.

A similar process occurs in the non-Ricardian regime if the policy rule prescribes a sufficiently aggressive response to inflation which dominates the stabilizing wealth effects of real debt on inflation expectations, as described in the simple example. Proposition 2 also implies that for $\phi_\pi < 0.5$, stability obtains independently of ϕ_τ. For higher values of ϕ_π stability depends on the fiscal rule. Furthermore, a fiscal rule with $\phi_\tau > \phi_\tau^*$ can be shown to weaken the rational expectations equilibrium relation between real debt and inflation, making inflation expectations less responsive to the level of real debt. As a result, under learning dynamics, the wealth effects operating through the intertemporal budget constraint of the government that are embedded in household and firm beliefs, are weaker, and therefore less of a stabilizing force.[13]

[12] Eusepi and Preston (2007) discuss in detail the case of a Ricardian regime with zero net supply of bonds.

[13] It is useful to recall that the learning analysis is local to the rational expectations equilibrium of interest. Beliefs are close, but not exactly equal, to those that obtain under rational expectations.

5.4 The Role of Indebtedness and Nominal Rigidities

The conditions of proposition 2 are independent of the steady state structural surplus-to-output ratio. Only one dimension of fiscal policy represents a constraint on macroeconomic stabilization — the choice of tax rule. Given a choice of monetary and fiscal policy that satisfies the above conditions, whether an economy is debt free or heavily indebted is irrelevant.

In the special case that an economy is debt free the aggregate demand equation becomes

$$\begin{aligned}\hat{x}_t &= \delta\left(\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\hat{s}_t + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) - \left(1-\beta\right)\hat{s}_{T+1}\right]\right) \\ &\quad + \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[\left(1-\beta\right)\hat{x}_{T+1} - \left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + r_T\right] \\ &= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[\left(1-\beta\right)\hat{x}_{T+1} - \left(\hat{\imath}_T - \hat{\pi}_{T+1}\right) + r_T\right]. \end{aligned} \tag{19}$$

It follows that an economy with zero debt on average has identical dynamics to an economy in which households correctly understand the government accounts to be intertemporally solvent. That is, if households believe that the intertemporal budget constraint of the government,

$$\hat{b}_t - \hat{\pi}_t = E_t \sum_{T=t}^{\infty} \beta^{T-t}\left[\left(1-\beta\right)\hat{s}_T - \beta\left(\hat{\imath}_T - \hat{\pi}_{T+1}\right)\right],$$

is satisfied at all points in time (as in rational expectations equilibrium) then aggregate demand is similarly determined by (19). This observation underscores that it is the uncertainty about current and future monetary policy that is the primary source of instability in this economy under regime uncertainty.

In each of these two special cases, aggregate demand depends neither on average indebtedness nor on the precise choice of tax rule. Determinacy of rational expectations equilibrium is similarly independent of these objects. The sequel demonstrates that under non-rational expectations this is not generally true — the efficacy of stabilization policy can hinge on the indebtedness of the economy and, therefore, uncertainty about the intertemporal solvency of the government accounts. This discussion is summarized in the following corollary.

Corollary 3 *Under regime uncertainty, macroeconomic stabilization policy is independent of the average indebtedness of the economy.*

Finally, it is of some interest to understand the role of nominal rigidities in stabilization policy. The following analytic result is available.

Proposition 4 *Under regime uncertainty, if* $\phi_\pi = \phi_\tau = 0$*, then expectational stability obtains for all* $0 < \alpha < 1$*.*

This is a special case of the Leeper conditions under locally non-Ricardian fiscal policy. Regardless of the degree of nominal rigidity, this configuration of policy rules out expectations driven instability. More generally, numerical analysis demonstrates higher degree of nominal frictions assists stabilization policy. Because prices tend to fluctuate less, agents can more easily discern the true dynamics of prices.

6 Improving Stabilization Policy

These instability results naturally raise the question of how can expectations be managed more effectively in the pursuit of macroeconomic stabilization. The model has two key information frictions. First, the central bank responds to information about the true state of the economy with a delay. This is an implication of the forecast-based monetary policy rule. Second, households and firms have an incomplete model of the macroeconomy and need to learn about the reduced-form dynamics of aggregate prices. A consequence of this assumption is that agents are uncertain about the policy regime and face statistical uncertainty about the true data generating process describing the evolution of nominal interest rates and taxes. Resolving these informational frictions may mitigate expectations driven instability.

6.1 Resolving Uncertainty About the State

In regards to the policymaker's uncertainty, suppose the central bank has perfect information about the current inflation rate. It can then implement the policy rule

$$i_t = \phi_\pi \pi_t. \tag{20}$$

The following result obtains.

Proposition 5 *With perfect information about the state of the economy, stabilization policy ensures expectational stability if and only if either*

1. *Monetary policy is active and fiscal policy is locally Ricardian such that*

$$\phi_\pi > 1 \text{ and } 1 < \phi_\tau < \frac{1+\beta}{1-\beta}; \text{ or}$$

2. *Monetary policy is passive and fiscal policy is locally non-Ricardian such that*

$$0 \leq \phi_\pi < 1 \text{ and either } 0 \leq \phi_\tau < 1 \text{ or } \phi_\tau > \frac{1+\beta}{1-\beta}.$$

Hence timely information about the state of the economy is invaluable to expectations stabilization. By responding to contemporaneous observations of the inflation rate the Leeper conditions are restored. Hence, the full menu of policy choices that deliver determinacy of rational expectations equilibrium similarly deliver expectational stability when agents face regime certainty. Having perfect information about the aggregate state reduces the delay in the adjustment of monetary policy, allowing the central bank to anticipate shifts in expectations and enabling households to better predict the path of the real interest rate. In general, though, accurate information about the current state is not a panacea for expectational instability in this model. The result depends on the assumption that only one period debt is issued. Eusepi and Preston (2007b) demonstrate that issuance of longer maturity debt can render stabilization policy prone to self-fulfilling expectations even when the Taylor principle is satisfied and the state is accurately observed.

As described in the previous section, in the case of imperfect information, aggressive monetary policy can destabilize expectations. Indeed, by not responding to inflation expectations and implementing a non-Ricardian fiscal policy, stability can be restored.

Corollary 6 *Consider the non-Ricardian regime with $0 \leq \phi_\tau < 1$. Assume imperfect information about the state of the economy. If $\phi_\pi = 0$, the stability conditions are the same as in the case of perfect information about inflation with $0 \leq \phi_\pi < 1$.*

Given that central banks are unlikely in practice to have complete information about the current state of the economy, it is worth considering other approaches to effective management of expectations. The remainder of the paper therefore explores whether resolving uncertainty about the policy regime jointly adopted by the monetary and fiscal authorities assists macroeconomic stabilization.

6.2 Resolving Uncertainty About the Policy Regime

The second key informational friction stems from households and firms having an incomplete model of the macroeconomy. Because they do not know the true statistical laws describing the evolution of exogenous state variables relevant to their decision problems, these variables must be projected using atheoretical models. An important dimension of this uncertainty is that the adopted monetary and fiscal policies are unknown to private agents. Hence, stabilization policy may be more effective if this uncertainty regarding the policy regime can be mitigated.

To explore the role of policy uncertainty, we follow Eusepi and Preston (2007a), and consider the benefits of credibly communicating the monetary and fiscal policy rules to firms and households. Hence, the precise details of the monetary and fiscal policy rules are announced, including the policy coefficients and conditioning variables. Knowledge of these rules serves to simplify firms' and households' forecasting problems. Indeed, agents need only forecast inflation and real debt: policy consistent forecasts of future nominal interest rates and taxes can then be determined directly from the announced policy rules. It follows that credible announcements have the property that expectations about future macroeconomic conditions are consistent with the policy strategy of the monetary and fiscal authorities.

Under communication of the policy regime, agents know that the policy regime is determined by the policy rules (5) and (7). The aggregate demand equation becomes

$$\begin{aligned} \hat{x}_t &= \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) - \beta^{-1}\delta\phi_\tau \hat{b}_t - (1-\delta)\,\phi_\pi \hat{E}_{t-1}\hat{\pi}_t \\ & \quad \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\left[(1-\beta)\left(\hat{x}_{T+1} - \delta\phi_\tau \hat{b}_{T+1}\right) - (1-\delta)\left(\phi_\pi\beta - 1\right)\hat{\pi}_{T+1} + r_T\right] \end{aligned} \qquad (21)$$

determined by direct substitution of the policy rules into equation (9). The remaining model equations are unchanged with the exception of beliefs. As nominal interest rates and taxes need not be forecast, an agent's vector autoregression model is estimated on the restricted state vector

$$X_t = \begin{bmatrix} x_t \\ \pi_t \\ b_{t+1} \end{bmatrix}.$$

It is clear that knowledge of the regime does not eliminate uncertainty about the statistical laws determining state variables, as future output, inflation and real debt must still be forecasted to make spending and pricing decisions.

Proposition 7 *Under knowledge of the policy regime, stabilization policy ensures expectational stability if the following conditions are satisfied: either*

1. *Monetary policy is active and fiscal policy is locally Ricardian such that*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \phi_\pi > \frac{1}{1-\beta\delta} \text{ ; or}$$

2. *Monetary policy is passive, $0 \leq \phi_\pi < 1$, and fiscal policy is non-Ricardian such that*

   (a)

$$0 \leq \phi_\tau < 1 \quad \text{and} \quad \delta < \min\left[\frac{\left(1-\beta+\beta^2\phi_\pi\right)\left(1-\phi_\pi\right)}{\phi_\pi\beta\left(1-\beta\phi_\pi\right)}, 1\right] \quad \text{or}$$

   (b)

$$\phi_\tau > \frac{1+\beta}{1-\beta}.$$

Remark 8 *The conditions in 1. and 2.(b) are also necessary conditions.*

Regardless of the regime, guarding against expectations driven instability for a given choice of tax rule, ϕ_τ, requires a choice of monetary policy rule that depends on two model parameters: the household's discount factor, β, and the steady state ratio of the primary surplus to output, δ (or equivalently the steady state debt-to-output ratio since $\bar{s} = (1-\beta)\bar{b}$). The choice of fiscal regime, reflected in the implied average debt-to-output ratio, imposes constraints on stabilization objectives. Less fiscally responsible governments have access to a smaller set of monetary policies to ensure learnability of rational expectations equilibrium. In the case of locally Ricardian fiscal policies, the higher is the average debt-to-output ratio, the more aggressive must monetary policy be to protect the economy from self-fulfilling expectations.

Similarly, under locally non-Ricardian fiscal policies, the choice of monetary policy is again constrained by the average level of indebtedness of the economy. The higher are average debt levels the more passive must be the adopted monetary policy rule. Regardless of the policy regime, for $0 < \delta < 1$, the menu of policies consistent with stabilizing expectations is larger than when agents are uncertain about the policy regime — compare proposition 2. This discussion is summarized in the following proposition which presents two special cases of the above results.

Proposition 9 *Communication unambiguously improves stabilization policy under learning dynamics. That is, for* $0 < \delta < 1$*, a larger menu of fiscal and monetary policies is consistent with expectations stabilization under knowledge of the policy regime than under regime uncertainty. When* $\delta = 1$*, the regions of stability in the communication and no communication cases coincide. When* $\delta = 0$*, the Leeper conditions are restored.*

That the stability of expectations depends on a steady state quantity through δ is surprising when compared to a rational expectations analysis. Indeed, the model indicates determinacy of equilibrium conditions to be independent of this quantity. What then is the source of this dependence?

Propositions 7 and 9 make clear that the choice of monetary policy, ϕ_π, and the steady state structural surplus-to-output ratio, δ, play a crucial role in determining stability, in both Ricardian and non-Ricardian policy regimes. The main source of instability are wealth effects arising from violations of Ricardian equivalence: agents perceive real bonds as net wealth out of rational expectations equilibrium — in contrast to Barro (1974). To provide intuition, consider a regime with active monetary policy and passive fiscal policy. Again, suppose that inflation expectations increase. Agents correctly predict a steeper path of the nominal interest rate and aggregate demand decreases leading to lower actual inflation. In an economy with zero net debt, this would decrease expectations driving the economy back to equilibrium. But with holdings of the public debt treated as net wealth, lower inflation generates a positive wealth effect, stimulating aggregate demand and increasing inflationary pressures. The increase in real debt is higher if the monetary authority does not observe current prices because the nominal interest rate does not immediately decrease with inflation. On the one hand, active policy restrains demand as agents expect future higher real rates. On the other hand, larger real debt and higher expected nominal interest rates generate wealth effects with inflationary consequences. If the monetary policy rule is not sufficiently active and the stock of government debt is large the latter prevail, leading to instability.

The same mechanism operates with passive monetary policy and active fiscal policy. Following an increase in inflation expectations, output and inflation increase stimulated by a decline in real interest rates. As in the simple example discussed above, the positive relation between real debt and inflation drives the economy back to equilibrium. But higher inflation can also have a destabilizing effect because it leads to a higher expected path for the nominal interest rate, increasing the real value of interest payments on outstanding government debt. This positive wealth effect increases aggregate demand and inflation. If the latter effect is sufficiently strong the combination of monetary and fiscal policy can be destabilizing.[14] That is, if monetary policy is sufficiently aggressive and the steady state level of real debt is sufficiently high, then inflationary effects dominate, leading to instability.

As a final note, it is worth comparing these findings to Preston (2006). That paper considered a model in which monetary policy was given by a Taylor rule

$$i_t = \phi_\pi \hat{E}_{t-1}\pi_t + \phi_x \hat{E}_{t-1}x_t.$$

The remaining model features were identical with two exceptions. First, an arbitrary degree of nominal rigidity, $0 < \alpha < 1$, was permitted. Second, it was assumed that households and firms correctly understood that the government pursued a zero debt policy each period and, therefore, that no taxes would be levied in the present or indefinite future. Thus a Ricardian fiscal policy is assumed and households need not forecast future taxes to make spending plans.

Under these assumptions the model is given by (10), (5) and (19). If agents do not know the monetary policy strategy of the central bank and $\phi_x = 0$, then a requirement for expectational stability is

$$\phi_\pi > \frac{1}{1-\beta} - \frac{\alpha\left(2-\beta-\alpha\beta\right)}{\left(1-\alpha\right)\left(1-\alpha\beta\right)^2}.$$

This condition collapses to that obtained in proposition 2 on noting that as $\alpha \to 0$ the second term vanishes. If the monetary policy strategy of the central bank is communicated, so there is no regime uncertainty, then the requirement for expectational stability is

$$\phi_\pi > 1.$$

[14] It can be shown that the higher ϕ_τ, the smaller the parameter set for which we have stability. In fact the higher ϕ_τ the weake the realtion between real debt and inflation, and the more importan the wealth effects from higher nominal rates.

Again, this is a special case of proposition 7. Differences emerge from differing assumptions about fiscal policy in the present paper. By allowing for a much broader set of fiscal policies in the analysis here, including locally non-Ricardian fiscal policy in addition to locally Ricardian, a richer set of predictions about stabilization policy obtains. Most importantly, permitting the fiscal authority to run positive average debt levels has additional implications for stabilization policy. Because of the failure of Ricardian equivalence out of rational expectations equilibrium, the resulting wealth effects on private spending serve to constrain the choice of monetary policy rule for a given tax rule.

7 Alternative Models of Learning Dynamics

Many recent papers have proposed analyses of learning dynamics in the context of models where agents solve infinite horizon decision problems, but without requiring that agents make forecasts more than one period into the future. In these papers, agents' decisions depend only on forecasts of future variables that appear in Euler equations used to characterize rational expectations equilibrium. Important contributions include Bullard and Mitra (2002) and Evans and Honkapohja (2003). Of most relevance to the present study is Evans and Honkapohja (2007) which similarly studies the interaction of monetary and fiscal policy, but in a model of learning dynamics in which only one period ahead expectations matter to expenditure and pricing plans of households and firms. The following section replicates part of their analysis in the context of the model developed here, and contrasts the resulting findings with those of sections 5 and 6.

Since the optimal decision rules for households and firms presented in section 2 are valid under arbitrary assumptions on expectations formation, they are satisfied under the rational expectations assumption. Application of this assumption implies the law of iterated expectations to hold for the aggregate expectations operator and permits simplification of relations

(9) and (10) to the following aggregate Euler equation and Phillips curve:[15]

$$
\begin{aligned}
x_t &= E_t x_{t+1} - (i_t - E_t \pi_{t+1} - r_t) \\
\pi_t &= \kappa x_t + \beta E_t \pi_{t+1}.
\end{aligned}
$$

Under learning dynamics, with only one period ahead expectations, it is assumed that aggregate demand and supply conditions are determined by

$$
x_t = \hat{E}_t x_{t+1} - \left(i_t - \hat{E}_t \pi_{t+1} - r_t \right) \tag{22}
$$

$$
\pi_t = \kappa x_t + \beta \hat{E}_t \pi_{t+1}. \tag{23}
$$

Identical assumptions are made on monetary and fiscal policy — relations (5), (6) and (7) continue to hold. The model is closed with a description of beliefs. As nominal interest rates and taxes need not be forecast, an agent's vector autoregression model is estimated on the restricted state vector

$$
X_t = \begin{bmatrix} x_t \\ \pi_t \\ b_{t+1} \end{bmatrix}.
$$

Two points should be underscored. First, the assumption that only one period ahead forecasts matter, implies that households and firms do not take account of transversality conditions in making their spending and pricing plans. This model feature might be thought to have implications in the present context as the fiscal theory of the price level is a theory grounded on shifting evaluations of various variables related precisely by this constraint. Furthermore, by considering a model of consumer behavior that does not account for the implications of the intertemporal budget constraint, fiscal policy has no direct impact on spending and pricing decisions. Neither forecasts of future taxes nor the average indebtedness of the macroeconomy matter for aggregate dynamics. Second, and related, is that because households do not need to forecast future nominal interest rates or taxes there is no uncertainty about the policy rules adopted by the monetary and fiscal authority – there is no regime uncertainty and no role for communication of the joint policy strategy. It seems worth

[15] See Preston (2005a, 2005b) for a detailed discussion.

exploring the consequences of these alternative modeling assumptions, and learning whether they elucidate earlier results.

In the model given by relations (22), (23), (5), (6) and (7) under learning dynamics the follow stability results obtain.

Proposition 10 *For $0 < \alpha < 1$, stabilization policy ensures expectational stability if and only if the following conditions are satisfied: either*

1. *Monetary policy is active and fiscal policy is locally Ricardian such that*

$$\phi_\pi > 1 \text{ and } 1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ ; or}$$

2. *Monetary policy is passive and fiscal policy is non-Ricardian such that*

$$0 \leq \phi_\pi < 1 \text{ and either } 0 \leq \phi_\tau < 1 \text{ or } \phi_\tau > \frac{1+\beta}{1-\beta}.$$

This generalizes the Evans and Honkapohja (2006) analysis to a model with nominal rigidities.[16] When only one period ahead expectations matter, the Leeper conditions are sufficient to rule out expectations driven instability. In contrast, in a model of optimal decisions, these conditions obtain only if there is no regime uncertainty — i.e. the policy rules are credibly communicated to households and firms — and either agents believe the government accounts to be intertemporally solvent or the fiscal authority chooses policy so that the steady state debt-to-output ratio is zero. If neither of these conditions is met, the analysis of this paper suggests a smaller menu of policies is consistent with expectations stabilization.

8 Conclusions

A model is developed to explore the constraints imposed on stabilization policy by expectations formation. Specific emphasis is given to household and firm uncertainty about the prevailing policy regime adopted by the central bank and fiscal authority.

Two central results emerge. First, when agents have no knowledge about the policy regime, stabilization policy is more difficult than under a rational expectations analysis of the model. The set of policies consistent with expectations stabilization is substantially reduced.

[16] The proof is available on request.

Indeed, for a class of monetary policies satisfying the Taylor principle, there is no choice of fiscal policy that prevents self-fulfilling expectations. However, for passive monetary policy, there is always a choice of non-Ricardian fiscal policy that ensures stability. However, the precise choice depends on the specific monetary policy being implemented. An implication is that under non-rational expectations, tighter coordination of monetary and fiscal policy is desirable. That non-Ricardian fiscal policies emerge to be relatively robust to expectational instability stems from two model properties: i) passive monetary policies minimize uncertainty about the future path of nominal interest rates and ii) stabilizing wealth effects that operate through the intertemporal budget constraint of the government.

Second, under full knowledge of the policy regime, stabilization policy is unambiguously improved across both Ricardian and non-Ricardian regimes. That active monetary policies are no longer a source of instability is a direct consequence of households being able to accurately project the future path of real interest rates when the monetary policy strategy is known. Similarly, under non-Ricardian fiscal policies, passive monetary policy induces less uncertainty about the path of nominal interest rates, enhancing the effectiveness of stabilization policy.

However, complete knowledge of the monetary and fiscal policy strategy does not ensure that all policies consistent with determinacy of rational expectations are similarly consistent with expectational stability under learning dynamics. Whether they are or not, depends on the average level of indebtedness of the economy. Because households imperfectly forecast future tax obligations, holdings of the public debt are perceived as net wealth. As a result, variations in outstanding debt lead to Keynesian expenditure effects, and these effects can be destabilizing. The magnitude of these wealth effects are proportional to the steady state debt-to-output ratio. The more heavily indebted the economy the more difficult it is to stabilize the macroeconomy.

Two special cases present themselves. If the economy is heavily indebted so that the structural surplus-to-output ratio is unity, then the same set of policies under both presence and absence of knowledge of the regime are consistent with stability. In contrast, if the economy is debt free on average so that the structural surplus to output ratio is zero, then the Leeper conditions are restored: all policies consistent with determinacy of rational expectations equilibrium deliver expectational stability.

A Appendix

A.1 Model Derivation

The following describes only the derivation of the aggregate demand equation. The derivation of the generalized Phillips curve can be found in Preston (2005b). The household's optimality conditions imply:

$$E_0^i \sum_{t=0}^{\infty} \beta^t \frac{U_c\left(C_t^i + g; \xi_t\right)}{U\left(C_0^i + g; \xi_0\right)} C_t^i = \frac{B_0^i}{P_0} + E_0^i \sum_{t=0}^{\infty} \beta^t \frac{U_c\left(C_t^i + g; \xi_t\right)}{U_c\left(C_0^i + g; \xi_0\right)} \left[Y_t - \frac{T_t}{P_t}\right]$$

which can be rewritten as

$$b_0^i \pi_0^{-1} = E_0^i \sum_{t=0}^{\infty} \beta^t \frac{U_c\left(C_t^i + g; \xi_t\right)}{U\left(C_0^i + g; \xi_0\right)} \left[C_t^i - Y_t + \tau_t\right] \tag{24}$$

where $\tau_t = T_t/P_t$ and $b_t^i = B_t^i/P_t$. In steady $\bar{s} = (1-\beta)\,\bar{b}$ where $s_t = T_t/P_t - g$ defines the structural surplus and market clearing implies $\bar{Y} = \bar{C} + g$.

Approximating (24) provides

$$\begin{aligned} \bar{b}\left(\hat{b}_0^i - \hat{\pi}_0\right) &= \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t [\bar{C}\hat{C}_t^i - \bar{Y}\hat{Y}_t + \bar{\tau}\hat{\tau}_t] + \bar{s}\hat{E}_0^i \sum_{t=0}^{\infty} \beta^t \left[\frac{U_{cc}}{U_c}\bar{C}\hat{C}_t^i - \frac{U_{cc}}{U_c}\bar{C}\hat{C}_0^i\right] \\ &= \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t [\left(\bar{C} - \bar{s}\tilde{\sigma}^{-1}\right)\hat{C}_t^i + \bar{s}\tilde{\sigma}^{-1}\hat{C}_0^i\,(1-\beta)^{-1} + \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t [-\bar{Y}\hat{Y}_t + \bar{\tau}\hat{\tau}_t] \end{aligned}$$

where $\tilde{\sigma} = -U_c/\left(U_{cc}\bar{C}\right)$.

The consumption Euler equation satisfies the log-linear approximation

$$\hat{C}_t^i = \hat{E}_t^i \hat{C}_{t+1}^i - \tilde{\sigma}\left(\hat{\imath}_t - \hat{E}_t^i \hat{\pi}_{t+1}\right).$$

Solving recursively backwards and taking expectations at time zero provides

$$\hat{E}_0^i \hat{C}_t^i = \hat{C}_0^i + \tilde{\sigma}\hat{E}_0^i \sum_{s=0}^{t-1} \left(\hat{\imath}_s - \hat{\pi}_{s+1}\right).$$

This determines the infinite sum

$$\begin{aligned} \hat{E}_0^i \sum_{t=1}^{\infty} \beta^t \hat{C}_t^i &= \frac{\beta\hat{C}_0^i}{(1-\beta)} + \tilde{\sigma}\hat{E}_0^i \sum_{t=1}^{\infty} \beta^t \sum_{s=0}^{t-1} \left(i_s - \pi_{s+1}\right) \\ &= \frac{\beta\hat{C}_0^i}{(1-\beta)} + \frac{\tilde{\sigma}\beta}{(1-\beta)} \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1}\right). \end{aligned}$$

Substituting into the intertemporal budget constraint

$$\begin{aligned}
\bar{b}\left(\hat{b}_0^i - \hat{\pi}_0\right) &= \hat{E}_0^i \sum_{t=1}^{\infty} \beta^t \left(\bar{C} - \bar{s}\tilde{\sigma}^{-1}\right) \hat{C}_t^i + \left(\bar{C} - \bar{s}\tilde{\sigma}^{-1}\right) \hat{C}_0^i + \bar{s}\tilde{\sigma}^{-1} \frac{\hat{C}_0^i}{(1-\beta)} + \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t [-\bar{Y}\hat{Y}_t + \bar{\tau}\hat{\tau}_t] \\
&= \frac{\bar{C}\hat{C}_0}{(1-\beta)} + \left(\bar{C} - \bar{s}\tilde{\sigma}^{-1}\right) \frac{\tilde{\sigma}\beta}{(1-\beta)} \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t \left(i_t - \pi_{t+1}\right) + \hat{E}_0^i \sum_{t=0}^{\infty} \beta^t [-\bar{Y}\hat{Y}_t + \bar{\tau}\hat{\tau}_t]
\end{aligned}$$

Divide through by $\bar{Y}\ (1-\beta)^{-1}$ and rearranging gives the optimal consumption rule

$$\hat{C}_0^i = s_c^{-1}\delta\left(\hat{b}_0 - \hat{\pi}_0\right) + s_C^{-1}\hat{E}_0^i \sum_{t=0}^{\infty} \beta^t \left[(1-\beta)\left(\hat{Y}_t - \delta\hat{s}_t\right) - (\sigma - \delta)\,\beta\,(i_t - \pi_{t+1})\right]$$

where $s_C = \bar{C}/\bar{Y}$, $\delta = \bar{s}/\bar{Y}$, $\bar{s}\hat{s}_t = \bar{\tau}\hat{\tau}$, $\sigma = s_c\tilde{\sigma}$ and using $\bar{s} = (1-\beta)\,\bar{b}$.

Finally, note that market clearing implies the log-linear approximations

$$\hat{Y}_t = s_c \int_0^1 \hat{C}_t^i di \text{ and } \hat{b}_t = \int_0^1 \hat{b}_t^i di.$$

Hence the aggregate demand equation is

$$\hat{Y}_0 = \delta\left(\hat{b}_0 - \hat{\pi}_0\right) + \hat{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\beta)\left(\hat{Y}_t - \delta\hat{s}_t\right) - (\sigma-\delta)\,\beta\,(i_t - \pi_{t+1}) - (1-\beta)\, s_C \bar{C}_t\right]$$

where $\hat{E}_t = \int_0^1 \hat{E}_t^i di$ defines average beliefs of households.

Define the output gap as $Y_t - Y_t^n$ where the latter is the natural rate of output under rational expectations permits

$$x_t = \delta\beta^{-1}\left(\hat{b}_t - \hat{\pi}_t\right) + \hat{E}_t \sum_{T=t}^{\infty} \beta^t \left[(1-\beta)\left(\hat{x}_{T+1} - \delta\beta^{-1}\hat{s}_T\right) - (\sigma-\delta)\left(i_T - \pi_{T+1}\right) + r_T\right]$$

where

$$r_t = Y_{t+1}^n - Y_t^n.$$

A.2 Proof of Proposition 1

TO BE ADDED

A.3 Proof of Proposition 2

Let λ_1 be the only eigenvalue in the model under rational expectations to be inside the unit circle. Then $\alpha \rightarrow 0$ implies $\lambda_1 \rightarrow \phi_\pi$. Stability under learning dynamics is determined by local stability of the associated ODE. The ODE can be broken in two separate sub-systems, describing the evolution of the constant coefficients and the coefficients on debt respectively. That is:

$$\begin{aligned} \dot{a} &= (J_a - I_5)\, a = Aa \\ \dot{b} &= (J_b - I_5)\, b = Bb \end{aligned}$$

Ricardian fiscal policy. Stability requires all five eigenvalues of each Jacobian to be negative. Because a) two eigenvalues are -1 and b) the eigenvalues are independent of δ and the *same* $\delta \in [0,1]$ consider the case $\delta = 0$. The three remaining eigenvalues can then be computed from three-dimensional sub-matrices $\tilde{A}$ and $\tilde{B}$, where, for example, the stability of the constant dynamics are determine by the matrix:

$$A = \begin{bmatrix} \tilde{A} & 0 \\ P & -1 \end{bmatrix}.$$

Regression intercept. In order for the real parts of the three eigenvalues to be negative we require

$$tr(\tilde{A}) < 0, \quad \det(\tilde{A}) < 0 \text{ and } M_{\tilde{A}} = -Sm(\tilde{A}) \cdot Tr(\tilde{A}) + \det(\tilde{A}) > 0$$

where $Sm(\tilde{A})$ denotes the sum of all principles minors of $\tilde{A}$. Taking the limit case $\alpha \rightarrow 0$, the trace, determinant and $M_{\tilde{A}}$ become arbitrarily large. To sign these objects, consider instead the limit

$$\lim_{\alpha \rightarrow 0^+} \alpha \cdot Tr(\tilde{A}) = -\left(\phi_\pi - 1 - \phi_\pi \beta\right)(1 - \beta)^{-1}$$

which is negative if and only if

$$\phi_\pi > \frac{1}{1-\beta}. \tag{25}$$

Likewise, the determinant can be shown to be negative if and only if $\phi_\pi > 1$. For $M_{\tilde{A}}$ we have

$$\lim_{\alpha \rightarrow 0^+} \alpha^2 \cdot M_{\tilde{A}} = \frac{2 - 2\phi_\pi + \phi_\pi \beta}{1 - 1\phi_\pi + \phi_\pi \beta}$$

which is positive provided (25) is satisfied.

Coefficients on debt. The trace satisfies

$$\lim_{\alpha \to 0^+} \alpha \cdot Tr(\tilde{B}) = \frac{1 - \phi_\tau (1-\beta)(\beta\phi_\pi + 1)}{\phi_\tau \beta(1-\beta)}$$

which is decreasing in ϕ_τ. In a Ricardian regime, $\phi_\tau > 1$. Evaluating the expression at $\phi_\tau = 1$ we get that if (25) then the trace of the $\tilde{B}$ matrix is negative. Evaluating the determinant gives

$$\lim_{\alpha \to 0^+} \alpha \cdot \det(\tilde{B}) = \frac{1 - \beta\phi_\pi - \beta^{-1}\phi_\tau \beta(1-\beta)}{\phi_\tau \beta(1-\beta)}$$

Again, imposing $\phi_\tau = 1$ we get $\lim_{\alpha \to 0^+} \alpha \cdot \det(\tilde{B}) = -(\phi_\pi - 1)(\phi_\tau(1-\beta))^{-1} < 0$.Finally,

$$\lim_{\alpha \to 0^+} \alpha^2 \cdot M_{\tilde{B}} = \frac{[\phi_\tau (\beta - 1)(\beta\phi_\pi + 2) - \beta\phi_\pi + 2][\phi_\tau (\beta - 1)(\beta\phi_\pi + 1) + 1]}{\beta^2 \phi_\tau^2 (1-\beta)^2}.$$

Imposing $\phi_\tau = 1$ we get

$$\lim_{\alpha \to 0^+} \alpha^2 \cdot M_{\tilde{B}} = \frac{(2 - 2\phi_\pi + \beta\phi_\pi)(1 - \phi_\pi + \beta\phi_\pi)}{\beta \phi_\tau^2 (1-\beta)^2}$$

which is positive if (25) is satisfied.

*Non-Ricardian fiscal Policy.*Consider the Jacobian corresponding to the matrix of the constant coefficients. For $\alpha = 0$ three eigenvalues are equal to -1 for all parameter values The remaining two eigenvalues z_1 and z_2 are negative if

$$tr(A) = z_1 + z_2 - 3 < 0 \text{ and } -\det(A) = -z_1 z_2 > 0.$$

The trace is

$$tr(A) = -\left[1 + \frac{1 - [1 - \beta\phi_\pi(1-\beta)]\phi_\tau - \beta\phi_\pi}{1 - (1-\beta)\phi_\tau - \beta\phi_\pi}\right].$$

When $0 \leq \phi_\tau < 1$, the condition $tr(A) = 0$ imposes the restriction on ϕ_τ and ϕ_π:

$$\phi_\tau = \frac{2}{[(1 - \beta\phi_\pi)^{-1} + (1-\beta)]}.$$

Hence $0 \leq \phi_\tau < \min[\phi_\tau^*(\phi_\pi), 1]$, where

$$\phi_\tau^*(\phi_\pi) = \frac{2}{[(1 - \beta\phi_\pi)^{-1} + (1-\beta)]}, \tag{26}$$

since $\partial tr\left(A\right)/\partial\phi_{\tau}>0$ for $\phi_{\tau}\in[0,1)$. The determinant is

$$-\det\left(A\right)=\frac{\left(1-\phi_{\tau}\right)\left(\beta\phi_{\pi}-1\right)}{\left(1-\beta\right)\phi_{\tau}+\beta\phi_{\pi}-1}>0.$$

Finally, consider the B matrix relating to the coefficients on real debt. The trace satisfies

$$tr(B)=-2-\frac{(1-\beta)\beta^{2}\phi_{\pi}^{2}\phi_{\tau}}{(-(1-\beta)\phi_{\tau}-\beta\phi_{\pi}+1)(\beta\phi_{\pi}-1)}.$$

which gives the expression

$$\phi_{\tau}^{**}\left(\phi_{\pi}\right)=\frac{2}{\left(\beta^{2}\phi_{\pi}^{2}+2\left(1-\beta\phi_{\pi}\right)\right)\frac{(1-\beta)}{(1-\beta\phi_{\pi})^{2}}}$$

for $tr(B)=0$ (also shown to have positive derivative with respect to ϕ_{τ}). It can be shown that $\phi_{\tau}^{**}\left(\phi_{\pi}\right)>\phi_{\tau}^{*}\left(\phi_{\pi}\right)$.[17] The determinant of the B matrix is equal to -1 for every parameter value.

(b) Straightforward algebraic manipulations shown that the stability condition holds for all parameter values with $\phi_{\tau}>\left(1+\beta\right)/(1-\beta)$.

A.4 Proof of Proposition 4

The rational expectations property that $\lambda_{2}=\alpha$ implies that two eigenvalues are always equal to -1 in the matrices A and B. The trace of the A matrix describing the dynamics of the intercept becomes

$$tr(A)=-\frac{\left[\beta^{2}\alpha^{2}-\alpha\left(\beta^{2}+2\beta\right)+2\right]}{1-\alpha\beta}<0$$

while the determinant of the matrix A is equal to 1 for all parameter values. For stability of the debt coefficients, the trace is

$$tr(B)=G(\alpha)=\frac{\alpha^{4}\beta^{3}-\alpha^{4}\beta^{2}-\alpha^{3}\beta^{3}-\alpha^{3}\beta^{2}+2\alpha^{2}\beta+2\alpha\beta-2}{\left(1-\alpha\beta\right)\left(1-\alpha^{2}\beta\right)}.$$

It is straightforward to show

$$G(0)=-2\text{ and }G(1)=\frac{-2\beta^{3}-2\beta^{2}+2\beta+2\beta-2}{\left(1-\beta\right)\left(1-\beta\right)}<0.$$

[17] It can be shown that the difference betwen the denominator of τ^{*} and the denominator in τ^{**} is equal to

$$\left(\beta\phi_{\pi}-1\right)^{-2}\left(1-\phi_{\pi}\right)\beta>0.$$

Finally, consider the *numerator*

$$\begin{aligned}\tilde{G}'(\alpha) &= -4\alpha^3\beta^2(1-\beta) - 4\alpha^2\beta^2 + \alpha^2\beta^2 + 4\alpha\beta + 2\beta - 3\alpha^2\beta^3 \\ &> \left[-4\alpha^3\beta^2(1-\beta) + 4\alpha\beta\left(1-\alpha\beta\right)\right] + \alpha^2\beta^2 + 2\alpha^2\beta^2 - 3\alpha^2\beta^3 \\ &= \left[-4\alpha^3\beta^2(1-\beta) + 4\alpha\beta\left(1-\alpha\beta\right)\right] + 3\alpha^2\beta^2 - 3\alpha^2\beta^3 > 0\end{aligned}$$

showing that the trace is always negative. The determinant is

$$\det(B) = \frac{(1-\alpha^2\beta - \alpha\beta + \alpha^3\beta^2)}{(1-\alpha\beta)(1-\alpha^2\beta)} > 0.$$

Let $D(\alpha) = (1-\alpha^2\beta - \alpha\beta + \alpha^3\beta^2)$,so that $D(0) = 1$, $D(1) = (1-\beta)^2$ and

$$D'(\alpha) = -\beta\left(2\alpha + 1 - 3\alpha^2\beta\right) < 0$$

for $0 < a < 1$. This completes the proof.

A.5 Proof of Proposition 5

Ricardian fiscal policy. It is easy to show that the matrices $\tilde{A}$ and $\tilde{B}$ have two eigenvalues equal to -1 for every parameter value. Inspecting the trace and determinant of the matrix A it is trivial to show that necessary and sufficient conditions for stability, as $\alpha \to 0$, is $\phi_\pi > 1$. Consider matrix $\tilde{B}$. The trace is

$$tr(\tilde{B}) = \frac{-\phi_\tau(1-\beta)(\beta\phi_\pi + 1) - \beta\phi_\pi + 1}{\phi_\pi\beta(1-\beta)\phi_\tau}$$

which can be verified to be negative if $\phi_\tau > 1$ (as required in the Ricardian regime) and $\phi_\pi > 1$. Finally, the determinant is

$$\det(\tilde{B}) = \frac{-\phi_\tau(1-\beta) + 1 - \beta\phi_\pi}{\phi_\pi\beta(1-\beta)\phi_\tau}$$

is negative if $\phi_\tau > 1$ (as required in the Ricardian regime) and $\phi_\pi > 1$.

Non-Ricardian fiscal policy. For the constant coefficients evaluating the Jacobian has one eigenvalue equal to -1. The remaining two eigenvalues are negative if

$$tr(A) = z_1 + z_2 - 1 < 0 \quad \text{and} \quad -\det(A) = -z_1 z_2 > 0.$$

Solving for the determinant we get

$$-\det(A) = \frac{(1-\phi_\tau)}{(1-\phi_\tau+\phi_\tau\beta)}.$$

Values of ϕ_τ consistent with a non-Ricardian fiscal rule satisfy

$$-1 < H(\phi_\tau) = \frac{\beta}{1-(1-\beta)\phi_\tau} < 1.$$

Multiplying the determinant by β (which leaves its sign unchanged) yields

$$\frac{\beta(1-\phi_\tau)}{(1-\phi_\tau+\phi_\tau\beta)} > 0. \tag{27}$$

For $\phi_\tau < 1$, $0 < H(\phi_\tau) < 1$, using (27) delivers $H(\phi_\tau)(1-\phi_\tau) > 0$ which implies that the determinant is negative. For $\phi_\tau > (1+\beta)\,/\,(1-\beta)$, $-1 < H(\phi_\tau) < 0$. Again, using (27) it is straightforward to show that the determinant is negative. Considering the trace

$$tr(A) = -\frac{(\phi_\tau\beta+2-2\phi_\tau)}{(1-\phi_\tau+\phi_\tau\beta)} = -\left[1+\frac{1-\phi_\tau}{(1-\phi_\tau+\phi_\tau\beta)}\right] < 0$$

by using the results above.

The Jacobian corresponding to the matrix of the coefficients on real debt gives the trace

$$tr(B) = G^T(\phi_\pi) - \frac{(\beta^2\phi_\pi\phi_\tau - \beta\phi_\pi\phi_\tau + 2\beta\phi_\pi - 2\phi_\tau\beta + 2\phi_\tau - 2)}{(\beta\phi_\pi - 1)(1-\phi_\tau+\phi_\tau\beta)}.$$

When $\phi_\pi = 0$

$$G^T(0) = -\frac{2(1-\phi_\tau+\phi_\tau\beta)}{(1-\beta)(1-\phi_\tau+\phi_\tau\beta)} = -\frac{2}{(1-\beta)} < 0$$

and for $\phi_\pi = 1$

$$G^T(1) = -\frac{(\beta^2\phi_\tau - \beta\phi_\tau + 2\beta - 2\phi_\tau\beta + 2\phi_\tau - 2)}{(\beta-1)(1-\phi_\tau+\phi_\tau\beta)} = -\left[1+\frac{1-\phi_\tau}{(1-\phi_\tau+\phi_\tau\beta)}\right] < 0$$

Lastly,

$$G'^T(\phi_\pi) = \frac{\beta\phi_\pi(\beta-1)}{(1-\phi_\tau+\phi_\tau\beta)^2(\beta\phi_\pi-1)} \geq 0 \text{ for every } \phi_\pi \in [0,1].$$

The trace is therefore negative for every value of ϕ_π and ϕ_τ consistent with determinacy of rational expectations equilibrium. Consider the determinant

$$\begin{aligned} -\det(J) \;\; = \;\; G^D(\phi_\pi) = & \frac{(1+\beta^3\phi_\pi^2\phi_\tau + \phi_\tau^2\beta^2 - \beta^2\phi_\pi^2\phi_\tau - 2\phi_\tau + 2\beta^2\phi_\pi\phi_\tau^2 - \beta\phi_\pi\phi_\tau^2 - 3\beta^2\phi_\pi\phi_\tau)}{(\beta^2\phi_\pi\phi_\tau - \beta\phi_\pi\phi_\tau + \beta\phi_\pi - \phi_\tau\beta - 1 + \phi_\tau)^2} \\ & + \frac{(3\beta\phi_\pi\phi_\tau + 2\phi_\tau\beta - 2\beta\phi_\pi + \beta^2\phi_\pi^2 - \beta^3\phi_\pi\phi_\tau^2 + \phi_\tau^2 - 2\phi_\tau^2\beta)}{(\beta^2\phi_\pi\phi_\tau - \beta\phi_\pi\phi_\tau + \beta\phi_\pi - \phi_\tau\beta - 1 + \phi_\tau)^2} \end{aligned}$$

For $\phi_\pi = 0$,

$$G^D(0) = \frac{(1+\phi_\tau^2\beta^2 - 2\phi_\tau + 2\phi_\tau\beta + \phi_\tau^2 - 2\phi_\tau^2\beta)}{(-\phi_\tau\beta - 1 + \phi_\tau)^2} = 1 > 0,$$

and imposing $\phi_\pi = 1$ gives $G^D(1) = (1-\phi_\tau)\left[(1-\phi_\tau+\phi_\tau\beta)\right]^{-1} > 0$.Calculating $G^{D\prime}(\phi_\pi) = -\beta\phi_\pi(1-\beta)\left[(1-\phi_\tau+\phi_\tau\beta)^2(1-\beta\phi_\pi)\right]^{-1} < 0$ obtains so that the determinant is negative for all parameter values.

A.6 Proof of Proposition 6

Consider again the stability conditions in the proof of Proposition 2. For the constant dynamics, when $\phi_\pi = 0$ the trace collapses to the same as in the case of perfect information.

$$tr\,(A) = -\left[1 + \frac{1-\phi_\tau}{1-(1-\beta)\phi_\tau}\right] < 0$$

and the same for the determinant

$$-\det(A) = \frac{(1-\phi_\tau)}{(1-(1-\beta)\phi_\tau)} > 0.$$

For the coefficient on real debt, the trace becomes

$$\mathrm{tr}\,(B) = \frac{2\,(1-\beta)\,\phi_\tau - 2}{(1-(1-\beta)\phi_\tau)} = -2$$

while the determinant of B is equal to -1 for all parameter values.

A.7 Proof of Proposition 7

Consider the case of a Ricardian fiscal policy and active monetary policy. For the constant coefficients, for three negative eigenvalues requires

$$tr(A) < 0, \quad \det(A) < 0 \text{ and } -Sm(A) * Tr(A) + \det(A) > 0$$

where $Sm(A)$ denotes the sum of all principles minors of A. For $\alpha \to 0$, the sign of the trace depends on the following expression

$$\lim_{\alpha \to 0^+} \alpha \cdot tr(A) = 1 + (\beta\delta - 1)\phi_\pi < \infty \tag{28}$$

which gives the expression in the text. Thus, for $\delta = 0$, the Taylor principle obtains. Using $\delta = (1-\beta)\frac{b}{y}$ we can rewrite the stability condition as

$$\phi_\pi(1 - \beta(1-\beta)\frac{b}{y}) - 1 > 0$$

so that for high levels of debt to output ratios and for intermediate values of the discount factor instability is likely to arise. As $\alpha \to 0$, the determinant is negative provided $(\phi_\pi - 1) > 1$. Finally,

$$\lim_{\alpha \to 0^+} \alpha^2 \left[-Sm(A) * Tr(A) + \det(A)\right] = \frac{\phi_\pi (2 - \beta\delta) - 2}{\phi_\pi (1 - \beta\delta) - 1}$$

which is positive, provided (28) is satisfied.

For the debt coefficients, as $\alpha \to 0$, the trace is negative if

$$(-1 + \beta + \beta^2\phi_\pi\delta - \beta\phi_\pi\delta)\phi_\tau - (1-\delta)\beta\phi_\pi + 1 < 0$$

which is negative provided the trace of the matrix for the constants is negative ($\phi_\tau > 1$ in the Ricardian fiscal regime). For $\alpha \to 0$, the determinant is always negative, satisfying

$$-\frac{(1-\beta)\phi_\tau - 1 + \beta\phi_\pi}{\text{positive number}} < 0.$$

Finally, the sum of all principle minors becomes

$$\frac{\left[(-2\beta - \beta^2\phi_\pi\delta + \beta\phi_\pi\delta + 2)\phi_\tau - 2 - \beta\phi_\pi\delta + 2\beta\phi_\pi\right]\left[(1 + \beta\phi_\pi\delta - \beta - \beta^2\phi_\pi\delta)\phi_\tau - 1 + \beta\phi_\pi - \beta\phi_\pi\delta\right]}{\phi_\tau^2\beta^2(1-\beta)^2}$$

which is positive provided $\phi_\tau > 1$ and (28) is satisfied.

As in the case of no-communication, one eigenvalue is always equal to -1. Consider the Jacobian of the associated ODE for the intercept coefficients. The trace is $tr(A) = \Phi^A(\phi_\tau, \phi_\pi, \delta)$ and has the properties

$$\Phi^A_\delta(\phi_\tau, \phi_\pi, \delta) = \frac{(1 - \beta\phi_\pi)\beta^2\phi_\pi}{(1-\beta)(1 - (1-\beta)\phi_\tau - \beta\phi_\pi)} > 0$$

if $0 \le \phi_\tau < 1$ and

$$\Phi^A_{\phi_\tau}(\phi_\tau, \phi_\pi, \delta) = \frac{(1 - \beta\phi_\pi)(\beta\phi_\pi\delta - \phi_\pi + 1)\beta}{(-1 + \phi_\tau - \phi_\tau\beta + \beta\phi_\pi)^2} > 0$$

for all admissible values of δ, ϕ_π, and ϕ_τ, where Φ_x^A denotes the derivative of Φ^A with respect to the argument x. Next we show that for values of $\delta < \delta^{TA}$ the trace is negative. Suppose $\phi_\tau < 1$. Using the inequality above, δ^{TA} satisfies

$$\Phi^A(1, \phi_\pi, \delta^{TA}) = -\frac{(\beta^2\phi_\pi - \beta^2\phi_\pi^2 + \beta^2\phi_\pi^2\delta^{TA} + \beta\phi_\pi - \beta\phi_\pi\delta^{TA} - \beta - \phi_\pi + 1)}{(1-\beta)/(1-\phi_\pi)} = 0.$$

Hence

$$\delta^{TA} = \frac{(1-\beta+\phi_\pi\beta^2)(1-\phi_\pi)}{\phi_\pi\beta(1-\beta\phi_\pi)} > 0.$$

For $\phi_\tau > (1+\beta)/(1-\beta)$, it can be shown that $\Phi_\delta^A(\phi_\tau, \phi_\pi, \delta) < 0$. Impose $\delta = 0$ which gives

$$\Phi_{\delta=0}^A(\phi_\tau, \phi_\pi) = \frac{(C_{\phi_\tau}\phi_\tau - \beta^3\phi_\pi^2 + 3\beta^2\phi_\pi - 2\beta\phi_\pi - 2\beta + 2)}{(1-\beta)((1-\beta)\phi_\tau - 1 + \beta\phi_\pi)}$$

where

$$C_{\phi_\tau} = (\beta^3\phi_\pi - 2\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2)$$

Substituting $\phi_\tau = ((1+\beta)/(1-\beta))$ yields

$$(1-\beta)((1-\beta)\phi_\tau - 1 + \beta\phi_\pi)\Phi_{\delta=0}^A(\phi_\tau, \phi_\pi) = \left(\beta^2 - \beta\right) + \left(\beta^2\phi_\pi - \beta\phi_\pi\right) + (2\beta^2\phi_\pi - 2\beta) - \beta^3\phi_\pi - \beta^3\phi_\pi^2 < 0$$

Last, we show that the coefficient C_{ϕ_τ} is positive, that is

$$\left(\beta^3\phi_\pi - \beta^2\phi_\pi\right) + R(\phi_\pi, \beta) < 0$$

where

$$R(\phi_\pi, \beta) = -\beta^2\phi_\pi - \beta^2 + \beta\phi_\pi + 3\beta - 2$$

$$R(0, \beta) = -(\beta-1)^2 - 1 + \beta < 0, \qquad R(1, \beta) = -2(\beta-1)^2 < 0$$

and

$$R_{\phi_\pi}(\phi_\pi, \beta) = -\beta^2 + \beta > 0.$$

Finally, the determinant of the Jacobian is

$$\det(A) = \frac{(1-\phi_\tau)(1-\beta\phi_\pi)}{1-(1-\beta)\,\phi_\tau - \beta\phi_\pi} > 0.$$

The proof for the debt the debt coefficients follows the same structure. It can be shown that

$$\delta^{TB} = \frac{\left(1-\beta\phi_\pi+\phi_\pi\beta^2+1-\beta\phi_\pi-\phi_\pi\beta^2(1-\phi_\pi)\right)(1-\phi_\pi)}{\beta\phi_\pi^2\,(1-\beta\phi_\pi)} > \delta^{TA}.$$

Moreover, for $\phi_\tau > (1+\beta)/(1-\beta)$

$$tr(B) = \Phi_\delta^B(\phi_\tau,\phi_\pi,\delta) < 0, \quad \text{and} \quad \Phi_{\delta=0}^B(\phi_\tau,\phi_\pi) = (\beta^2\phi_\pi^2 - 2\beta\phi_\pi + 2)/(1-\beta\phi_\pi) < 0$$

Finally, the determinant is equal to one for all parameter values.

A.8 Proof of Proposition 8

Active monetary policy and passive fiscal policy. Consider the case of $\delta = 1$. Then the trace of the constants' matrix becomes

$$1 + (\beta - 1)\phi_\pi$$

which coincides with the stability condition in the case where the agents have no knowledge about the policy rule. Thus, communication is always stability enhancing. The case of $\delta = 0$ is obvious. *Active fiscal policy and passive monetary policy.* Setting $\delta = 1$ we have that

$$tr\,(A) = -\left[1 + \frac{1-[1-\beta\phi_\pi(1-\beta)]\,\phi_\tau - \beta\phi_\pi}{1-(1-\beta)\phi_\tau-\beta\phi_\pi}\right]$$

$$\begin{aligned} tr(A) &= \Phi_{\delta=1}^A(\phi_\tau,\phi_\pi) = \frac{(\beta^2\phi_\pi\phi_\tau + 2\beta\phi_\pi - \beta\phi_\pi\phi_\tau - \phi_\tau\beta + 2\phi_\tau - 2)}{1-(1-\beta)\phi_\tau-\beta\phi_\pi} \\ &= -\left[1 + \frac{1-[1-\beta\phi_\pi(1-\beta)]\,\phi_\tau - \beta\phi_\pi}{1-(1-\beta)\phi_\tau-\beta\phi_\pi}\right] \end{aligned}$$

which is the trace obtained about for the case of no communication. Since we know that $\Phi_\delta^A < 0$ for $0 < \phi_\tau < 1$ we have that $\Phi^A < tr\,(A)$ for $\delta < 1$. [Insert the case with $\delta = 0$.]

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