Inflation, Money Demand and Portfolio Choice*

Kosuke Aoki†
London School of Economics

Alexander Michaelides‡
Central Bank of Cyprus and
London School of Economics, CEPR, FMG and Netspar

March 2010

*We thank the Central Bank of Cyprus and the Bank of England for hospitality when this paper was written. We are responsible for any remaining errors.

†London School of Economics, Houghton Street, WC2A 2AE, UK. E-mail: k.aoki@lse.ac.uk

‡LSE, Houghton Street, London, WC2A 2AE, UK, CEPR and FMG. Email: A.Michaelides@lse.ac.uk.

Abstract

We investigate the effects of inflation on life-cycle saving and asset allocation in a partial equilibrium setting. To achieve this goal, we estimate the preference parameters that can generate plausible holdings of money, bonds and stocks over the life cycle. The model allows us to quantify the effects of inflation on money demand, saving and asset allocation. The model predicts that the share of wealth invested in stocks increases as the household becomes richer, in line with the data. When inflation is i.i.d. (as in the recent few years) hedging demands are small in magnitude for reasonable values of mean and volatility of inflation. When mean inflation approaches Friedman's optimal rule, the demand for bonds is eliminated and the share of financial wealth in stocks decreases.

JEL Classification: E41, G11.

Key Words: Life Cycle Models, Portfolio Choice, Inflation, Money Demand, Liquidity Constraints, Uninsurable Labor Income Risk.

1 Introduction

In the recent large literature on portfolio choice[1] households are assumed to invest between different real assets (typically bonds, stocks and/or housing), ignoring the fact that all transactions in the data are actually done in nominal terms. These models therefore cannot study the effects of inflation on the real economy, money demand or household portfolio choice decisions. On the other hand, the monetary economics literature starts out with a nominal model and inflation becomes a key driver of results and implications for policy. Nevertheless, canonical models in the money demand literature follow the Baumol-Tobin analysis and typically focus on the distinction between money and bonds as a proxy for all other assets in the household portfolio.

However, a potentially more important decision (especially for the richest part of the population) involves the asset allocation between transaction-type balances (like money) and stocks (that have a substantially different risk-return tradeoff from bonds). Moreover, as documented by **?**, inflation changes the real value of nominal assets causing large redistributionary effects in the macroeconomy. Thus, both the level and volatility of inflation have the potential of affecting household consumption and portfolio choice by changing the real returns of assets.

What model can we use to analyze the effect of inflation on asset allocation choices? To proceed we first examine the empirical properties of portfolio choices of households over the life cycle in three broad asset classes: money (transaction accounts), bonds and stocks. We use the US 2001 SCF for this purpose and find that 90% of households have liquid accounts and 52% hold stocks and 63% hold bonds. On the other hand, very few households have no liquid accounts but simultaneously invest in either bonds or stocks. These facts

[1] See **?** for a recent excellent survey.

suggest that limited participation to any interest-bearing assets (as opposed to limited participation in the stock market emphasized in the portfolio choice literature) is a key feature of the data. Nevertheless, in this paper we focus on the saving and asset allocation choices of the subset of households that hold all assets. We follow this route to better explain the intuition behind the non-linear model we eventually solve and leave to future work the interesting extension of incorporating the extensive margin in the analysis.

In the SCF data, wealthier households tend to hold a mixture of money, bonds and stocks with money always featuring in the portfolio. Moreover, wealthier households tend to hold less money and more stocks as a percentage of their total financial assets. To analyze the effect of inflation on the real economy we therefore first need to introduce a role for money balances in the model. Introducing money can vary in complexity from the micro-founded **?** setup to the more reduced form models such as the cash-in-advance (**?**), money-in-utility function (**?**) and the shopping-time approach (**?**).

Given that our purpose is to develop a tractable model that can be confronted with the data, we use a reduced form approach similar to shopping time models.[2] Specifically, we assume that money provides liquidity services and therefore a higher amount of money lowers the cost from having to undertake a given transaction for consumption purposes, other things being equal. Everything else we assume is similar to recent life-cycle models that feature intermediate consumption and stochastic uninsurable labor income in the tradition of **?** and **?**[3], and as extended in the life cycle portfolio choice literature by **?**, for instance. One nice feature of our setting is that it nests the life cycle portfolio models where bonds and stocks are real assets and money does not

[2] For recent applications of shopping time models, see, for example, **?**.

[3] **?**, **?** and **?** extend this tradition and estimate the structural parameters of life cycle models with a single real asset (a riskless bond).

circulate in the economy.

Due to the lack of guidance for picking the shopping technology parameters, we estimate these using a method of simulated moments (MSM) estimation technique. We estimate the structural parameters by matching moments from the 2001 Survey of Consumer Finances data and simulated data from the model. Specifically, we match mean financial wealth over mean labor income over the life cycle, and the portfolio shares across money, bonds and stocks for the investors in the model. Our model can replicate the fact that the share of wealth in stocks increases as financial wealth rises, which is consistent with the data, while this stylized fact has been a priori inconsistent with recent models of household portfolio choice, as pointed out by **?**. The young (who start out with low financial wealth) have higher liquidity needs and economize on shopping costs by holding liquid balances in the form of money. As financial wealth increases over the life cycle, they diversify into stocks and then hold a mixture of all three assets later on in the life cycle.

We next use the estimated model to provide answers to interesting questions. What are the effects of both the level and volatility of inflation on money demand and asset allocation? High inflation causes a reallocation away from money into stocks in the model. This effect is stronger for the younger agents. On the other hand the portfolio choice of the older agents is relatively insensitive. Alternatively, introducing deflation to approach Friedman's optimal rule of a zero nominal interest rate, increases dramatically the demand for money and bonds are completely crowded out from the household portfolio.

What are the hedging demands generated by inflation? We find that hedging demands are small for low to moderate rates (10%) of inflation when inflation is perceived to be i.i.d. (as in recent years). We find this surprising but given the level of idiosyncratic uncertainty faced by households, we think it is a reasonable conclusion. Further research can ascertain whether the introduction of a persistent inflation process or money illusion can change this conclusion.

Can inflation have real, aggregate demand effects in the model? Mean inflation affects not only aggregate money demand but also aggregate consumption demand and wealth accumulation. As mentioned above, even though the young substitutes money for stocks, the money demand of the old is relatively insensitive. Then the decrease in real return of money due to high inflation decreases the overall return of their portfolio and hence decreases their wealth accumulation and consumption.

In terms of the literature, we view the paper as contributing towards understanding money demand and portfolio choice in the presence of nominal assets. Typically, research on money demand focusses on the distinction between money and safe bonds (see, for example, **?**, **?**). In our model we make explicit the choice between money (that earns a zero nominal return) and other assets like bonds and stocks that earn the historically observed rates of return. Moreover, we estimate the structural parameters of a life cycle model that can replicate the observed demands and therefore we offer some guidance into how the structural model can be extended in the future to address interesting macroeconomic questions.

The other strand of the literature that the model relates to is the recent life cycle saving and portfolio choice literature (**?**, **?**, **?** and **?** to name some examples). In all these papers, however, the choice is between real assets (real bonds and real stocks) and therefore the effects of inflation on consumption, money demand and portfolio choices cannot be analyzed.

The rest of the paper is organized as follows. Section 2 generates some stylized facts with regards to money holdings over the life cycle. Section 3 presents the model, and Section 4 reports the estimated parameters. Section 5 presents the benchmark numerical results and Section 6 conducts several comparative statics. Section 7 investigates implications of mean inflation for aggregate money, consumption demand and wealth accumulation. Section 8 concludes.

2 Empirical Evidence on Life Cycle Asset Allocation and Participation in Different Asset Markets

We focus on using a single cross section, the 2001 Survey of Consumer Finances, for establishing certain stylized facts about the holdings of money in household portfolios over the life cycle. We have repeated the analysis below for all triennial surveys between 1989 and 2007 and we can report that the results from the 1998, 2001, 2004 and 2007 surveys are very similar along the dimensions we report below. Earlier surveys (the 1989 for example) feature lower stock market participation and higher shares of money in the portfolios. The rise of the equity culture in the 1990s is probably responsible for this change. This points towards having to come up with identifying assumptions to decompose the cross sectional results into age, time and cohort effects in this earlier period. Instead of following this approach, we compare the results across the 1998 and 2007 surveys and find that our stylized facts are robust both qualitatively and quantitatively across these four surveys. Cohort effects seem to be less important in this period and we therefore interpret the cross sectional evidence as life cycle implications a good monetary model will need to explain. We therefore leave to future work this decomposition that could be quite important in understanding the evolution of money demand in the last three decades.

In the data, most households have a liquid account to undertake their transactions. In the 2001 SCF 91% of all households had a transactions/liquidity account, 63% had a positive amount of bonds and 52% participated in the equity market (including participation through retirement plans). Recent work explains these facts using a fixed cost to prevent households from participating in the bond and stock market after opening a transactions account (see **?** and **?**, among others, for further exposition). We do not follow that approach in this paper but instead focus on the intensive margin because we view portfolio choices in the presence of money sufficiently interesting to warrant its own analysis.

One of the well-known stylized facts in the life cycle portfolio choice literature is that financial wealth is correlated with stock market participation (see **?** for a recent survey). We estimate the mean amount of financial wealth for households with no bonds or stocks but just liquid accounts. The mean amount of financial wealth for this group equals 4162 US$, whereas for the group that holds either bonds or stocks (and typically also holds a liquid account) mean financial wealth equals 260206 US$ illustrating the stark dichotomy between households that hold bonds and stocks and households that just hold transaction accounts. Table 1 reports the levels of financial assets across the two groups over five broad age categories (four during working life and one during retirement).

This table shows that poor households tend to hold just liquid balances in the form of money and deposits, while richer households tend to invest in higher return assets. We can also compute the mean asset allocations across money, bonds and stocks for the households that hold all three assets. We

Life Cycle Financial Wealth Accumulation

Age Group	Mean (Median) Wealth No Bonds/Stocks	Mean (Median) Wealth With either Bonds or Stocks
20-34	1454 (290)	69386 (13450)
35-45	3086 (400)	137095 (42100)
46-55	3333 (400)	296958 (72700)
56-65	4458 (400)	471997 (103400)
66-75	9249 (1000)	399217 (97350)

Table 1: Mean (median) financial wealth for the two main groups (bond/stockholders and households with only a transaction account) from the 2001 SCF data. The precise definitions for the different variables are in Appendix A.

find that the share of wealth in stocks is 37.6% (with a standard deviation of 34.4%), the share of wealth in money is 22.4% (with a standard deviation of 22.5%) and finally the share of wealth in bonds is 40% (with a standard deviation of 33.4%).

A second issue that is well known in the literature that comes out from Table 1 is the skewed distribution of financial wealth which affects the choices researchers need to make when bringing models to the data. In general, there are three main mechanisms being used to match the observed wealth distribution: heterogeneous discount rates (**?**), bequests (**?**), and a combination of bequests and entrepreneurship (**?**). These are general equilibrium models with a single asset, whereas we want to eventually solve a model with three different assets and different rates of return. Rather than complicating the model further we abstract from matching the wealth distribution exactly. Instead we focus on matching the ratio of mean financial wealth to mean labor income, the idea being that a general equilibrium model can be calibrated to match these magnitudes eventually, after the demands for different assets have been pinned down.[4] We leave again the more ambitious task of matching the wealth

[4] This is the approach advocated by **?**. Solving and understanding the intuition behind general equilibrium heterogeneous agent models is difficult. One useful (or intermediate) step

Life Cycle Financial Wealth Accumulation Relative to Mean Labor Income

Age Group	Mean Wealth / Mean Income No Bonds/Stocks	Mean Wealth/Income With either bonds or stocks
20-34	0.07	1.37
35-45	0.11	1.85
46-55	0.13	3.51
56-65	0.18	6.45
66-75	0.60	11.9

Table 2: Mean financial wealth relative to mean labor income for the two main groups (bond/stockholders and households with only a transaction account) from the 2001 SCF data. The definitions for the different variables are in Appendix A

distribution in the context of this monetary model to future work. The targets of the model estimation in Section 4 are given in table 2.

We next go deeper into the role of money in the household portfolio and how money allocations change over the life cycle. Table 3 reports (for households holding either bonds or stocks) the portfolio shares for money (α_m), bonds (α_b) and stocks (α_s) for the five age groups. The life-cycle profiles in Table 3 do not show any substantial variations, even though there is a small tendency for the share of wealth in money balances to decrease over the working life cycle and increase after retirement. What is immediately apparent, however, is that money are a key feature of the household portfolio, despite the rate of return dominance of other assets with all age groups devoting a substantial percentage of their financial wealth in money holdings.

We next sort the asset allocation decisions by age group and total financial wealth and report the results in Table 4. Table 4 illustrates how less wealthy households tend to allocate a larger fraction of their wealth in liquid balances and reduce this dependence as they get wealhier. For every age group the share of financial wealth allocated to liquid balances decreases as the household gets

involves matching the demand side holding asset returns exogenous and at their observed historical values before a general equilibrium model that matches the data is constructed.

Life Cycle Portfolio Choice

Age Group	α_m	α_b	α_s
20-34	27.7	33.2	39.1
35-45	21.6	36.0	42.4
46-55	18.9	41.3	39.8
56-65	18.6	41.7	39.7
66-75	25.0	48.0	27.0

Table 3: Mean shares of financial wealth allocated to money, bonds and stocks from the 2001 SCF data. The precise definitions for the different variables are in Appendix A.

wealthier. This decrease is primarily taken up by an increase in the share of wealth allocated to stocks since the share of wealth allocated to bonds tends to be more balanced and exhibits fewer changes. This stylized fact is a priori inconsistent with recent models of household portfolio choice, as pointed out by **?**. In recent models of household portfolio choice the young tend to be endowed with high human capital and to the extent that this is not correlated with the stock market the prediction is that younger households should be more heavily invested in the stock market. As they grow older (and financially richer), the share of wealth in stocks should decrease. **?** rely on explaining these facts through a clever use of the utility function and preferences across goods. We argue that the implicit assumption in recent household portfolio choice models that money and bonds are perfect substitutes is not innocuous. To understand money demand we argue for the need to build a motive for holding money and the need to include inflation explicitly in the model. This is the approach we take in constructing our model in the next section.

3 The Model

The model is a nominal version of life-cycle models that are extensively used in the household portfolio literature. Agents work while they are young, and

Life Cycle Portfolio Choice by Age and Financial Wealth

Age Group and Financial Wealth quartile	α_m	α_b	α_s
20-34 and One	35.5	30.0	34.5
20-34 and Two	25.3	35.1	39.6
20-34 and Three	15.6	36.5	47.9
20-34 and Four	10.7	40.6	48.7
35-44 and One	33.3	42.5	24.2
35-44 and Two	21.9	40.7	37.4
35-44 and Three	16.3	30.6	53.1
35-44 and Four	12.4	27.9	59.7
45-54 and One	34.5	46.0	19.5
45-54 and Two	23.0	46.7	30.3
45-54 and Three	14.2	44.6	41.2
45-54 and Four	10.7	31.2	58.1
55-64 and One	31.0	56.4	12.6
55-64 and Two	23.3	44.2	32.5
55-64 and Three	19.0	37.1	43.9
55-64 and Three	10.2	36.8	53.0
65 plus and One	43.0	52.5	4.5
65 plus and Two	34.8	57.0	8.2
65 plus and Three	21.9	56.8	21.3
65 plus and Four	13.9	34.1	52.0

Table 4: Mean shares of financial wealth allocated to money, bonds and stocks from the 2001 SCF data. The portfolio choice decision is sorted by the four quartiles of financial wealth and the five age groups. The definitions for the variables can be found in Appendix A.

receive a pension after retirement. They are subject to uninsurable labor income risk and borrowing constraints. There are three assets in the economy, money, bonds and stocks, and they are traded in nominal terms. In order to introduce money, we extend the model by introducing nominal assets and transaction frictions.

3.1 Preferences

Time is discrete and t denotes adult age which, following the typical convention in the literature, corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 (T) periods (age 100). The probability that a consumer/investor is alive at time $(t+1)$ conditional on being alive at time t is denoted by p_t ($p_0 = 1$). Finally, the consumer/investor has bequest motive.

Households have Epstein-Zin-Weil utility functions (?, ?) defined over one single non-durable consumption good. Let $C_{i,t}$ and $X_{i,t}$ denote respectively real consumption level and nominal wealth (cash on hand) of agent i at time t. Then the real cash on hand is defined as $X_{i,t}/P_t$ where P_t denotes the price level at time t. The preferences of household i are defined by

$$V_{i,t} = \left\{ (1-\beta)C_{i,t}^{1-1/\psi} + \beta \left(E_t \left[p_t V_{i,t+1}^{1-\rho} + (1-p_t) b (X_{i,t+1}/P_{t+1})^{1-\rho} \right] \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}} \tag{1}$$

where ρ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution, β is the discount factor, and b determines the strength of the bequest motive.

3.2 Labor Income Process

Following the standard specification in the literature, the labor income process before retirement is given by

$$Y_{i,t} = Y_{i,t}^{p} U_{i,t} \tag{2}$$

$$Y_{i,t}^{p} = \exp(f(t, Z_{i,t})) Y_{i,t-1}^{p} N_{i,t} \tag{3}$$

where $f(t, Z_{i,t})$ is a deterministic function of age and household characteristics $Z_{i,t}$, $Y_{i,t}^{p}$ is a permanent component with innovation $N_{i,t}$, and $U_{i,t}$ a transitory component. We assume that $\ln U_{i,t}$ and $\ln N_{i,t}$ are independent and identically distributed with mean $\{-.5 * \sigma_u^2, -.5 * \sigma_n^2\}$, and variances σ_u^2 and σ_n^2, respectively. The log of $Y_{i,t}^{p}$ evolves as a random walk with a deterministic drift, $f(t, Z_{i,t})$. For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period K, corresponding to age 65 ($K = 46$). Earnings in retirement ($t > K$) are given by $Y_{i,t} = \lambda Y_{i,K}^{p}$, where λ is the replacement ratio (a scalar between zero and one).

Due to the absence of empirical studies at the micro level that estimate separate processes for stockholders and non-stockholders, we use the fact (surveyed for instance in **?**) that financial wealth is correlated with stock market participation and education is correlated with both. We therefore use a different labor income parametrization depending on the household's education. We provide further details in the calibration section.

3.3 Financial Assets and constraints

The agent has options to hold three kinds of assets: fiat money ($M_{i,t}$) , nominal bonds ($B_{i,t}$) and nominal stocks ($S_{i,t}$). As in the standard literature, let $X_{i,t}$ be nominal "cash on hand" that the agent can use for consumption and portfolio decisions. The budget constraint is given by

$$X_{i,t} = P_t C_{i,t} + S_{i,t} + B_{i,t} + M_{i,t}. \tag{4}$$

In order to motivate money holdings, we assume transaction frictions. Our approach is related to shopping time models, first proposed by **?**, but we modify them to more easily incorporate in the portfolio choice literature. In shopping time models, transaction costs are modeled in terms of foregone time: money can help reduce transaction time. As is shown in **?**, there is a connection between the shopping time models and the inventory-theoretic studies of money (**?**,**?**).[5] More broadly speaking, the transaction cost can include not only a shopping cost but also a cost of selling illiquid assets to finance consumption. Different versions assume different trade-offs in the presence of transactions frictions. For example, **?** assumes that agents face a trade-off between hours spent on production and transactions. **?** (Ch. 24) assume a trade-off between transaction time and leisure.

In this paper, we model transaction costs as a direct physical cost in terms of consumption goods. An advantage of our approach is that we can treat money by exactly the same way as we treat bonds and stocks because there is no additional margin between money holding decisions and leisure (or labor supply) decisions. Therefore our model maintains the basic structure of the models used in the portfolio choice literature, making the model computationally tractable and making its results easily comparable to those obtained in the literature. Also, our modeling approach maintains the basic properties of the shopping time models — money demand will be increasing in consumption

[5] See **?** for recent developments.

and decreasing in nominal interest rates.

Let $H_{i,t}$ denote the transaction cost in terms of consumption goods associated with consumption expenditure at time t. We assume that this cost is deducted at the beginning of the next period. This timing assumption ensures that $X_{i,t}$ is a state variable, as in the portfolio choice literature. Then, the evolution of $X_{i,t}$ is given by

$$X_{i,t+1} = R^s_{t+1} S_{i,t} + R^b_{t+1} B_{i,t} + M_{i,t} + P_{t+1} Y_{i,t+1} - P_{t+1} H_{i,t}, \tag{5}$$

where R^s_{t+1} and R^b_{t+1} respectively denote the nominal returns of stocks and bonds. Note that the nominal return of fiat money is unity. Finally, $Y_{i,t+1}$ is real income at time $t+1$ that is discussed in Section 3.2.

We assume that the transaction cost function is given by

$$H_{i,t} = H(C_{i,t}, Z_{i,t}; \xi_{i,t}), \quad H_c > 0, \quad H_z < 0, \tag{6}$$

where $Z_{i,t}$ represents the "liquid" part of household cash on hand, and $\xi_{i,t}$ is a vector of exogenous variables that affect the cost. We assume that $H_{i,t}$ is homogeneous of degree one. This assumption ensures that the size of the shopping cost relative to the household's consumption does not go zero as the households accumulate financial wealth. We assume that $Z_{i,t}$ is given by

$$Z_{i,t} = M_{i,t-1} - P_t H_{i,t-1}. \tag{7}$$

Finally, as in the portfolio choice literature, we prevent households from borrowing against their future labor income. More specifically we impose the following restrictions:

$$B_{i,t} \geq 0$$

$$S_{i,t} \geq 0$$

$$M_{i,t} \geq 0$$

We have two state variables: $X_{i,t}$ and $Z_{i,t}$, and the control variables are $C_{i,t}$, $M_{i,t}$, $S_{i,t}$ and $B_{i,t}$.

3.4 Normalizing by Prices and Growth

Let lower case letters denote real variables normalized by the permanent component of labor income ($Y^p_{i,t}$). For example the normalized real cash on hand is defined as $x_{i,t} = X_{i,t}/(Y^p_{i,t}P_t)$. Similarly, the normalized transaction cost is $h_{i,t} \equiv H_{i,t}/Y^p_{i,t}$ and so on. The evolution of the state variables is given by

$$x_{i,t+1} = \frac{r^s_{t+1}}{g_{i,t+1}} s_{i,t} + \frac{r^b_{t+1}}{g_{i,t+1}} b_{i,t} + \frac{r^m_{t+1}}{g_{i,t+1}} m_{i,t} + y_{i,t+1} - \frac{h_{i,t}}{g_{i,t+1}}, \tag{8}$$

where

$$r^s_{t+1} \equiv R^s_{t+1}\pi^{-1}_{t+1}, \quad r^b_{t+1} \equiv R^b_{t+1}\pi^{-1}_{t+1}, \quad r^m_{t+1} \equiv \pi^{-1}_{t+1}$$

are respectively the real returns of stocks, nominal bonds and money, where $\pi_{t+1} \equiv P_{t+1}/P_t$ denotes gross inflation, and $g_{i,t+1} \equiv Y^p_{i,t+1}/Y^p_{i,t}$ is the gross growth rate of permanent income. Similarly, $z_{i,t}$ evolves according to

$$z_{i,t+1} = \frac{r^m_{t+1}}{g_{i,t+1}} m_{i,t} - \frac{h_{i,t}}{g_{i,t+1}}. \tag{9}$$

3.5 Normalised recursive utility

Let $v_{i,t} \equiv V_{i,t}/Y^p_{i,t}$ be normalised value, and $g_{i,t+1} \equiv Y^p_{i,t+1}/Y^p_{i,t}$. Then, by dividing both sides of equation (1) by $Y^p_{i,t}$, we obtain

$$\begin{aligned} v_{i,t} = & \Bigg[(1-\beta)c_{i,t}^{1-1/\psi} \\ & + \beta\left\{E_t\left[p_t(v_{i,t+1})^{1-\rho}(Y^p_{i,t+1}/Y^p_{i,t})^{1-\rho} + (1-p_t)b\frac{(x_{i,t+1})^{1-\rho}(Y^p_{i,t+1}/Y^p_{i,t})^{1-\rho}}{1-\rho}\right]\right\}^{\frac{1-1/\psi}{1-\rho}}\Bigg]^{\frac{1}{1-1/\psi}} \\ = & \left[(1-\beta)c_{i,t}^{1-1/\psi} + \beta\left\{E_t\left[p_t(v_{i,t+1}g_{i,t+1})^{1-\rho} + (1-p_t)b\frac{(x_{i,t+1}g_{i,t+1})^{1-\rho}}{1-\rho}\right]\right\}^{\frac{1-1/\psi}{1-\rho}}\right]^{\frac{1}{1-1/\psi}}. \end{aligned} \tag{10}$$

The two states are $x_{i,t}$ and $z_{i,t}$ and their evolutions are given by (8) and (9).

3.6 Specification of transaction technology

In the benchmark simulation we assume that

$$H_{i,t} = \varepsilon Y^p_{i,t}\frac{C_{i,t}}{Z_{i,t}}, \quad \varepsilon > 0. \tag{11}$$

In this case, $h_{i,t}$ is given by

$$h_{i,t} = \varepsilon\frac{c_{i,t}}{z_{i,t}}, \quad \varepsilon > 0. \tag{12}$$

Our preferred interpretation is that the transaction cost represents an opportunity cost of time and is therefore proportional to the permanent component of labor income. The functional form (11) is consistent with **?** who shows that the implied money demand function is consistent with the demand function of **?** and **?**. In our model the opportunity cost maps into monetary units as specified in (11). Parameter ε measures the severity of transaction frictions. A large ε means it takes more resources to do transactions and it can be different over the life cycle or across agents. For example, older people may have more spare time to undertake transactions, therefore having a smaller ε compared with the young. Another example might be educated households that have better ability to manage nonmonetary assets, also having a smaller ε.

3.7 Specification of aggregate exogenous processes

We will use exogenous processes for stock and bond returns, inflation and the aggregate component of labor income. Given that we calibrate the cross sectional model to decisions taken in 2001, we use the period 1995 to 2008 to compute descriptive statistics and correlations between these variables and provide comparative statics experiments later on based on historical experience.

4 Parameter Estimation

We will estimate the preference parameters for the rich households (the stockholders). Given the large number of parameters in the model we will calibrate certain parameters and then estimate the preference and shopping cost parameters. The calibration for labor income uses the estimates in **?** so that $\sigma_u = 0.1$, $\sigma_n = 0.08$, and $\lambda = 0.68$. Given the positive correlation between education, financial wealth and the probability to participate in the stock market, we use the hump shape process for households with a college degree from **?**.

We use annual CRSP data for the U.S. from 1926 to 2008 for inflation, stock returns, long and short bond returns. Given that we estimate a cross sectional model based on 2001 SCF data we focus on the returns and correla-

Means and Standard Deviations

Variable	Mean	S. D.
Inflation	2.5	1.0
Bond Returns	2.4	2.6
Stock Returns	6.8	22.0
Wage growth	2.7	2.0

Table 5: We report the means and standard deviations (S.D.) of key inputs in the decision model. All variables are real, and the bond return is the return on the one-year bond. Details about the data can be found in Appendix A.

Correlation Matrix

Variable	Inflation	Bond Returns	Stock Returns	Wage Growth
Inflation	1.0	-0.49	0.25	-0.06
Bond Returns		1.0	-0.1	0.37
Stock Returns			1.0	0.44
Wage growth				1.0

Table 6: We report the correlation matrix of key inputs in the decision model. All variables are real, and the bond return is the return on the one-year bond. This is for the period between 1995 and 2008. Details about the data can be found in Appendix A.

tions from 1995 to 2008 but provide extensive comparative statics with regards to the main parameters to reflect other historical episodes with different return characteristics. The table below reports the descriptive statistics for the variables of interest.

We also assume an i.i.d process for stock returns with a mean real return equal to six percent and a standard deviation equal to 22%. The bond return process is similarly calibrated with a mean return equal to two percent and a standard deviation equal to three percent.

We also need to take a stance on the correlations across these variables. The correlations are set according to Table 6 from the 1995-2008 correlations in the data. Based on this table, and for this period, we set the correlations between bond and stock returns equal to zero, as well as the correlation between inflation and the real wage growth.

To use the method of simulated moments we need to decide which moments to match. The key variables of interest for our purposes are the mean holdings of financial wealth over the life cycle and the asset allocations between money, bonds and stocks sorted by age and financial wealth. For the rich households that find it optimal to participate in all asset markets, we pick the structural parameters to minimize the distance between five moments of wealth from the simulated model and the same five moments reported in 2. At the same time we sort financial portfolios for money, bonds and stocks by age and financial wealth and match the simulated data to the ones reported in 4. This gives a total of forty five moment conditions. The structural parameters are $\{b, \psi, \rho, \varepsilon_w, \varepsilon_r\}$ where ε_w denotes the shopping cost for workers and ε_r the one for retirees.[6]

5 Results

The estimated parameters for the households that participate in at least one market other than the money market are given in table 7.

The results are consistent with previous estimates of preference parameters that exist in the literature. A relatively high risk aversion is needed to generate

[6] We provide estimates of the structural parameters using Method of Simulated Moments Estimator (MSM) of **?**. The structural parameters collected in a vector $\hat{\theta}$ are determined as:

$$\hat{\theta} = Argmin_{\theta} D' S^{-1} D.$$

Let Y_t and $\tilde{Y}_t$ denote the observations at time t of the actual and simulated endogenous variables, respectively. Let T be the sample size of the observed series whereas $T \cdot H$ data points are simulated to compute moments from the structural model. For the latter, let $Y_{[T]}$ and $\tilde{Y}_{[TH]}$ denote the vectors of actual and simulated endogenous variables of length T and TH, respectively. We have:

$$D = \left(\frac{1}{T} \sum_{t=1}^{T} moments(Y_t) - \frac{1}{TH} \sum_{t=1}^{TH} moments(\tilde{Y}_t) \right).$$

where moments() denotes a particular moment. The asymptotically efficient optimal weighting matrix S^{-1} equals the inverse of the variance-covariance matrix of the data. Following Appendix B in **?**, we use a diagonal weighting matrix for S^{-1} with the elements along the diagonals being the variance of each moment from the data.

Estimated Structural Parameters for the Rich		
Parameter	Estimate	Standard error
b	0.1	
ψ	0.6	
ρ	6.0	
ε_w	0.005	
ε_r	0.0125	

Table 7: Estimated structural parameters for the rich households.

balanced portfolios between bonds and stocks given the high equity premium, while the EIS at 0.6 is consistent with the estimated parameters in **?**. There is some evidence for a bequest motive needed because financial wealth is not fully decumulated during retirement, while there are no micro estimates of the shopping cost parameters against which we can compare our results (this was also one of the reasons for performing structural estimation). The implied shopping cost varies between 0.5 and 2.0 percent of mean annual labor income that we view as a reasonable transaction cost and is consistent with **?**.

What are the policy functions and life cycle profiles implied by these parameter estimates? Figure 1 shows some policy functions of portfolio choice against the two continuous state variables (money holdings z and cash on hand x) for the young (age 25), middle-aged (age 55) and retirees (age 85). The vertical axis plots portfolio shares as a percentage of financial wealth invested in each asset (between zero and one due to the no borrowing/no short sale constraints). The three age groups mainly hold money when their cash on hand is small. Especially, the young and the middle-aged agents invest almost their entire assets in money when they are poor. This is consistent with the data that shows that the young and poor agents tend to hold more money. In line with the portfolio literature, the bond share is increasing in cash on hand while the stock share is decreasing in cash on hand.

Figure 2 shows the simulated paths of consumption, financial wealth and income over the life cycle. We simulate the model economy with 1000 individuals starting with zero financial wealth and take the mean of each variable. As discussed above, there is some overaccumulation in wealth levels relative to the financial wealth present in the data. Finally, Figure 3 shows simulated portfolio choice over the life cycle. Consistently with Figure 1, the young hold mainly money, and the money share decreases as the agents become older. At the same time they gradually start investing in stocks but not in bonds. As they get older they finally invest in bonds. It should be noted that the share of wealth allocated to stocks increases for the youngest age group as households get richer. Liquidity needs are stronger early in life when financial wealth is low and therefore money figures prominently in the portfolio to minimize shopping costs. As financial wealth begins to be accumulated, the household begins investing in stocks due to the equity premium and therefore the share of wealth in transaction accounts decreases.

How do the predicted moments compare with the actual ones? We first go through the mean wealth to mean labor income ratios which are given in table 8. We observe that the model predicts some overaccumulation in wealth levels relative to the financial wealth present in the data. We will provide robustness checks of these predictions by adjusting the discount rate to allow lower wealth accumulation in future work.

We next present the moments for the portfolio shares. The results illustrate the strong demand for stocks early in life as labor income is mostly seen like a riksless asset. Nevertheless, money is held in the portfolio for transaction purposes, thereby dramatically changing the composition of the portfolio relative to other models in the portfolio choice literature that lump money and bonds in the same category. Specifically, in these models the standard prediction is that stockholders should allocate all financial wealth in stockholding while

Predicted vs Actual Life Cycle Financial Wealth to Labor Income		
Age Group	Mean Wealth/Mean Income Data	Mean Wealth/Mean Income Predicted Moments
20-34	1.37	1.86
35-45	1.85	4.57
46-55	3.51	8.38
56-65	6.45	12.78
66-75	11.9	19.76

Table 8: Actual versus predicted moments for mean financial wealth relative to mean labor income for the bond/stockholders. The model is compared to the 2001 SCF data. The definitions for the different variables are in Appendix A

in this setup the very young (ages 25-34) allocate between 65 to 80 percent of their financial wealth in stocks. The allocation to bonds is still underpredicted relative to the data but we view the model as getting one step closer to matching observed behavior in the data.

6 Comparative Statics

6.1 No money $\varepsilon_w = \varepsilon_r = 0$

To understand the predictions of the model better, we next perform a series of comparative statics. The first model we can compare our results to is the standard portfolio choice model where money does not circulate. In our model this specification is nested by setting the shopping technology parameters equal to zero. Figure 4 shows the prediction of the model in which the shopping parameter ε is set to zero. This is an interesting case because money does not circulate in this economy and the model becomes identical with the recent models on household portfolio choice like **?** or **?**. For comparison purposes Figure 4 also shows the benchmark case with a dotted line. We can see that that the models that treat money and bonds as perfect substitutes generate

Predicted and Actual Moments	Actual Moments			Predicted Moments		
Age Group and Financial Wealth quartile	α_m	α_b	α_s	$\widehat{\alpha}_m$	$\widehat{\alpha}_b$	$\widehat{\alpha}_s$
20-34 and One	35.5	30.0	34.5	34.9	0.0	65.1
20-34 and Two	25.3	35.1	39.6	39.5	0.0	60.5
20-34 and Three	15.6	36.5	47.9	24.4	0.0	75.6
20-34 and Four	10.7	40.6	48.7	20.1	0.0	79.9
35-44 and One	33.3	42.5	24.2	19.5	0.1	80.4
35-44 and Two	21.9	40.7	37.4	20.0	1.5	78.5
35-44 and Three	16.3	30.6	53.1	20.0	22.7	57.3
35-44 and Four	12.4	27.9	59.7	20.0	34.1	45.9
45-54 and One	34.5	46.0	19.5	20.0	39.5	40.5
45-54 and Two	23.0	46.7	30.3	18.5	41.7	39.8
45-54 and Three	14.2	44.6	41.2	19.2	45.2	35.6
45-54 and Four	10.7	31.2	58.1	20.0	57.8	22.2
55-64 and One	31.0	56.4	12.6	20.0	51.2	28.8
55-64 and Two	23.3	44.2	32.5	19.7	59.5	20.8
55-64 and Three	19.0	37.1	43.9	16.7	63.3	20.0
55-64 and Four	10.2	36.8	53.0	19.6	60.4	20.0
65 plus and One	43.0	52.5	4.5	35.7	44.2	20.1
65 plus and Two	34.8	57.0	8.2	28.4	50.2	21.4
65 plus and Three	21.9	56.8	21.3	20.5	58.5	21.0
65 plus and Four	13.9	34.1	52.0	20.0	60.5	19.5

Table 9: Predicted versus Actual mean shares of financial wealth allocated to money, bonds and stocks. Data are from the 2001 SCF. The portfolio choice decision is sorted by the four quartiles of financial wealth and the five age groups. The definitions for the variables can be found in Appendix A.

a large demand for stocks early in life because future labor income is treated like a bond: all saving is done through the stock market. On the other hand, the shopping technology model generates a demand for money that generates an upward sloping share of wealth in stocks over the early years of the lifecycle and also reduces substantially the demand for stocks early in life.

The model without money clearly predicts that the share of wealth in stocks decreases as financial wealth rises, a counterfactual prediction. Recently **?** argue that non-separabilities in the utility function across different goods can generate the upward sloping shape for the share of wealth in stocks as financial wealth increases. Our model provides an alternative explanation that relies on the determinants of money demand and treating transaction accounts and bond investments as assets with different risk/return characteristics.

6.2 Effects of mean inflation

Figure 5 shows the results when mean inflation is substantially increased (we set annual inflation equal to ten percent). A high mean inflation decreases the mean rate of return of holding money, and as a result, households reduce money holdings, which is in line with the money demand literature. Interestingly, Figure 5 shows that the portfolio choice of young agents is particularly affected by inflation, but older agents do not change their portfolio choice significantly. This arises because older agents have more predictable streams of income and therefore their money holdings are stable as a proportion of their financial wealth. Younger households, however, devote a higher share of their financial wealth in liquid balances and therefore are more keen to reallocate out of money in the presence of higher inflation. Another interesting implication of the model is that younger households hedge inflation by reallocating their money holdings towards stocks rather than bonds, in contrast to the money demand literature that focuses on the choice between money and bonds.[7]

We next examine the implications of Friedman's rule on saving and portfolio choices. Friedman's rule is defined as a zero mean nominal interest rate, which implies a mean inflation of -2.4% given our choice of the mean real return on bonds of 2.4%. For computational reasons we report the case in which mean inflation is -2.3%, which is shown in Figure 6. In this case, the mean returns on bonds and money become almost identical. As is shown in Figure 6, the demand for money significantly increases, in line with the literature. What is striking is the fact the bonds are almost perfectly substituted out by money. Given the large increase in the share of wealth allocated to money generated by the deflationary expectations, there is a substantial decrease in the demand for stock by younger cohorts who (otherwise) hold most money balances.

6.3 Hedging Demands

We next examine how various volatilities and correlations of shocks affect portfolio choice over life cycle. When mean inflation is low at 2.5% as in the benchmark case, we find that increasing inflation volatility has little effects on portfolio choice. More specifically, increasing the standard deviation of inflation by 5 percentage points, even with mean inflation at 2.5% does not have significant effects of household behavior.

We also examine the effects of changing the correlation of inflation with bonds and stock returns through two experiments. First we set the correlation of inflation with stock returns equal to zero, keeping the other correlations the same as in the benchmark case. In the second, we set the correlation of inflation with bond returns equal to zero. Both experiments show that those

[7] This is also related with the so-called Tobin effect that argues that inflation has potential of inducing capital accumulation because agents substitute money for stocks. It would be interesting to analyze this prediction more fully in a general equilibrium setting.

correlations have negligible effects. Overall, we found that when inflation is low at 2.5%, hedging demands due to inflation volatility are very small. One hypothesis might be that hedging demands due to volatile inflation may be important only when inflation is high. Data show that inflation volatility is positively correlated with the mean inflation rate. For this purpose, we consider the economy with a mean inflation rate at 10%, and examine the effects of higher inflation volatility in this regime. Comparing the cases in which the standard deviation of inflation is 1% and 5% respectively barely affects portfolio choice. In the interest of space we do not report those figures.

7 Implications for Aggregate Money Demand, Consumption and Wealth

What is the partial equilibrium relationship between money demand and the nominal interest rate on bonds? Figure 7 shows aggregate money demand, consumption and wealth accumulation against different values of the mean nominal interest rate.[8] Consistent with the literature, aggregate money demand is decreasing in the nominal interest rate, and increases rapidly as the nominal rate approaches zero. Note that in this figure we do not keep aggregate consumption constant. Therefore, the change in money demand comes both from the increase in the nominal interest rate and from endogenous changes in consumption. Aggregate consumption also decreases as the nominal rate increases. Our model implies that aggregate consumption demand decreases by some 6% as the nominal rate increases from zero to 12.4%. This is due to the decrease in aggregate wealth, which is also shown in the figure. Aggregate

[8] We calibrate the mean net real return on bonds as 2.4%, so the mean net nominal interest rate shown in the Y-axis of Figure 7 is the mean net inflation plus 0.024.

wealth accumulation decreases by the same magnitude as consumption. The decrease in wealth is much larger than the increase in the shopping cost. Even when the nominal interest rate is 12.4%, the flow shopping cost per permanent income is on average 1.78%.[9] In **?**, the shopping cost per income when the nominal interest rate is 12.4% can be computed as 1.76%, which is very similar to ours. Also, unlike the standard shopping time models, we keep the labor income process exogenously given. Therefore the decrease in wealth is mainly due to the effects of inflation on agents' consumption-saving and portfolio decisions. As mentioned in Section 6, the young substitutes money for stocks when inflation rises. Therefore the wealth of the young does not necessarily decrease when inflation rises. However, the portfolio choice of the old is relatively insensitive to inflation unless it is close to the Friedman's rule. Then the decrease in real return of money due to high inflation decreases the overall return of their portfolio and hence decreases their wealth accumulation and consumption.

8 Conclusion

We estimate the preference parameters of a life cycle money demand and portfolio choice model. The predictions of the model are consistent with the data and the model can be therefore used to analyze how inflation or deflation affects money demand and asset allocation. While we focus on households that hold all three types of assets, our finding in Section 2 implies that the poorer households hold only money in their financial portfolio. In future work we plan to model the extensive margin to analyze the behavior of these households. Future work can also extend the analysis in a general equilibrium setting

[9] On the other hand, when the nominal interest rate is 0.1%, the mean shopping cost per permanent income in our model is 0.23%.

to address policy questions such as the effects of changes in the relative supply of money and bonds through open market operations by the central bank. Relatedly, a general equilibrium model will be more useful in computing the welfare cost of inflation across different households, and in the aggregate.

Appendix A The Data

A.1 Survey of Consumer Finances

We use repeated cross sections from the U.S. Survey of Consumer Finances to establish certain robust facts with regards to household choices across liquid accounts (money), bonds and stocks. Total financial assets are broken up into the three broad categories the model has implications for: liquid resources (LIQ), stock (EQUITY) and nonequity (BOND) investments. In the 2001 public extract of the SCF data set, LIQ is defined as the sum of all checking, saving, money market deposit and call accounts. We follow the same convention and LIQ becomes our measure of money when confronting the model implications to the data. EQUITY is defined in the same extract as all financial assets invested in stocks and this comprises the following categories:

1) directly held stock

2) stock mutual funds (the full value is assigned if the fund is described as a stock mutual fund, and half the value for combination mutual funds)

3) IRAs/Keoghs invested in stock (full value if mostly invested in stock, half value if split between stocks/bonds or stocks/money market, one third value if split between stocks/bonds/money market),

4) other managed assets with equity interest (annuities, trusts, MIAs) (where again the full value is used if mostly invested in stock, half value if split between stocks/MFs & bonds/CDs, or "mixed/diversified," and one third

value if "other")

5) thrift-type retirement accounts invested in stock (full value if mostly invested in stock and half value if split between stocks and interest earning assets) and

6) savings accounts classified as 529 or other accounts that may be invested in stocks. We classify the remaining financial assets as BOND and interpret them as capturing the bond investments in the model (both government and corporate bonds are lumped together in this category).

A.2 Aggregate Data

We used the CRSP data base to download annual US inflation, bond and stock returns from 1925 to 2008. We report empirical results for long and short bond yields in the paper. More details.

For the aggregate component of labor income we use the NIPA wages and salary disbursement series and we deflate using the inflation rate from CRSP.