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Alternative Approaches to Commercial Property Price Indexes for Tokyo

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Abstract

The paper studies the problems associated with the construction of price indexes for commercial properties that could be used in the System of National Accounts. Property price indexes are required for the stocks of commercial properties in the Balance Sheets of the country and related price indexes for the land and structure components of a commercial property are required in the Balance Sheet accounts of the country for the calculation of the Multifactor Productivity of the Commercial Property Industry. The paper uses a variant of the builder's model that has been used to construct Residential Property Price Indexes. Geometric depreciation rates are estimated for commercial offices in Tokyo using assessment data for REITs. The problems associated with the decomposition of asset value into land and structure components are addressed. The problems associated with depreciating capital expenditures on buildings and with measuring the loss of asset value due to early retirement of the structure are also addressed.

Key Words:
Commercial property price indexes, System of National Accounts, Balance Sheets, methods of depreciation, land and structure price indexes, demolition depreciation.

Journal of Economic Literature Classification Numbers:
C2, C23, C43, D12, E31, R21.

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1 Introduction

In this paper, we will use quarterly data on the performance of 50 Real Estate Income Trusts (REITs) that have single location commercial office buildings in Tokyo. The period covered is the first quarter of 2007 through the second quarter of 2012 or 22 quarters in all. We will make use of the quarterly assessed value information that is required for REIT properties and treat these end of quarter assessed property values as approximations to the beginning of the quarter market value of the properties. In addition to assessed value information, we also have information on the age of the building, the floor space area of the structure and the area of the land plot. We also have data on some other characteristics of the property but for this paper, we will only use information on assessed values, age of structure, floor space area, land space area and two other variables: quarterly capital expenditures on the property and an exogenous construction price index for the construction of new office buildings in Tokyo.

Our goal is to obtain not only an overall *commercial property price index* for this group of 50 properties but to have a decomposition of the overall index into structure and land components. This decomposition is required in order to construct industry balance sheets and to measure the Total Factor Productivity of a commercial building.

In section 2, we briefly describe our data set.

In section 3, we construct our first (overall) price index that requires only information on assessed values of the properties. This index is conceptually flawed because it does not take into account depreciation of the building or capital expenditures that have been made to the property. However, as we shall see, this very simple index does provide a useful approximation to a more accurate index.[*1]

In section 4, we develop an overall price index and component subindexes for capital expenditures, the basic structure and the land area of the properties using the same type of techniques that national income accountants use to construct estimates of the capital stock. "Reasonable" assumptions about the form of structure depreciation are required in order to implement this method.

In section 5, we try out the traditional approach to hedonic regressions where the logarithm of the selling price of a property is the dependent variable and the various characteristics of the property are used as explanatory variables. For our hedonic regressions, we use property assessed values in place of selling prices.

In sections 6 and 7, we move away from the traditional hedonic regression approach and use assessed value as the dependent variable (in place of the logarithm of assessed value) and we are able to decompose overall value into separate land and structure components. In section 6, we use a geometric model of structure depreciation where there is only one constant over time depreciation rate that is estimated by the hedonic regression. In section 7, we generalize this model to allow for changing geometric depreciation rates as the building ages.

The models described in sections 6 and 7 provide a decomposition of the assessed value of a commercial property into the sum of a land plot value plus the value of the structure. However, our sample of properties includes only properties where the same structure continued to exist throughout the sample period. Our models capture the decline in structure value throughout the sample period but they do not capture the (unanticipated) depreciation of structures that

[*1] This index is an assessed value counterpart to a repeat sales index, which also suffers from the same conceptual problems.

are prematurely demolished during the sample period. This unanticipated decline in structure asset value needs to be estimated separately. In section 8, we show how this can be done with the help of historical data on the demolition of commercial office structures in Japan.

Section 9 concludes.

2 The Tokyo REIT Data

This paper uses published information on the Japanese Real Estate Investment Trust (REIT) market in the Tokyo area.[*2] We used a balanced panel of observations on 50 REITs for 22 quarters, starting in Q1 of 2007 and ending in Q2 of 2012. The variables that were used in this paper were V, the assessed value of the property;[*3] CE, the quarterly capital expenditures made on the property during the quarter; L, the area of the land plot in square meters (m^2); S, the total floor area of the structure in m^2 and A, the age of the structure in quarters. V and CE were reported in yen. In order to reduce the size of these variables, we divided by one million so the units of measurement for these financial variables is in millions of yen. The basic descriptive statistics for the above variables are listed in Table 1 below.[*4]

Table. 1 Descriptive Statistics for the Variables

Name	No. of Obs.	Mean	Std. Dev.	Minimum	Maximum
V	1100	4984.8	3417.8	984.3	18600.0
S	1100	5924.8	3568.1	2099.0	18552.0
L	1100	1106.3	718.2	294.5	3355.0
A	1100	83.9	25.2	16.7	156.7
CE	1100	6.08	11.94	0.06	85.49

Thus over the sample period, the sample average assessed value of the properties was approximately 4985 million yen, the average structure area was $5925m^2$, the average lot size was $1106m^2$, the average age of the structure was 84 quarters or 21 years and the average quarterly capital expenditure was about 6 million yen.

There were fairly high correlations between the V, S and L variables. The correlations of the selling price V with structure and lot area S and L were 0.725 and 0.532 respectively and the correlation between S and L was 0.840. Given the large amount of variability in the data and the relatively high correlations between V, S and L, we can expect multicollinearity problems in a simple linear regression of V on S and L.[*5]

In order to eliminate the multicollinearity problem between the lot size L and floor space area S for an individual REIT property when running hedonic regressions in later sections, we will assume that the value of a new structure in any quarter is proportional to a Construction

[*2] REIT data were supplied by MSCI-IPD, Japan. The authors thank Toshiro Nishioka and Hideaki Suzuki for their assistance.

[*3] The REITs were chosen so that each REIT consisted of a single commercial property located somewhere in Tokyo. The assessed values are reported at the end of each quarter. However, the actual assessments take place either during the quarter or prior to it. We will regard the published assessed values as approximations to the true market values of the property as of the beginning of the relevant quarter.

[*4] Additional variables were made available to us such as (quarterly) net operating income, property taxes, rentable floor space, number of basement floors, number of above ground floors and the distance to the Tokyo main station. We did not use these property characteristics in the present paper.

[*5] See Diewert, de Haan and Hendriks (2011a)[13] (2011b)[14] for evidence on this multicollinearity problem for house prices using Dutch data.

Cost Price Index for Tokyo.[*6] In order to approximate beginning of the quarter values for this construction cost index, we will lag the official index by one quarter. In section 4 below, we also will require an estimate of construction cost per square meter for the first quarter in our sample. We obtained a starting value for construction cost at the beginning of our sample period from a commercial provider of data, Turner and Townsend (2012)[35].[*7]

3 The Asset Value Price Index for Commercial Properties in Tokyo

Denote the estimated asset value for REIT n during quarter t by V_{tn} for $t = 1, ..., 22$ and $n = 1, ..., 50$ where $t = 1$ corresponds to the first quarter of 2007 and $t = 22$ corresponds to the second quarter of 2012. If we ignore capital expenditures and depreciation of the structures on the properties, each property can be regarded as having a constant quality over the sample period.[*8] Thus each property value at time t for REIT n, V_{tn}, can be decomposed into a price component, P_{tn}, times a quantity component, Q_{tn}, which can be regarded as being constant over time. We can choose units of measurement so that each quantity is set equal to unity. Thus the price and quantity data for the 50 REITs has the following structure: $Q_{tn} \equiv 1$; $P_{tn} = V_{tn}$ for $t = 1, ..., 22$ and $n = 1, ..., 50$. The *asset value price index* for period t for this group of REITs is the following Lowe (1823)[27][*9] index:

$$P_A^t \equiv \frac{\sum_{n=1}^{50} P_{tn} Q_{1n}}{\sum_{n=1}^{50} P_{1n} Q_{1n}} = \frac{\sum_{n=1}^{50} V_{tn}}{\sum_{n=1}^{50} V_{1n}}; \qquad t = 1, ..., 22. \tag{1}$$

Thus the asset value price index for period t is simply the total asset value for the 50 REITs in period t divided by the corresponding total asset value for sample period 1. The series P_A^t is graphed in Figure 1 in the following section and the series is listed in Table 2 in the Appendix.

This index is very much analogous to a *repeat sales index*,[*10] except instead of using actual sales of properties, the index uses the assessed values for the properties that are supplied by professional assessors.

There are three major problems with the assessed value price index:

- The index relies on assessed values for the properties and there is some evidence that assessed values are smoother and lag behind indexes that are based strictly on sales at market values;[*11]

[*6] This index, denoted as P_{St} for quarter t, was constructed by the Construction Price Research Association which is now an independent agency but prior to 2012 was part of the Ministry of Land, Infrastructure, Transport and Tourism (MLIT), a ministry of the Government of Japan. The quarterly values for this index are listed in Table 2 in the Appendix; see the listing for the variable P_S. The quarterly values were constructed from the Monthly Commercial Construction Cost index for Tokyo for reinforced concrete buildings.

[*7] On page 20 of Turner and Townsend (2012)[35], the 2011 construction cost for a prestige CBD office in Japan is listed as $303,800$ yen per m^2. Since construction prices in 2011 were very close to construction prices in 2007, in section 4 we will assume that the construction cost of a new commercial office was approximately $300,000$ yen per m^2 at the start of our sample period.

[*8] We are also ignoring changes in the amenities around the property over the sample period.

[*9] A Lowe index is a fixed basket price index where the quantity basket remains fixed over the sample period.

[*10] The Repeat Sales Method for measuring property prices dates back to Bailey, Muth and Nourse (1963)[1]. See Shimizu, Nishimura and Watanabe (2010)[32] for a comparison of the Repeat Sales Method and hedonic regression methods. Clapp and Giaccotto (1992)[4] and Gatzlaff and Ling (1994)[19] noted the structural similarity of an assessed value index to a repeat sales index in the housing context.

[*11] See for example, Shimizu and Nishimura (2006)[31].

- The index does not take into account that *capital expenditures* will generally change the quality of each property over time (so that the Q_{tn} are not in fact constant) and
- The index does not take into account *depreciation* of the underlying structure, which of course also changes the quality of each property.

The last problem mentioned above will generally impart a downward bias to the asset value indexes, P_{At}.*12 We cannot address the first problem mentioned above but in the following section, we will attempt to address problems 2 and 3 listed above.

4 A National Balance Sheet Accounting Approach to the Construction of Commercial Property Price Indexes

In this section, we will implement an approach to the construction of commercial property price indexes that is similar to the approach used by national income accountants to construct capital stock estimates.*13 National income accountants build up capital stock estimates for a production sector by deflating investments by asset and then adding up depreciated real investments made in prior periods. For commercial property capital expenditures and for the expenditures on the initial structure, we will more or less follow national income capital stock construction procedures. Next, we will assume that the assessed values for each property represents a good estimate for the total value of the structure and the land that the structure sits on. Once we have formed estimates for the stock values for capital expenditures and the value of the initial structure on the property, the value of land is set equal to assessed value of the property less our imputed value for the initial structure and the capital improvements made to the structure. The weakness in this approach is that one must make estimates for the structure depreciation rates based on limited information.

We postulate that the assessed asset value of REIT n in quarter t, V_{tn}, is equal to the sum of three components:

- The *value of the land plot* V_{Ltn} for the property;
- The *value of the initial structure* on the property, V_{Stn}, and
- The *value of the cumulated (but also depreciated) capital expenditures* on the property made in prior periods, V_{CEtn}.

Thus we assume that the following asset value decomposition holds for property n in period t:*14

$$V_{tn} = V_{Ltn} + V_{Stn} + V_{CEtn}; \qquad n = 1, ..., 50;\ t = 1, ..., 22. \tag{2}$$

We know the assessed values, V_{tn}, on the left hand side of equations (2) and our strategy will be to determine the components of the values on the right hand side of equations (2) by making plausible assumptions about the prices and quantities involved in the right hand side values. We start off by considering the decomposition of the property land values, V_{Ltn}, into

*12 Repeat sales price indexes are also subject to this downward bias due to the neglect of depreciation but this downward bias can often be negated by an upward bias due to sample selectivity problems associated with the repeat sales index. In any case, the repeat sales method is in general not very workable for the construction of a commercial property price index due to the infrequency of sales of commercial properties (and their heterogeneity).

*13 See Schreyer (2001)[29] (2009)[30] and Diewert (2005)[9] for a detailed explanation of these techniques.

*14 This assumption is a strong one. In particular, we are assuming that capital expenditures immediately add to asset value, an assumption that is unlikely to hold precisely.

price and quantity components; i.e., we assume that the following equations hold:

$$V_{Ltn} = P_{Ltn}Q_{Ltn};\ Q_{Ltn} = L_{tn} = L_n; \qquad n = 1, ..., 50;\ t = 1, ..., 22 \tag{3}$$

where L_n (which is equal to L_{tn}) is the area of the land plot for REIT n, which is part of our data base (and constant from period to period), and P_{Ltn} is the price of a square meter of land for REIT n in quarter t (which is not known yet).

Turn now to the value of the structure for property n in period t. If the structure is a new one, its value should be approximately equal to its cost of construction. Recall that an approximation to the cost of a square meter of new commercial property construction in quarter n is $300,000$ times $P_{St}S_{tn}$ where P_{St} is the construction price index per m^2 for Tokyo for quarter t (normalized to equal one in quarter 1) and $S_{tn} = S_n$ is the floor area for property n in period t. Upon noting that V_{tn} has been rescaled to units of million yen from a single yen, if the structure for REIT n is new in period t, then its value in millions of yen, V_{Stn}, should be approximately equal to $.3P_{St}S_{tn}$. We now assume that the *quarterly geometric (or declining balance) depreciation rate* for the structure is $\delta_S \equiv 0.005$ or 0.5% per quarter.*15 Thus the structure value for REIT n in quarter t (where the age of the structure in quarters at time t is A_{tn}) should be approximately equal to:

$$V_{Stn} = .3P_{St}S_{tn}(1 - \delta_S)^{A(t,n)}; \qquad n = 1, ..., 50;\ t = 1, ..., 22 \tag{4}$$

where $A(t, n) \equiv A_{tn}$. Thus we obtain the following decomposition of V_{Stn} into price and quantity components:

$$V_{Stn} = P_{Stn}Q_{Stn};\ P_{Stn} \equiv P_{St};\ Q_{Stn} \equiv .3S_{tn}(1 - \delta_S)^{A(t,n)}; \quad n = 1, ..., 50;\ t = 1, ..., 22 \tag{5}$$

where P_{St} is the known official construction price index for quarter t (lagged one quarter), S_{tn} is the known floor space for REIT n in quarter t (this is almost always constant across quarters), $A(t, n)$ is the known age of REIT n in quarter t and $\delta_S = 0.005$ is the assumed known quarterly geometric structure depreciation rate. Thus V_{Stn} can be calculated.

Finally, we need to determine the contribution of capital expenditures to REIT asset values. This is a more difficult task.*16 Define the capital expenditures of REIT n in quarter t as CE_{tn}. We need a deflator to convert these nominal expenditures into real expenditures. It is difficult to know precisely what the appropriate deflator should be. We will simply assume that the official structure price index, P_{St}, is a suitable deflator. Thus define *real capital expenditures* for REIT n in quarter t, q_{CEtn}, as follows:

$$q_{CEtn} \equiv \frac{CE_{tn}}{P_{St}}; \qquad n = 1, ..., 50;\ t = 1, ..., 22. \tag{6}$$

We know both series on the right hand side of (6) so the q_{CEtn} can also be determined. Now we require starting capital stocks for these capital expenditures and a geometric depreciation

*15 Hulten and Wykoff (1981)[22] obtain annual geometric depreciation rates for office buildings in the U.S. around 1% for continuing structures and around 2.5% when premature retirement is taken into account. Other studies often obtain higher rates. Our later analysis in section 6 below justifies our assumption of a 1/2 percent quarterly depreciation rate.

*16 Crosby, Devaney and Law acknowledge the importance of capital expenditures in explaining property value but they also point out the scarcity of research on this topic: "Other important issues are the roles of maintenance expenditure and replacement investment. ... Thus, expenditure is central to interpreting depreciation rates but it has received little attention in much of the commercial real estate literature." Neil Crosby, Steven Devaney and Vicki Law (2012; 230)[6].

rate that determine how these capital expenditures are written off over time. It is difficult to determine an appropriate depreciation rate for capital expenditures since this problem has not been studied very extensively (if at all) in the literature. In section 7 below, we will bring some limited econometric evidence to bear on this issue and using this evidence, we assume that the quarterly geometric depreciation rate for capital expenditures is $\delta_{CE} = 0.10$ or 10% per quarter.*17 The next problem is the problem of determining the starting stock of capital expenditures for each REIT, given that we do not know what capital expenditures were before the sample period. We provide a solution to this problem in two stages. First, we generate *sample average real capital expenditures* for each REIT n, q_{CEn}, as follows:

$$q_{CEn} \equiv \frac{1}{22}\sum_{t=1}^{22} q_{CEtn}; \qquad n = 1, ..., 50. \tag{7}$$

Our next assumption is that each REIT n has a starting stock of capital expenditures equal to depreciated investments for 20 quarters (or 5 years) equal to the REIT n sample average investment, q_{CEn}, defined above by (7).*18 Thus the *starting stock of CE capital* for REIT n is Q_{CE1n} defined as follows:

$$Q_{CE1n} \equiv q_{CEn} \cdot \frac{1-(1-\delta_{CE})^{21}}{\delta_{CE}}; \qquad n = 1, ..., 50. \tag{8}$$

The REIT capital stocks for capital expenditures can be generated for quarters subsequent to quarter 1 using the usual geometric model of depreciation recommended by Hulten and Wykoff (1981)[22], Jorgenson (1989)[24] and Schreyer (2001)[29] (2009)[30] as follows:

$$Q_{CEtn} \equiv (1-\delta_{CE})Q_{CE,t-1,n} + q_{CE,t-1,n}; \qquad t = 2, 3, ..., 22;\ n = 1, ..., 50. \tag{9}$$

Note that Q_{CEtn} is now completely determined for $t = 1, ..., 22$ and $n = 1, ..., 50$ and the corresponding price P_{St} is also determined. Thus an estimated value for the stock of capital expenditures of REIT n for the beginning of period t, V_{CEtn}, can be determined by multiplying P_{St} by Q_{CEtn}; i.e., we have:

$$V_{CEtn} \equiv P_{CEtn}Q_{CEtn};\ P_{CEtn} \equiv P_{St}; \qquad t = 1, ..., 22;\ n = 1, ..., 50 \tag{10}$$

where the Q_{CEtn} are defined by (8) and (9).

Now that the asset values V_{tn}, V_{Stn} and V_{CEtn} have all been determined, the price of land for REIT n in quarter t, P_{Ltn}, can be determined residually using equations (2) and (3):

$$P_{Ltn} \equiv \frac{V_{tn} - V_{Stn} - V_{CEtn}}{L_n}; \qquad n = 1, ..., 50;\ t = 1, ..., 22. \tag{11}$$

The above material shows how to construct estimates for the price of land, structures and capital expenditures for each REIT n for each quarter t (P_{Ltn}, P_{Stn} and P_{CEtn}) and the corresponding quantities (Q_{Ltn}, Q_{Stn} and Q_{CEtn}). Now use this price and quantity information in order to construct *quarterly value aggregates* (over all 50 REITs in our sample) for the

*17 After 20 quarters or 5 years, only 12% of a initial real investment in capital expenditures contributes to asset value; after 40 quarters or 10 years, only 1.5% of a initial real investment in capital expenditures contributes to asset value.

*18 The smallest age of structure in our sample is 4 years and so virtually all structures in our sample are at least 5 years old.

properties and for the land, structure and capital expenditure components; i.e., make the following definitions:*[19]

$$V^t \equiv \sum_{n=1}^{50} V_{tn};\ V_L^t \equiv \sum_{n=1}^{50} V_{Ltn};\ V_S^t \equiv \sum_{n=1}^{50} V_{Stn};\ V_{CE}^t \equiv \sum_{n=1}^{50} V_{CEtn}; \quad t = 1, ..., 22. \tag{12}$$

We form aggregate overall and component price and quantity indexes using chained Fisher (1922)[16] ideal indexes.*[20] In order to define these indexes, it is necessary to define Laspeyres and Paasche indexes and their chain link components. We will indicate how this is done when constructing aggregate land price indexes for the group of 50 REITs for each quarter. Define the *Laspeyres chain link land index* going from quarter $t-1$ to quarter t, $P_{\text{L,Land}}^{t-1,t}$, as follows:

$$P_{\text{L,Land}}^{t-1,t} \equiv \frac{\sum_{n=1}^{50} P_{Ltn} Q_{L,t-1,n}}{\sum_{n=1}^{50} P_{L,t-1,n} Q_{L,t-1,n}}; \qquad t = 2, 3, ..., 22. \tag{13}$$

The above chain links are used in order to define the *overall Laspeyres land price indexes*, $P_{\text{L,Land}}^t$, as follows:

$$P_{\text{L,Land}}^1 \equiv 1;\ P_{\text{L,Land}}^t \equiv P_{\text{L,Land}}^{t-1} P_{\text{L,Land}}^{t-1,t}; \qquad t = 2, 3, ..., 22. \tag{14}$$

Thus the Laspeyres price index starts out at 1 in period 1 and then we form the index for the next period by updating the index for the previous period by the chain link indexes defined by (13). A similar procedure is used in order to define the sequence of *Paasche chained indexes* for land, $P_{\text{P,Land}}^t$. First Define the *Paasche chain link land index* going from quarter $t-1$ to quarter t, $P_{\text{P,Land}}^{t-1,t}$, as follows:

$$P_{\text{P,Land}}^{t-1,t} \equiv \frac{\sum_{n=1}^{50} P_{Ltn} Q_{Ltn}}{\sum_{n=1}^{50} P_{L,t-1,n} Q_{Ltn}}; \qquad t = 2, 3, ..., 22. \tag{15}$$

The above chain links are used in order to define the overall Paasche land price indexes, $P_{\text{P,Land}}^t$, as follows:

$$P_{\text{P,Land}}^1 \equiv 1;\ P_{\text{P,Land}}^t \equiv P_{\text{P,Land}}^{t-1} P_{\text{P,Land}}^{t-1,t}; \qquad t = 2, 3, ..., 22. \tag{16}$$

Once the sequences of Laspeyres and Paasche land price indexes, $P_{\text{L,Land}}^t$ and $P_{\text{P,Land}}^t$, have been constructed, the *Fisher ideal land price index* for quarter t, $P_{\text{F,Land}}^t$, is defined as the geometric mean of the corresponding Laspeyres and Paasche indexes; i.e., define

$$P_{\text{F,Land}}^t \equiv \left[P_{\text{L,Land}}^t P_{\text{P,Land}}^t\right]^{1/2}; \qquad t = 1, ..., 22. \tag{17}$$

The Fisher chained price indexes for structures and capital expenditures, $P_{\text{F},S}^t$ and $P_{\text{F},CE}^t$, are constructed in an entirely analogous way, except that the REIT micro price and quantity data on land, P_{Ltn} and Q_{Ltn}, are replaced by the corresponding REIT micro price and quantity data on structures, P_{Stn} and Q_{Stn}, or on capital expenditures, P_{CEtn} and Q_{CEtn}, in equations (13)-(17).

*[19] These aggregate value series are listed in the Appendix in Table 2.

*[20] Laspeyres, Paasche and Fisher indexes are explained in much more detail in Fisher (1922)[16] and in the 2004 *Consumer Price Index Manual* [23]. The Fisher indexes have very good axiomatic and economic properties.

Finally, an *overall chained Fisher property price index*, $P^t_{\rm F}$, can be constructed in the same way except that the summations in the numerators and denominators of (13) and (15) above sum over 150 separate price components (all of the P_{Ltn}, P_{Stn} and P_{CEtn}) instead of just 50 price components. The Fisher price indexes $P^t_{\rm F}, P^t_{\rm F,Land}, P^t_{{\rm F},S}$ and $P^t_{{\rm F},CE}$ are listed in Table 2 in the Appendix, except that we dropped the subscript F; i.e., in what follows, denote these series by P^t, P^t_L, P^t_S and P^t_{CE} respectively.

The price series P^t, P^t_L, P^t_S and P^t_{CE} can be used to deflate the corresponding aggregate value series defined above by (12), V^t, V^t_L, V^t_S and V^t_{CE}, in order to form *implicit quantity or volume indexes*; i.e., define the following aggregate quantity indexes:

$$Q^t \equiv \frac{V^t}{P^t};\ Q^t_L \equiv \frac{V^t_L}{P^t_L};\ Q^t_S \equiv \frac{V^t_S}{P^t_S};\ Q^t_{CE} \equiv \frac{V^t_{CE}}{P^t_{CE}}; \qquad t = 1, ..., 22. \tag{18}$$

Q^t can be interpreted as an estimate of the real stock of assets across all 50 REITs at the beginning of quarter t, Q^t_L is an estimate of the aggregate real land stock used by the REITs*21 and Q^t_{CE} is an estimate of the real stock of capital improvements made by the REITs since they were constructed.*22

Because the price of structures for each REIT is proportional to the exogenous official construction price index for Tokyo, the aggregate structure price index, P^t_S, defined above as a Fisher index turns out to equal the official price index, P_{St} defined earlier.*23 Similarly, the Fisher price index of capital expenditures, P^t_{CE}, defined above also turns out to equal the official index, P_{St}. Thus the fairly complicated construction of the Fisher implicit quantity indexes that was explained above can be replaced by the following very simple shortcut equations:

$$Q^t_S = \frac{V^t_S}{P_{St}};\ Q^t_{CE} \equiv \frac{V^t_{CE}}{P_{St}}; \qquad t = 1, ..., 22. \tag{19}$$

The asset value (or repeat sales) overall price index, P^t_A, is graphed in Figure 1 below along with the overall commercial property price index P^t, where the method used to construct P^t might be termed a "national accounts" method for constructing a capital stock price index. We also show the "national accounts" land price index P^t_L and the official structures construction cost price index P^t_S which we have used as a price deflator for both capital expenditures and the estimated value of the structure.*24

It can be seen that the asset value price index P^t_A defined in the previous section is consistently below the more accurate economic accounting index P^t and the gap widens over time.*25 In our Japanese sample of commercial properties, our estimated average land value divided by total property value turned out to be 74.7%; i.e., approximately 75% of the property value is due to land value. In the U.S., the land ratio is very much less so that the bias in the asset value price index would be correspondingly much larger since it is the neglect of structure depreciation

*21 This remains constant over time since the quantity of land used by each REIT remained constant over time.

*22 The four implicit quantity series defined by (18) are also listed in Table 2 of the Appendix.

*23 The chained Laspeyres and Paasche price indexes for structures are also equal to the official index (and so are the corresponding fixed base indexes). And since the quantity of land is fixed for each REIT, the chained (and fixed base) Laspeyres and Paasche land price indexes are also equal to the chained Fisher land price indexes.

*24 P^t_A, P^t, P^t_S and P^t_L are listed as PA, P, PS and PL in Figure 1.

*25 From Table 2 in the Appendix, we see that $P^{22}_A = 0.8798$ and $P^{22} = 0.9027$. This translates into an approximate (geometric) downward bias in the asset value price index of about .5 percentage points per year, which is fairly significant.

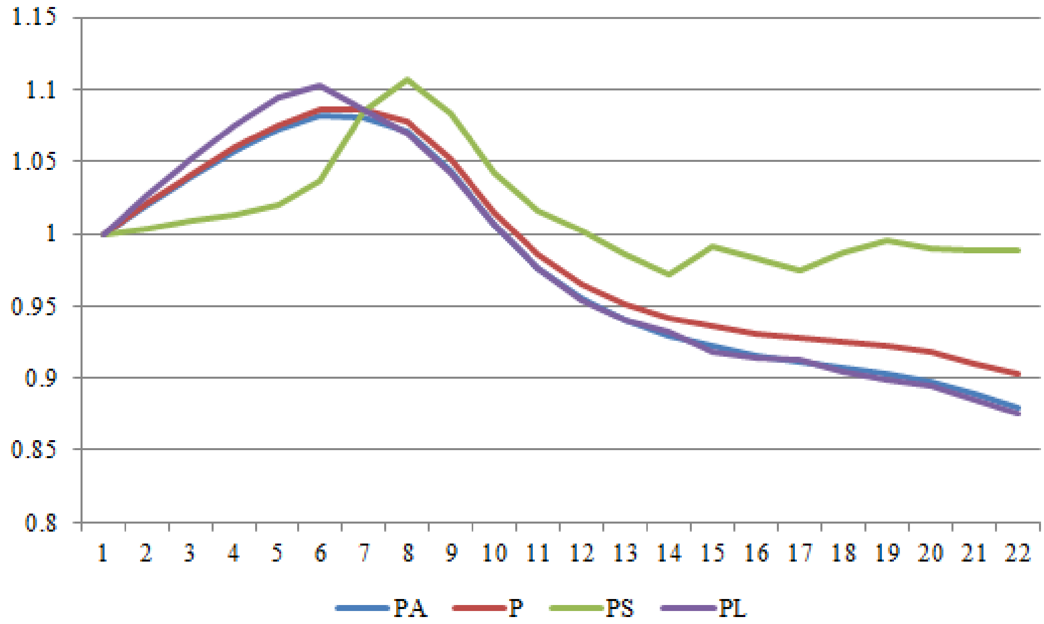

Figure 1 Asset Value Price Index P_A and Accounting Price Index P, Price of Structures P_S and Price Index for Land P_L

that causes the differences in P^t and P_A^t.*26 The movements in the price of structures, P_S^t, versus the price of land, P_L^t, are also of some interest. It appears that land prices peaked in period 6 (Q2-2008) while construction prices peaked somewhat later in period 8 (Q4-2008). Land prices continued to fall steadily after Q2-2008, ending up at 0.8752. Structure prices fell from Q8-2008 until Q2-2010 and remained more or less steady until the end of the sample period to end up at 0.9887.

In the following sections, we will construct alternative price indexes using hedonic regression techniques rather than using assumptions about depreciation rates (and the form of depreciation) along with assessed value information.

5 Traditional Hedonic Regression Approaches to Index Construction

Most hedonic commercial property regression models are based on the *time dummy approach* where the log of the selling price of the property is regressed on either a linear function of the characteristics or on the logs of the characteristics of the property along with time dummy variables.*27 In this section, instead of using selling prices for commercial properties, we will use the quarterly assessed values for the properties. The time dummy method does not generate decompositions of the asset value into land and structure components and so it is not suitable when such decompositions are required but the time dummy method can be used to generate overall property price indexes, which can then be compared with the overall price

*26 Since the asset value price index is a variant of the repeat sales index that is frequently used to construct property price indexes, we expect that these repeat sales indexes also have a substantial downward biases compared to indexes that take structure depreciation into account.

*27 This methodology was developed by Court (1939; 109-111)[5] as his Hedonic Suggestion Number Two.

indexes P_A^t and P^t that were described in the previous 2 sections.

Recall that V_{tn} is the assessed value for REIT n in quarter t, $L_{tn} = L_n$ is the area of the plot, $S_{tn} = S_n$ is the floor space area of the structure and A_{tn} is the age of the structure for REIT n in period t. In the time dummy linear regression defined below by (20), we have replaced V_{tn}, L_{tn} and S_{tn} by their logarithms, $\ln V_{tn}, \ln L_{tn}$ and $\ln S_{tn}$.[*28] Our first time dummy hedonic regression model is defined for $t = 1, ..., 22$ and $n = 1, ..., 50$ by the following equations:[*29]

$$\ln V_{tn} = \alpha + \alpha_t + \beta \ln L_{tn} + \gamma \ln S_{tn} + \delta A_{tn} + \varepsilon_{tn} \quad (20)$$

where $\alpha_1, \ldots, \alpha_{22}, \alpha, \beta, \gamma$ and δ are 25 unknown parameters to be estimated and the ε_{tn} are independently distributed normal error terms with mean 0 and constant variance. The α_t are the quarter t time coefficients which shift the hedonic surface during each quarter, α is a constant term, γ and β are parameters which adjust the asset value for the size of the lot and the floor space area respectively and δ is a parameter which adjusts the asset value for the age of the structure (essentially a depreciation parameter). We expect β and γ to be positive and δ to be negative. The time dummy variables associated with the α_t and the constant term α are linearly dependent and so we need to impose a normalization on the parameters in order to identify the remaining parameters. We choose the following normalization:

$$\alpha_1 = 0. \quad (21)$$

This normalization makes the overall commercial property price index equal to 1 in the first period.

The ordinary least squares estimates for the 25 remaining parameters in Model 1 are listed in Table 3 of the Appendix. For later reference, we note that the log likelihood for Model 1 was -583.955 and the R^2 between the dependent variable and the corresponding predicted variable was 0.6339. The estimated coefficients are listed in Table 3 of the Appendix. The estimated coefficient associated with the log of land area was $\beta = -0.1713$ (which is the wrong sign for this parameter) and with the log of the structure area was $\gamma = 1.1264$ and was highly significant (t statistic equal to 25.9). The estimated age coefficient was $\delta = 0.0020$, which is also the wrong sign for this parameter (t statistic equal to 3.9). The results for Model 1 were not very encouraging.

The *overall commercial property price indexes* for Model 1, P_1^t, are defined as the exponentials of the estimated time coefficients α_t:

$$P_1^t \equiv \exp[\alpha_t]; \qquad t = 1, ..., 22. \quad (22)$$

The resulting overall commercial property price indexes generated by Hedonic Model 1, the P_1^t, are graphed in Figure 2 below and are listed in Table 5 of the Appendix. We will discuss these estimated price indexes after we have presented the results for our second "traditional" hedonic regression model.

Our second time dummy hedonic regression model is defined for $t = 1, ..., 22$ and $n = 1, ..., 50$ by the following equations which introduce a *dummy variable* ω_n for each property n:

$$\ln V_{tn} = \alpha + \alpha_t + \beta \ln L_{tn} + \gamma \ln S_{tn} + \delta A_{tn} + \omega_n + \varepsilon_{tn} \quad (23)$$

[*28] This led to a better fitting regression model.

[*29] The hedonic regression models defined by (20) and (23) can be set up as linear regression models by defining suitable dummy variables for the α_t and ω_n parameters that appear in these equations.

where $\alpha_1, \ldots, \alpha_{22}, \omega_1, \ldots, \omega_{50}, \alpha, \beta, \gamma$ and δ are 76 unknown parameters to be estimated and the ε_{tn} are independently distributed normal error terms with mean 0 and constant variance. The linear regression model defined by equations (23) is the same as the model defined by equations (20) except that we have now added 50 additional property dummy variables where ω_n is the parameter which shifts the hedonic surface when the dependent variable is the logarithm of property value for property n. As before, the α_t are the quarter t time coefficients which shift the hedonic surface during each quarter, α is a constant term, γ and β are parameters which adjust the asset value for the size of the lot and the floor space area respectively and δ is a parameter which adjusts the asset value for the age of the structure (essentially a depreciation parameter). However, not all parameters can be identified in this model. Since $L_{tn} = L_n$ and $S_{tn} = S_n$ (so that the floor area and land areas of each REIT in our sample are constant over our sample period), it can be seen that the effects of the $\beta \ln L_{tn}$ and $\gamma \ln S_{tn}$ terms in (23) can be absorbed into the REIT specific parameters ω_n. Thus we set $\beta = \gamma = 0$. As was the case with (20), the dummy variables associated with the α_t and the constant term α are also linearly dependent, so as before, we set $\alpha_1 = 0$. However, the dummy variables associated with $\alpha, \alpha_2, \ldots, \alpha_{22}$ when combined with the dummy variables associated with $\omega_1, \ldots, \omega_{50}$ are also linearly dependent so to eliminate this linear dependence, we set $\alpha = 0$. Finally, it turns out that the age variable A_{tn} is also linearly dependent on the dummy variables associated with $\alpha_2, \ldots, \alpha_{22}$ and $\omega_1, \ldots, \omega_{50}$. In order to eliminate this linear dependence in the regression model, we could set $\delta = 0$. However, if we replace A_{tn} by the logarithm of A_{tn}, this leads to a regression model where all of the parameters are identified. Thus our *second linear regression model* is the following one which has 72 independent parameters:[*30]

$$\ln V_{tn} = \alpha_t + \omega_n + \delta \ln A_{tn} + \varepsilon_{tn}; \qquad t = 1, ..., 22; \; n = 1, ..., 50. \tag{24}$$

Equations (24) and (21) define Hedonic Model 2. The α_t parameters explain how, on average, the property values of the REIT sample shift over time and the REIT specific parameters, the ω_n, reflect the effect on REIT value of the size of the structure and the size of the land plot as well as any locational characteristics that can be attributed to each REIT. The δ parameter reflects the effects of aging of the structure on property value (we would expect this parameter to be negative: the value of the structure should decline as it ages).[*31]

The ordinary least squares estimates for the 72 parameters in Model 2 are listed in Table 4 of the Appendix. The log likelihood for Model 2 was 1687.33, a massive increase from the Model 1 log likelihood which was -583.955. The Model 2 R^2 between the dependent variable and the corresponding predicted variable was 0.9941, a big increase over the Model 1 R^2 which was 0.6339. The estimated coefficients for Model 2 have relatively small standard errors and high T statistics. However the estimated age coefficient for this model was a huge $\delta = 0.2896$ (with a standard error of 0.0476 and t statistic equal to 6.1), which is the wrong sign for this parameter.

The *overall commercial property price indexes* for Model 2, P_2^t, were defined as the exponentials of the estimated time coefficients α_t:

$$P_2^t \equiv \exp[\alpha_t]; \qquad t = 1, ..., 22. \tag{25}$$

The P_2^t, are graphed in Figure 2 below and are listed in Table 5 of the Appendix.

[*30] We still impose the normalization (21) on the parameters in (24); i.e., we set $\alpha_1 = 1$.

[*31] The problem is that the parameter δ will be an imperfect indicator of the effects of structure aging, due to the fact that the age variable (before transformation) will be subject to a multicollinearity problem in our original specification. We attempt to solve this problem by taking a nonlinear transform of the age variable in order to negate the exact multicollinearity but this solution does not really solve the problem.

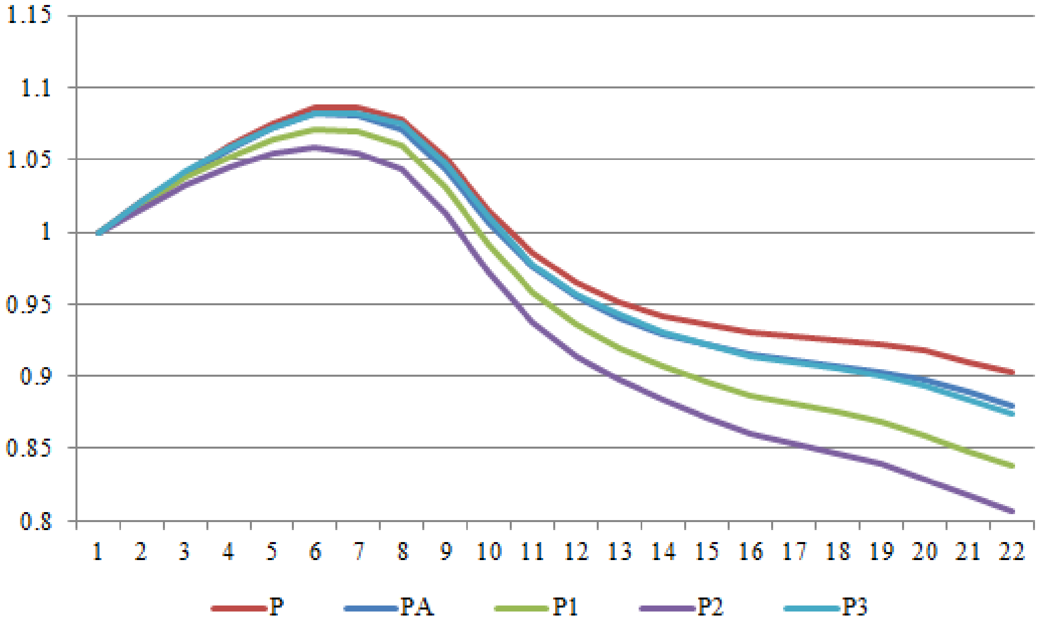

Figure 2 Accounting Price Index P, Asset Value Price Index P_A and Hedonic Price Indexes P_1, P_2 and P_3

From viewing Figure 2, it can be seen that our accounting based overall commercial property price index, P^t, shows the least amount of deflation over the sample period, ending up at an index value of 0.9027. The simple asset value price index, P_A^t, lies below P^t, ending up at 0.8798. Our first traditional log value hedonic regression model generated the index P_1^t which ended up at 0.8382 while the second model P_2^t ended up even lower at 0.8066. These large downward biases for Models 1 and 2 are due to the fact that the estimated coefficient δ for the age variable was positive in both regressions rather than the expected negative coefficients; i.e., as the structure ages, other factors held constant, we would expect asset value to fall. Thus Models 1 and 2 fail for our particular application.

It is of interest to rerun Model 2 after setting the age parameter δ equal to 0, which results in Model 3. It turns out that Model 3 is identical to the *Country Product Dummy regression model* that was originally introduced by Summers (1973)[34] in the context of making international comparisons between countries.[*32] The R^2 for Model 3 turned out to be 0.9939 and the log likelihood was 1667.89, a drop of about 20 from the previous Model 2. We constructed the resulting Commercial Property Price index P_3^t in the usual way (use the counterparts to equations (23) above).[*33] The index values for P_3^t are listed in Table 5 and the series is graphed on Figure 2 above. It can be seen that P_3^t is virtually identical to the asset value series P_A^t. This is perhaps not too surprising since the two indexes simply aggregate up the individual REIT asset prices into an overall price index; the form of aggregation is somewhat different but the basic ingredients are the same. Of course, the problem with both P_A^t and P_3^t is that they make no allowance for structure depreciation (or for capital expenditures) and

[*32] In the original Country Product Dummy (CPD) model, the two categories were countries and commodities. In our present context, the two categories are REITs and time. In the time series context, the CPD model also has an application as the Time Product Dummy (TPD) model.

[*33] We did not list the coefficient estimates in the Appendix for Model 3.

thus both of these indexes will generally have a downward bias relative to an index that takes structure depreciation into account.

There are two major problems with traditional log value hedonic regression models applied to property prices:

- These models often do not generate reasonable estimates for structure depreciation and
- These models essentially allow for only *one factor* that shifts the hedonic regression surface over time (the α_t) when in fact, there are generally *two major shift factors*: the price of structures and the price of land. Unless these two price factors move in a proportional manner over time, the usual hedonic approach will not generate accurate overall price indexes.

In the following section, we will estimate two alternative hedonic regression models that will address the above two difficulties.

6 The Builder's Model Applied to Commercial Property Assessed Values

The *builder's model* for valuing a residential property postulates that the value of a residential property is the sum of two components: the value of the land which the structure sits on plus the value of the residential structure.*34

In order to justify the model, consider a property developer n who builds a structure on a particular property that is ready for commercial use at the beginning of quarter t. The total cost of the property after the structure is completed will be equal to the floor space area of the structure, say S_{tn} square meters, times the building cost per square meter, β_t say, plus the cost of the land, which will be equal to the land cost per square meter, γ_{tn} say, times the area of the land site, L_{tn}. Thus if REIT n has a new structure on it at the start of quarter t, the value of the property, V_{tn}, should be equal to the sum of the structure and land value, $\beta_t S_{tn} + \gamma_{tn} L_{tn}$.*35 Note that as in section 3 above, we assume that the building cost price β_t depends on time only and not on the location of the building. On the other hand, the property prices γ_{tn} will generally depend on both the time period t and the location of the property which is indexed by n.

The above model applies to new structures. But it is likely that a similar model applies to older structures as well. Older structures will be worth less than newer structures due to the depreciation of the structure. Assuming that we have information on the age of the structure n at time t, say $A_{tn} \equiv A(t,n)$ and assuming a *geometric depreciation model*, a more realistic

*34 This model has been applied to residential property sales by de Haan and Diewert (2011)[8], Diewert, de Haan and Hendriks (2011a)[13] (2011b)[14] and Diewert and Shimizu (2013)[15] except that straight line or piece-wise linear depreciation was used as the depreciation model for the structure whereas in the present paper, we will use geometric depreciation models. In the following section, we will estimate a more complex geometric depreciation model where the depreciation rates change as the building ages. Geometric depreciation models have the advantage that the implied structure asset values that the models generate always remain positive whereas piece-wise linear depreciation models can generate negative asset values.

*35 Other papers that have suggested hedonic regression models that lead to additive decompositions of property values into land and structure components include Clapp (1980)[3], Francke and Vos (2004)[18], Gyourko and Saiz (2004)[20], Bostic, Longhofer and Redfearn (2007)[2], Davis and Heathcote (2007)[7], Francke (2008)[17], Koev and Santos Silva (2008)[25], Statistics Portugal (2009)[33], Diewert (2010)[10] (2011)[11] and Rambaldi, McAllister, Collins and Fletcher (2010)[28].

hedonic regression model is the following *basic builder's model*:

$$V_{tn} = \beta_t S_{tn}[e^{\phi}]^{A(t,n)} + \gamma_{tn} L_{tn} + \varepsilon_{tn}; \qquad t = 1, ..., 22;\ n = 1, ..., 50 \tag{26}$$

where the parameter e^{ϕ} is defined to be $1 - \delta$ and δ in turn is defined as the quarterly depreciation rate for the structure.[*36] Note that (26) is now a nonlinear regression model (whereas all of the regression models in the previous section were linear in the unknown parameters).[*37] There are two problems with the model defined by (26):

- We have only 22 times 50 observations (1100 observations in all) on V but there are 1100 land price parameters γ_{tn} to be estimated;
- The above model does not take into account the capital expenditures that were made in order to improve the structure after its initial construction.

We deal with the second problem by subtracting our section 3 estimated period t capitalized value of capital expenditures estimate V_{CEtn} from total asset value V_{tn} in order to obtain a new dependent variable. Then we will use a hedonic regression to decompose $V_{tn} - V_{CEtn}$ into structure and land components. We deal with the first problem by applying the Country Product Dummy methodology to the land component on the right hand side of equations (26) above; i.e., we set

$$\gamma_{tn} = \alpha_t \omega_n; \qquad t = 1, ..., 22;\ n = 1, ..., 50. \tag{27}$$

We also set the new structure prices for each quarter t, β_t, equal to a single price of structure in quarter 1, say β, times our official construction cost index P_S^t described in earlier sections. Thus we have:

$$\beta_t = \beta P_S^t; \qquad t = 1, ..., 22. \tag{28}$$

Replacing V_{tn} by $V_{tn} - V_{CEtn}$ and substituting (27) and (28) into equations leads to the following nonlinear regression model:

$$V_{tn} - V_{CEtn} = \beta P_S^t S_{tn}[e^{\phi}]^{A(t,n)} + \alpha_t \omega_n L_{tn} + \varepsilon_{tn}; \quad t = 1, ..., 22;\ n = 1, ..., 50. \tag{29}$$

This nonlinear regression has one unknown structure price β, one unknown ϕ (where $\delta = 1 - e^{\phi}$ and δ is the quarterly geometric depreciation rate), 22 unknown α_t (the overall land price series for our sample) and 50 unknown ω_n (which reflect the relative discount or premium in the land price for REIT n relative to other REITs). This is a total of 74 parameters but not all of the α_t and ω_n can be identified so we impose the normalization (21), $\alpha_1 = 1$. Thus there are 73 independent parameters to be estimated with 1100 degrees of freedom.

Shazam had no trouble estimating the unknown parameters.[*38] At first glance, the results appeared to be satisfactory. The R^2 between the observed variable and the predicted variable turned out to be 0.9943 and the log likelihood was -7658.84. The estimated ϕ parameter turned out to be -0.00454 and the corresponding quarterly depreciation rate was 0.00453, which is very close to our assumed rate of 0.005 that was used in section 3. The land price series (the estimated α_t ended up at $\alpha_{22} = 0.8754$) turned out to be very similar to our accounting generated land price series P_L^t listed in Table 2 (which ended up at $P_L^{22} = 0.8752$).

[*36] Note that $\delta = 1 - e^{\phi}$.

[*37] We used the nonlinear option in Shazam to estimate the nonlinear regressions in this section and the OLS option to estimate the linear regressions in the previous section; see White (2004)[36].

[*38] It was necessary to define two sets of dummy variables (one set of dummy variables for the time periods and one set for the REITs) and then interact these dummy variables in order to set up the nonlinear regression. This was a straightforward exercise.

However, the estimated β coefficient turned out to be 0.1524, which is far below our estimated cost of construction for the first period in our sample which is around 0.3.

Thus we decided to set β equal to 0.3 and rerun the nonlinear regression model defined by equations (29) (and $\alpha_1 = 1$). Call the resulting hedonic regression model, *Model 4*. The R^2 between the observed variable and the predicted variable for this model turned out to be 0.9943 and the log likelihood was -7659.58, a very small drop in log likelihood of about 1.2 points due to the fact that we now set $\beta = 0.3$ rather than estimate it as in the previous regression. Thus the cost in terms of fit and log likelihood of imposing this parameter constraint appears to be small. The estimated ϕ parameter turned out to be -0.00515 and the corresponding quarterly depreciation rate was 0.00514, which is very close to our assumed rate of 0.005 that was used in section 3. The land price series for Model 4 is denoted by $P_{L4}^t \equiv \alpha_t^*$ and it is graphed in Figure 4 below and listed in Table 5 in the Appendix. The Model 4 land prices turned out to be very similar to our accounting generated land price series P_L^t listed in Table 2.

We need to explain how our new land price series P_{L4}^t can be combined with our structures (and capital expenditures) price series P_S^t. Denote the estimated Model 4 parameters as $\beta^*, \alpha_1^* \equiv 1, \alpha_2^*, \ldots, \alpha_{22}^*, \phi^*$ and $\omega_1^*, \ldots, \omega_{50}^*$. We can break up the fitted value on the right hand side of equation (29) for observation tn into a *fitted structures component*, V_{S4tn}^*, and a *fitted land component*, V_{L4tn}^*, for $n = 1, ..., 50$ and $t = 1, ..., 22$ as follows:

$$V_{S4tn}^* \equiv \beta^* P_S^t S_{tn} [e^{\phi^*}]^{A(t,n)}; \tag{30}$$

$$V_{L4tn}^* \equiv \alpha_t^* \omega_n^* L_{tn}. \tag{31}$$

Now form structures and capital expenditures aggregate (over all REITS), V_{S4t}^*, by adding up the fitted structure values V_{S4tn}^* defined by (30) and the capital expenditures capital stocks V_{CEtn} that were defined by equations (10) in section 4 for each quarter:

$$V_{S4t}^* \equiv \sum_{n=1}^{50} [V_{S4tn}^* + V_{CEtn}]; \qquad t = 1, ..., 22. \tag{32}$$

In a similar fashion, form a land value aggregate (over all REITS), V_{L4t}^*, by adding up the fitted land values V_{L4tn}^* defined by (31) for each quarter t:

$$V_{L4t}^* \equiv \sum_{n=1}^{50} V_{L4tn}^*; \qquad t = 1, ..., 22. \tag{33}$$

Now define the *period t aggregate structure* (including capital expenditures) *quantity* or volume, Q_{S4t}^*, by (34) and *the period t aggregate land quantity* or volume, Q_{L4t}^*, by (35):

$$Q_{S4t}^* \equiv \frac{V_{S4t}^*}{P_S^t}; \qquad t = 1, ..., 22; \tag{34}$$

$$Q_{L4t}^* \equiv \frac{V_{L4t}^*}{P_{L4}^t}; \qquad t = 1, ..., 22. \tag{35}$$

Thus for each period t, we have 2 prices, P_S^t and P_{L4}^t, and the corresponding 2 quantities, Q_{S4t}^* and Q_{L4t}^*. We form an overall commercial property price index, P_4^t, by calculating the chained Fisher price index of these two price components.[*39] This overall index P_4^t is graphed in Figure 3 below along with our accounting method overall index P^t and the asset value price index, P_A^t.

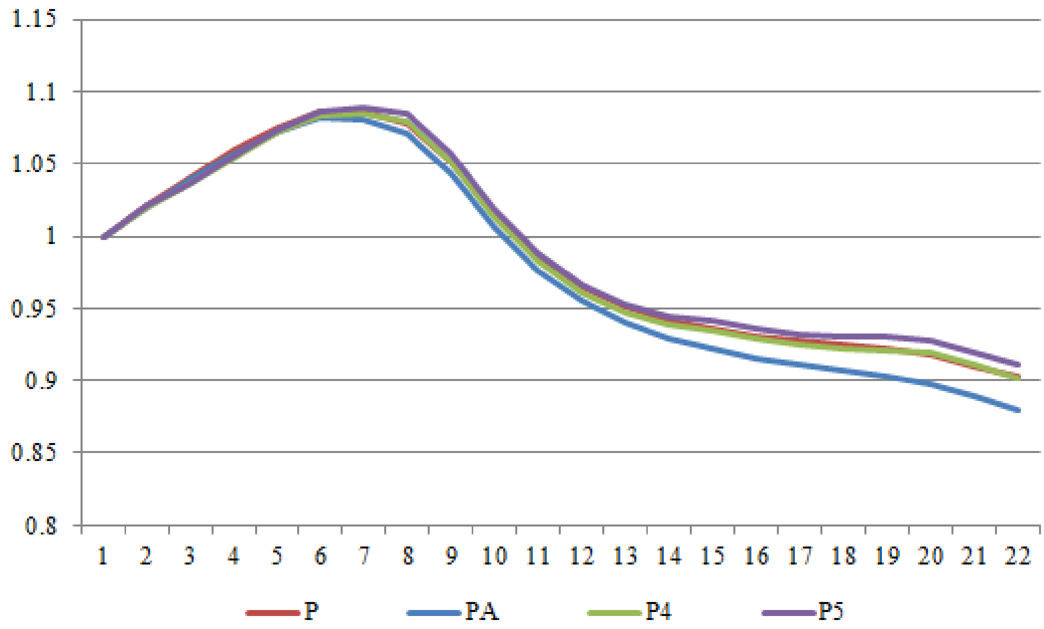

Figure 3 Accounting Method Price Index P, Asset Value Index P_A, Builder's Model Price Indexes P_4 and P_5

From Figure 3, it can be seen that our accounting method overall commercial property price index series, P^t, is extremely close to the builder's model hedonic regression approach index P_4^t that was just explained in this section. The geometric depreciation rate for capital expenditures is exactly the same (10% per quarter by assumption) in both models and the geometric depreciation rates for the main structure are almost identical in both models but the method of land price aggregation is different in the two approaches so the close correspondence between the two methods is a bit surprising. The asset value price index, P_A^t, lies well below P^t and P_4^t and the price index P_5^t lies a bit above P^t and P_4^t. The index P_5^t will be explained in the following section.

[*39] Our method for aggregating over REITs can be viewed as an application of Hicks' Aggregation Theorem; i.e., if the prices in a group of commodities vary in strict proportion over time, then the factor of proportionality can be taken as the price of the group and the deflated group expenditures will obey the usual properties of a microeconomic commodity. "Thus we have demonstrated mathematically the very important principle, used extensively in the text, that if the prices of a group of goods change in the same proportion, that group of goods behaves just as if it were a single commodity." J.R. Hicks (1946; 312-313)[21]. Our REIT structure (and capital expenditure) prices move in a proportional manner over time for all REITs, where each REITs' structure prices are proportional to the exogenous construction price index. Our REIT land prices also move in a manner that is proportional to the movements in the α_t because we have forced this movement by our choice of functional form in the regression model.

7 The Builder's Model with Geometric Depreciation Rates that Depend on the Age of the Structure

The age of the structures in our sample of Tokyo commercial office buildings ranges from about 4 years to 40 years. One might question whether the quarterly geometric depreciation rate does not change as the structure on the property ages. Thus in this section, we experimented with a model that allowed for different rates of geometric depreciation every 10 years. However, we found that there were not enough observations of "young" buildings to accurately determine separate depreciation rates for the first and second age groups so we divided observations up into three groups where the change in the depreciation rates occurred at ages (in quarters) 80 and 120, observations where the building was 0 to 80 quarters old, 80 to 120 quarters old and over 120 quarters old. Thus we found that 550 observations fell into the interval $0 \le A_{tn} < 80$, 424 observations fell into the interval $80 \le A_{tn} < 120$ and 126 observations fell into the interval $120 \le A_{tn} \le 160$. We label the three sets of observations that fall into the above three groups as groups 1-3. For each observation n in period t, we define the three *age dummy variables*, D_{tnm}, for $m = 1, 2, 3$ as follows:[*40]

$$\begin{aligned} D_{tnm} &\equiv 1 \text{ if observation } tn \text{ has a building whose age belongs to group } m; \\ &\equiv 0 \text{ if observation } tn \text{ has a building whose age is } \textit{not} \text{ in group } m. \end{aligned} \tag{36}$$

These dummy variables are used in the definition of the following function of age A_{tn}, $g(A_{tn})$, defined as follows where the break points, A_1 and A_2, are defined as $A_1 \equiv 80$ and $A_2 \equiv 120$:

$$\begin{aligned} g(A_{tn}) \equiv \exp\{&D_{tn1}\phi_1 A_{tn} + D_{tn2}[\phi_1 A_1 + \phi_2(A_{tn} - A_1)] \\ &+ D_{tn3}[\phi_1 A_1 + \phi_2(A_2 - A_1) + \phi_3(A_{tn} - A_2)]\} \end{aligned} \tag{37}$$

where ϕ_1, ϕ_2 and ϕ_3 are parameters to be estimated. As in the previous section, each ϕ_i can be converted into a depreciation rate δ_i where the δ_i are defined as follows:

$$\delta_i \equiv 1 - \exp[\phi_i]; \qquad i = 1, 2, 3. \tag{38}$$

Note that the logarithm of $g(A)$ is a piecewise linear function of the variable A. The economic meaning of all of this is as follows: first the first 80 quarters of a building's life, the constant price quantity of the structure declines at the quarterly geometric rate $(1 - \delta_1)$. Then for the next 40 quarters, the quarterly geometric rate of depreciation switches to $(1 - \delta_2)$. Finally after 120 quarters, the quarterly geometric rate of depreciation switches to $(1 - \delta_3)$.

Now we are ready to define our new nonlinear regression model that generalizes the model defined by (29) and (21) in the previous section. *Model 5* is the following nonlinear regression model:

$$V_{tn} - V_{CEtn} = \beta P_S^t S_{tn} g(A_{tn}) + \alpha_t \omega_n L_{tn} + \varepsilon_{tn}; \quad t = 1, ..., 22; \; n = 1, ..., 50 \tag{39}$$

where $g(A_{tn})$ is defined by (37). This nonlinear regression has one unknown structure price β, 3 unknown ϕ_i (where $\delta_i = 1 - \exp[\phi_i]$ and δ_i is a quarterly geometric depreciation rate), 22 unknown α_t (the overall land price series for our sample) and 50 unknown ω_n (which reflect

[*40] Note that for each observation, the Age dummy variables sum to one; i.e., for each tn, $D_{tn1} + D_{tn2} + D_{tn3} = 1$.

the relative discount or premium in the land price for REIT n relative to other REITs). This is a total of 76 parameters but not all of the α_t and ω_n can be identified so, as usual, we impose the normalization (21), $\alpha_1 = 1$. Thus there are 75 independent parameters to be estimated with 1100 degrees of freedom.

Again, Shazam had no trouble estimating the unknown parameters using the Nonlinear Regression option. The R^2 between the observed variable and the predicted variable turned out to be 0.9946 and the log likelihood was -7633.63, which is a large increase in log likelihood of 26 over Model 4 for the addition of two depreciation parameters and one structure price parameter β that sets the level of structure prices in quarter 1. The estimated parameters are listed in Table 6 in the Appendix. The estimated ϕ_i parameters turned out to be $-0.00328, -0.00705$ and -0.03623 and the corresponding quarterly depreciation rates turned out to be $\delta_1 = 0.00327, \delta_2 = 0.00702$ and $\delta_3 = 0.03558$. Compare these rates to the single quarterly geometric depreciation rate from Model 4, which was 0.00514. Thus the new results indicate that the quarterly depreciation rate is around 0.33% for the first 20 years of building life, increasing to 0.70% for the next 10 years and then finishing its useful life with a 3.6% per quarter depreciation rate. The estimated β turned out to be 0.2963 which is very close to the assumed rate of 0.3 that we have used in earlier sections of this paper. The land price series for Model 5 is denoted by $P_{L5}^t \equiv \alpha_t^*$ and it is graphed in Figure 4 below and listed in Table 5 in the Appendix. It can be seen that the new land price series P_{L5}^t lies a bit above the accounting land price index P_L^t and the previous builder's model land price index P_{L4}^t that was described in the previous section.

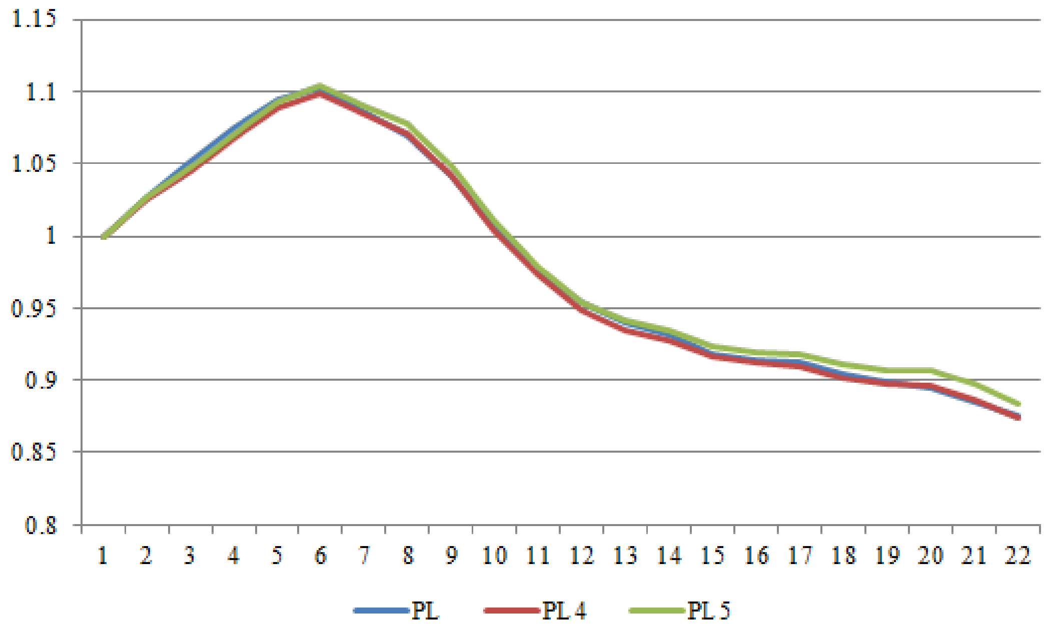

Figure 4 Accounting Method Price of Land P_L, Hedonic Regression Price Indexes for Land P_{L4} and P_{L5}

Finally, we can carry out the same procedure that was used in the previous section to generate an overall commercial property price index series, P_5^t, using the fitted values that are generated by Model 5. The series P_5^t are listed in Table 5 of the Appendix and are graphed in Figure 3 in the previous section. It can be seen that P_5^t lies slightly above P_4^t and our accounting method index P^t.

8 Estimating Demolition or Obsolescence Depreciation

The models that were described in the previous two sections are useful for national income accountants because they facilitate the accurate estimation of structure depreciation, which is required for the national accounts. The depreciation estimates that are generated by our models are *wear and tear depreciation* estimates that apply to structures that continue in existence over the sample period. However, there is another form of structure depreciation that we have not estimated; namely the loss of residual structure value that results from the *early demolition* of the structure. This problem was noticed and addressed by Hulten and Wykoff (1981)[22][*41] but we will propose a somewhat different solution to the problem.

Our suggested solution to the problem of measuring the effects of the early retirement of a building will draw on the framework suggested by Komatsu, Kato and Yashiro (1994)[26]. Their method requires the existence of data on the date of construction and the date of retirement of each building in the class of buildings under consideration and for the region that is in scope.[*42] Komatsu, Kato and Yashiro collected date of construction and date of retirement data for reinforced concrete office buildings in Japan for the reference year 1987. Thus for each age of building s (in years), they were able to calculate the number of office buildings of age s (in years), N_s, as of January 1, 1987 along with the number of office buildings of age s, n_s, that were demolished in 1987 for ages $s = 1, 2, ..., 75$. Given this information, they were able to calculate the *conditional probability*, ρ_s, that a surviving structure of age s at the beginning of the year would be demolished during 1987; i.e., they defined ρ_s as follows:

$$\rho_s \equiv \frac{n_s}{N_s}; \quad s = 1, ..., 75. \tag{40}$$

Under the assumption that the conditional probabilities defined by (40) have persisted through time, KKY defined the *unconditional probability* π_s that a building of age s is still in existence at the beginning of the year 1987 as follows:

$$\pi_0 \equiv 1; \ \pi_s \equiv \pi_{s-1}(1 - \rho_s); \quad s = 1, ..., 75. \tag{41}$$

It can be seen that the series π_s are a building counterpart to *life expectancy tables*; i.e., the births and deaths of a population of buildings are used to construct the probability of building survival as a function of age instead of the probability of individual survival as a function of age.

Using the Japanese data for the π_s for 1987 that is on Figure 7 in Komatsu, Kato and Yashiro (1994; 8)[26], we were able to construct (slightly smoothed) numerical estimates for their estimated survival probabilities, π_s. Once the probabilities of survival π_s have been determined, then the conditional probabilities of demolition ρ_s can be determined from the

[*41] "Any analysis based only on survivors will therefore tend to overstate both the value and productivity of estimated capital stocks." Charles Hulten and Frank Wykoff (1981; 377)[22]. Wear and tear depreciation is often called *deterioration* depreciation and demolition or early retirement depreciation is sometimes called *obsolescence* depreciation. Crosby, Devaney and Law (2012; 230)[6] distinguish the two types of depreciation and in addition, they provide a comprehensive survey of the depreciation literature as it applies to commercial properties.

[*42] Usually, land registry offices and/or municipal authorities issue building permits for the construction of new buildings and demolishment permits for the tearing down of buildings. It may be difficult to classify buildings into the desired economic categories.

π_s using equations (41) above.[*43] The resulting estimates for π_s and ρ_s are listed in Table 7 in the Appendix. See Figure 5 below for plots of these series.

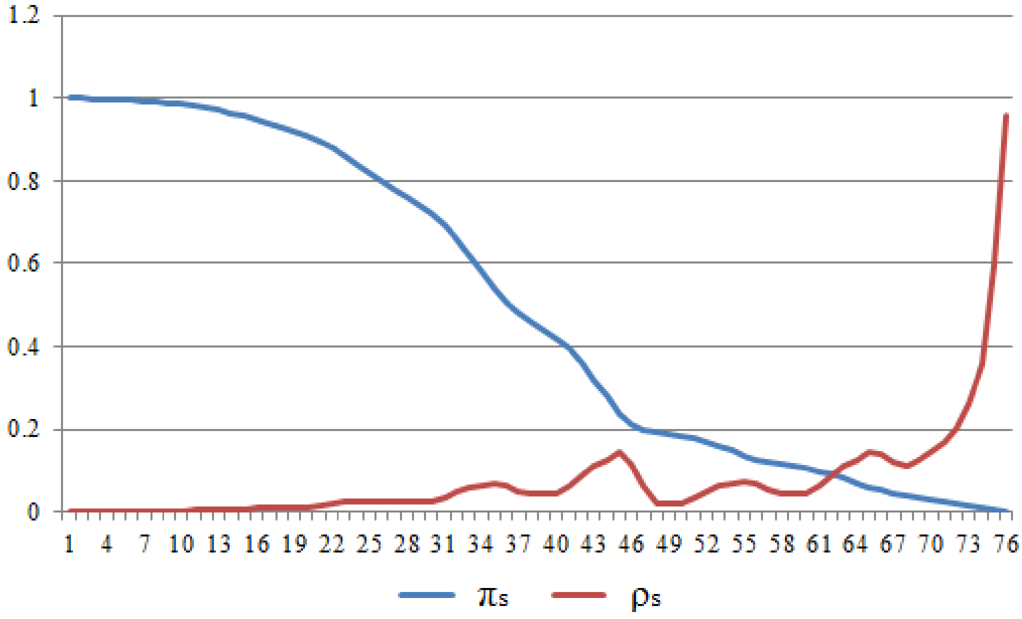

Figure 5 Unconditional Probabilities of Building Survival and Conditional Probabilities of Demolition

Note that as could be expected, the conditional probabilities of demolition are very small for the first 20 years or so of building life. From 20 to 42 years, these probabilities gradually increase from 1.4% to about 11% and then the probabilities fluctuate around the 10% level from age 43 to 67. Finally, after age 67, the conditional probabilities of demolition increase rapidly to end up close to unity at age 75.

It is likely that the underlying probabilities of demolition are smoother than the ρ_s exhibited in Figure 5. Thus a closer approximation to these underlying probabilities could be obtained by smoothing the above estimates.[*44] However, for our purposes in this section, the data listed in Table 7 in the Appendix and graphed above will suffice.

Recall that the wear and tear structure geometric depreciation rate that we estimated for our sample of continuing structures in section 6 above was about 0.5% per quarter. We want to form a rough idea of the possible magnitude of demolition depreciation using the information in Table 7. This component of depreciation is not included in our estimate of wear and tear depreciation.

Suppose that the *annual wear and tear geometric depreciation rate* is 2% so that we define $\delta \equiv 0.02$. Suppose further that investment in Tokyo office buildings has been constant for 75 years. We will normalize the annual structure investment to equal unity in constant yen units. Finally, suppose that the survival probabilities π_s listed in Table 7 apply to our hypothetical

[*43] Define $\rho_0 \equiv 0$.

[*44] Recall that Komatsu, Kato and Yashiro carried out their life table estimation exercise for the year 1987. Ideally, the national statistical agency could carry out a similar exercise every year. Then the panel of life tables could be smoothed, leading to more accurate estimates for the underlying conditional probabilities.

investment data. Thus after 75 years of steady investment, the constant yen value of the Tokyo commercial *office building stock* can be K defined as follows:

$$K \equiv \pi_0 + \pi_1(1-\delta) + \pi_2(1-\delta)^2 + \cdots + \pi_{75}(1-\delta)^{75}. \quad (42)$$

The corresponding real value of *wear and tear depreciation* Δ is defined as follows:

$$\Delta \equiv \delta\pi_0 + \delta\pi_1(1-\delta) + \delta\pi_2(1-\delta)^2 + \cdots + \delta\pi_{75}(1-\delta)^{75} = \delta K. \quad (43)$$

The corresponding amount of *demolition depreciation* D is defined as each component of the surviving capital stock on the right hand side of equation (42), $\pi_s(1-\delta)^s$, multiplied by the corresponding conditional probability of demolition, ρ_s; i.e., define D as follows:

$$D \equiv \rho_0\pi_0 + \rho_1\pi_1(1-\delta) + \rho_2\pi_2(1-\delta)^2 + \cdots + \rho_{75}\pi_{75}(1-\delta)^{75}. \quad (44)$$

Once the surviving capital stock K, the amounts of wear and tear depreciation Δ and demolition depreciation D have been defined, the *average wear and tear depreciation and demolition depreciation rates*, δ and d, are defined as the following ratios:

$$\delta \equiv \frac{\Delta}{K};\ d = \frac{D}{K}. \quad (45)$$

Of course, our assumed annual wear and tear depreciation rate of 2% turns out to equal the average wear and tear depreciation rate defined in (45) and the average demolition depreciation rate d turned out to equal 0.01795. Thus for the depreciation model considered in section 6 above, *it is likely that demolition depreciation is almost equal to wear and tear depreciation.* Note that the sum of the two depreciation rates is approximately 3.8% per year.*45

A similar set of calculations can be carried out for the more complex depreciation model defined in section 7 above. Recall that our three *quarterly* geometric depreciation rates were estimated as follows:

$$\delta_1^* \equiv 0.00327;\ \delta_2^* \equiv 0.00702;\ \delta_3^* \equiv 0.03558. \quad (46)$$

We need to convert these quarterly depreciation rates into *annual rates*. Define $\phi_i^* \equiv 1-\delta_i^*$ for $i = 1, 2, 3$. Define $\phi_i \equiv [\phi_i^*]^4$ and $\delta_i \equiv 1-\phi_i$ for $i = 1, 2, 3$. The δ_i turned out to be the following numbers:

$$\delta_1 \equiv 0.01302;\ \delta_2 \equiv 0.02779;\ \delta_3 \equiv 0.13490. \quad (47)$$

The geometric depreciation rates δ_i defined by (47) are the annualized counterparts to the quarterly rates defined by (46). Thus for the first 20 years of building life, annual wear and tear geometric depreciation is about 1.3% per year, about 2.8% per year for the next 10 years and about 13.5% per year for the remaining life of the building.

A hypothetical *capital stock component* that is s years old (adjusted for wear and tear depreciation), K_s, is defined as follows: $K_0 \equiv 1$; $K_s \equiv (1-\delta_1)K_{s-1}$ for $s = 1, 2, ..., 19$; $K_s \equiv (1-\delta_2)K_{s-1}$ for $s = 20, 21, ..., 29$ and $K_s \equiv (1-\delta_3)K_{s-1}$ for $s = 30, 31, ..., 75$. The

*45 Our method for adjusting wear and tear depreciation rates for the early retirement of assets is similar to the method suggested by Hulten and Wykoff. The main difference between our suggested method and their method is that we use a building life table to form estimates of building survivor probabilities whereas Hulten and Wykoff used somewhat arbitrary assumptions to form their estimates of survivor probabilities: "Our survivor probabilities are based upon the set of retirement distributions developed by Winfrey (1935)[37]."

aggregate constant yen capital stock (adjusted for survival and wear and tear depreciation), K, is defined as follows:

$$K \equiv \pi_0 K_0 + \pi_1 K_1 + \pi_2 K_2 + \cdots + \pi_{75} K_{75}. \tag{48}$$

Aggregate wear and tear constant yen depreciation, Δ, is defined as follows:

$$\Delta \equiv \delta_1 \sum_{s=0}^{19} \pi_s K_s + \delta_2 \sum_{s=20}^{29} \pi_s K_s + \delta_3 \sum_{s=30}^{75} \pi_s K_s. \tag{49}$$

Finally, *aggregate demolition depreciation* D is defined as follows:

$$D \equiv \sum_{s=0}^{75} \rho_s \pi_s K_s. \tag{50}$$

Once the surviving capital stock K, the amounts of wear and tear depreciation Δ and demolition depreciation D have been defined, the *average wear and tear depreciation and demolition depreciation rates*, δ and d, can again be defined by equations (45).

The annual wear and tear depreciation rate δ for our new model turned out to equal 0.02563 and the average demolition depreciation rate d turned out to equal 0.01234. Thus for the depreciation model considered in section 7 above, the "traditional" wear and tear depreciation rate is approximately 2.6% per year under our stationary state assumptions on building investment and the corresponding demolition depreciation rate is approximately 1.2% per year. Note that the sum of the two depreciation rates is approximately 3.8% per year, which is the same "total" depreciation rate that was generated by our section 6 model for wear and tear depreciation.[*46]

Our estimated demolition depreciation rates are only rough approximations to actual demolition depreciation rates. The actual rates of demolition depreciation depend on actual investments in commercial property office buildings in Tokyo for the past 75 years and this information is not available to us. However, the above calculations indicate that accounting for premature retirements of buildings adds significantly to the wear and tear depreciation rates that are estimated using hedonic regressions on continuing buildings. Thus it is important that national statistical agencies construct a data base for building births and retirements so that depreciation rates for buildings that are not retired can be adjusted to reflect the loss of building asset value that is due to premature retirement.

The analysis presented in this section does not invalidate our earlier analysis of alternative methods for constructing constant quality price indexes for commercial properties, since price indexes compare like to like and thus apply only to continuing structures. However, as a by product of our hedonic regressions in sections 6 and 7, we were able to form estimates of wear and tear depreciation for buildings that remained in use. The analysis in this section simply warns the reader that wear and tear depreciation[*47] is not the entire story: there is also a loss of asset value that results from the early retirement of a building that needs to be taken into account when constructing national income accounting estimates of depreciation.

[*46] For comparison purposes, Hulten and Wykoff (1981; 387)[22] found that their best fitting geometric model of depreciation for office buildings in the U.S. generated an estimated annual rate of 2.47%. This estimate includes early retirement or demolition depreciation and so is comparable to our rough estimate of 3.8% for Tokyo office buildings.

[*47] What we have labeled as wear and tear depreciation could be better described as *anticipated amortization of the structure* rather than wear and tear depreciation. Once a structure is built, it becomes a fixed asset which cannot be transferred to alternative uses (like a truck or machine). Thus amortization of the cost

9 Conclusion

Some conclusions that we can draw from the paper are as follows:

- The traditional time dummy approach to hedonic property price regressions does not always work well. The basic problem is that there are two main drivers of property prices over time: changes in the price of land and changes in the price of structures. The hedonic time dummy method allows for only one shifter of the hedonic surface when in fact there are at least two major shifters. Moreover, the traditional approach does not lead to sensible decompositions of overall price change into land and structure component changes.
- The simple asset value price index suggested in section 3 seemed to work better than indexes based on the traditional time dummy hedonic regression approach.
- The accounting method for constructing land, structure and overall property price indexes that was described in section 4 turned out to generate price indexes that were pretty close to the hedonic indexes based on the builder's model that were developed in sections 6 and 7.
- The methods suggested in sections 4, 6, 7 and 8 are practical and could be used by statistical agencies to improve their balance sheet estimates for commercial properties and their estimates of depreciation.

However, there are many additional avenues that could be explored.

- We experimented with capitalizing REIT Net Operating Income into capital stock indexes but the volatility in REIT cash flows presents practical problems in implementing this method. Even after smoothing cash flows, we could not generate sensible capital stock estimates with our data set.
- We also tried to use an econometric model to determine what an appropriate quarterly depreciation rate for *capital expenditures* should be but we found that the likelihood function was very flat over a very large range of depreciation rates so we simply settled on a quarterly rate of 10% without completely convincing evidence to back up this rate.
- The depreciation rates that we estimate in sections 6 and 7 understate the actual amount of structure depreciation that takes place. Our approach is fine as far as it goes but it applies only to continuing structures. Unfortunately, structures are not all demolished at the same age: many structures still generate cash flow but yet they are demolished before their initial cost of construction is fully amortized. We take this effect into account in section 8 and generate estimates of demolition (or premature retirement or obsolescence) depreciation.

Our overall conclusion is that constructing usable commercial property price indexes is a very challenging task; a much more difficult task than the construction of residential property price indexes.

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of the structure should be proportional to the cash flows that the building generates over its expected lifetime. The pattern of cash flows generated by a commercial property can be quite volatile but market based assessed values should be able to forecast these cash flows to some degree. However, technical progress, obsolescence or unanticipated market developments can cause the building to be demolished before it is fully amortized. See Diewert and Fox (2014)[12] for a more complete discussion of the fixity problem.

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Appendix A Model Estimated Coefficients and Index Number Tables

Table. 2 Aggregate Values of REIT Properties V^t and Component Capital Expenditure Stock, Structure and Land Values V_{CE}^t, V_S^t and V_L^t; Corresponding Volumes or Quantities Q^t, Q_{CE}^t, Q_S^t and Q_L^t; Asset Value Price Index P_A^t, Accounting Commercial Property Price Index P^t; Price Index for Structures and Capital Expenditures P_S^t and Land Price Index P_L^t

t : Q-Y	V^t	V_{CE}^t	V_S^t	V_L^t	Q^t	Q_{CE}^t	Q_S^t	Q_L^t	P_A^t	P^t	P_S^t	P_L^t
1 : Q1-07	255096	2695	62262	190139	255096	2695	62262	190139	1.0000	1.0000	1.0000	1.0000
2 : Q2-07	260300	2831	62149	195320	254913	2822	61951	190139	1.0204	1.0211	1.0032	1.0272
3 : Q3-07	265139	2979	62162	199998	254741	2954	61641	190139	1.0394	1.0408	1.0084	1.0518
4 : Q4-07	269661	3024	62117	204520	254474	2986	61333	190139	1.0571	1.0597	1.0128	1.0756
5 : Q1-08	273556	3149	62268	208139	254278	3086	61026	190139	1.0724	1.0758	1.0203	1.0947
6 : Q2-08	275886	3270	62920	209695	254054	3156	60721	190139	1.0815	1.0859	1.0362	1.1029
7 : Q3-08	275517	3452	65502	206563	253785	3184	60418	190139	1.0801	1.0856	1.0842	1.0864
8 : Q4-08	273372	3500	66543	203328	253457	3162	60116	190139	1.0716	1.0786	1.1069	1.0694
9 : Q1-09	266347	3424	64809	198114	253146	3160	59815	190139	1.0441	1.0521	1.0835	1.0419
10 : Q2-09	256628	3315	61991	191322	252862	3183	59516	190139	1.0060	1.0149	1.0416	1.0062
11 : Q3-09	248934	3288	60146	185500	252612	3237	59218	190139	0.9758	0.9854	1.0157	0.9756
12 : Q4-09	243575	3238	59052	181285	252299	3231	58922	190139	0.9548	0.9654	1.0022	0.9534
13 : Q1-10	239770	3189	57760	178821	251999	3236	58628	190139	0.9399	0.9515	0.9852	0.9405
14 : Q2-10	237035	3115	56654	177266	251666	3207	58334	190139	0.9292	0.9419	0.9712	0.9323
15 : Q3-10	235115	3111	57527	174477	251290	3139	58043	190139	0.9217	0.9356	0.9911	0.9176
16 : Q4-10	233465	2981	56768	173717	250870	3032	57753	190139	0.9152	0.9306	0.9829	0.9136
17 : Q1-11	232450	2908	56021	173521	250514	2983	57464	190139	0.9112	0.9279	0.9749	0.9126
18 : Q2-11	231272	2835	56422	172015	250093	2873	57176	190139	0.9066	0.9247	0.9868	0.9047
19 : Q3-11	230234	2766	56612	170855	249686	2780	56891	190139	0.9025	0.9221	0.9951	0.8986
20 : Q4-11	228861	2680	55994	170188	249303	2709	56606	190139	0.8972	0.918	0.9892	0.8951
21 : Q1-12	226650	2643	55639	168368	248961	2675	56323	190139	0.8885	0.9104	0.9879	0.8855
22 : Q2-12	224437	2621	55403	166414	248622	2651	56036	190139	0.8798	0.9027	0.9887	0.8752

Table. 3 Estimated Coefficients for Model 1

Name	Est Coef	T Stat	Name	Est Coef	T Stat
α	-0.2726	-1.2710	α_{14}	-0.09780	-1.171
α_2	0.0190	0.2281	α_{15}	-0.1098	-1.314
α_3	0.0371	0.4459	α_{16}	-0.1205	-1.441
α_4	0.0510	0.6126	α_{17}	-0.1270	-1.519
α_5	0.0624	0.7497	α_{18}	-0.1337	-1.597
α_6	0.0689	0.8276	α_{19}	-0.1414	-1.689
α_7	0.0672	0.8072	α_{20}	-0.1521	-1.814
α_8	0.0581	0.6976	α_{21}	-0.1649	-1.966
α_9	0.0308	0.3701	α_{22}	-0.1765	-2.103
α_{10}	-0.0085	-0.1018	β	-0.1713	-4.338
α_{11}	-0.0427	-0.5120	γ	1.1264	25.930
α_{12}	-0.0664	-0.7953	δ	0.0020	3.941
α_{13}	-0.0836	-1.0020			

Table. 4 Estimated Coefficients for Model 2

Name	Est Coef	T Stat	Name	Est Coef	T Stat	Name	Est Coef	T Stat
δ	0.2896	6.082	ω_3	7.3748	37.90	ω_{27}	8.0236	40.40
α_2	0.0165	1.524	ω_4	7.7596	37.61	ω_{28}	6.6051	31.46
α_3	0.0322	2.956	ω_5	7.8609	39.33	ω_{29}	7.8269	36.00
α_4	0.0438	3.973	ω_6	7.4669	38.65	ω_{30}	6.8843	33.74
α_5	0.0529	4.737	ω_7	7.4183	38.40	ω_{31}	5.8602	29.60
α_6	0.0573	5.035	ω_8	8.0808	39.60	ω_{32}	6.0074	27.26
α_7	0.0535	4.608	ω_9	7.4382	35.60	ω_{33}	6.2018	31.43
α_8	0.0424	3.568	ω_{10}	7.0035	31.61	ω_{34}	6.6439	33.34
α_9	0.0131	1.079	ω_{11}	7.2203	37.96	ω_{35}	6.2384	31.72
α_{10}	-0.0281	-2.255	ω_{12}	6.0359	29.83	ω_{36}	6.7646	34.52
α_{11}	-0.0642	-5.013	ω_{13}	6.0571	31.02	ω_{37}	6.1642	32.54
α_{12}	-0.0896	-6.816	ω_{14}	6.5959	29.56	ω_{38}	6.4691	31.44
α_{13}	-0.1086	-8.041	ω_{15}	7.0383	36.04	ω_{39}	7.4075	35.54
α_{14}	-0.1241	-8.963	ω_{16}	6.6352	32.99	ω_{40}	6.6638	34.25
α_{15}	-0.13802	-9.676	ω_{17}	6.8393	34.66	ω_{41}	6.7752	32.19
α_{16}	-0.15027	-10.25	ω_{18}	5.9573	28.73	ω_{42}	7.4138	39.14
α_{17}	-0.15833	-10.52	ω_{19}	8.3453	54.31	ω_{43}	6.4641	28.16
α_{18}	-0.16647	-10.77	ω_{20}	8.1428	54.23	ω_{44}	7.0462	35.83
α_{19}	-0.17561	-11.06	ω_{21}	7.7022	38.66	ω_{45}	7.0326	36.54
α_{20}	-0.18764	-11.52	ω_{22}	7.6575	39.08	ω_{46}	6.2644	29.29
α_{21}	-0.20182	-12.08	ω_{23}	7.4996	36.35	ω_{47}	7.0350	31.27
α_{22}	-0.21488	-12.55	ω_{24}	7.0084	34.15	ω_{48}	6.6572	32.18
ω_1	7.0285	34.07	ω_{25}	8.2189	39.73	ω_{49}	8.2911	35.91
ω_2	7.6391	33.28	ω_{26}	8.0236	40.40	ω_{50}	8.2489	43.37

Table. 5 Accounting Price Index P^t, Asset Value Index P_A^t, Hedonic Model Price Indexes, P_1^t - P_5^t, and Land Price Indexes P_L^t, P_{L4}^t and P_{L5}^t

t : Q-Y	P^t	P_A^t	P_1^t	P_2^t	P_3^t	P_4^t	P_5^t	P_L^t	P_{L4}^t	P_{L5}^t
1 : Q1-07	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2 : Q2-07	1.0211	1.0204	1.0192	1.0166	1.0212	1.0203	1.0208	1.0272	1.0260	1.0273
3 : Q3-07	1.0408	1.0394	1.0378	1.0327	1.0420	1.0362	1.0368	1.0518	1.0455	1.0473
4 : Q4-07	1.0597	1.0571	1.0523	1.0447	1.0588	1.0544	1.0553	1.0756	1.0683	1.0711
5 : Q1-08	1.0758	1.0724	1.0644	1.0544	1.0731	1.0720	1.0739	1.0947	1.0893	1.0937
6 : Q2-08	1.0859	1.0815	1.0714	1.0590	1.0823	1.0833	1.0861	1.1029	1.0991	1.1045
7 : Q3-08	1.0856	1.0800	1.0695	1.0550	1.0827	1.0845	1.0888	1.0864	1.0849	1.0908
8 : Q4-08	1.0786	1.0716	1.0598	1.0433	1.0750	1.0795	1.0850	1.0694	1.0707	1.0775
9 : Q1-09	1.0521	1.0441	1.0313	1.0132	1.0482	1.0517	1.0575	1.0419	1.0415	1.0484
10 : Q2-09	1.0149	1.0060	0.9915	0.9723	1.0099	1.0128	1.0184	1.0062	1.0036	1.0103
11 : Q3-09	0.9854	0.9758	0.9582	0.9379	0.9779	0.9831	0.9886	0.9756	0.9727	0.9792
12 : Q4-09	0.9654	0.9548	0.9358	0.9143	0.9570	0.9614	0.9669	0.9534	0.9482	0.9545
13 : Q1-10	0.9515	0.9399	0.9198	0.8971	0.9425	0.9472	0.9529	0.9405	0.9349	0.9415
14 : Q2-10	0.9419	0.9292	0.9068	0.8830	0.9311	0.9383	0.9443	0.9323	0.9277	0.9348
15 : Q3-10	0.9356	0.9217	0.8960	0.8711	0.9219	0.9344	0.9412	0.9176	0.9163	0.9238
16 : Q4-10	0.9306	0.9152	0.8865	0.8605	0.9139	0.9292	0.9362	0.9136	0.9120	0.9199
17 : Q1-11	0.9279	0.9112	0.8807	0.8536	0.9098	0.9254	0.9325	0.9126	0.9095	0.9177
18 : Q2-11	0.9247	0.9066	0.8748	0.8466	0.9056	0.9226	0.9306	0.9047	0.9022	0.9111
19 : Q3-11	0.9221	0.9025	0.8681	0.8389	0.9005	0.9215	0.9300	0.8986	0.8981	0.9076
20 : Q4-11	0.9180	0.8972	0.8589	0.8289	0.8927	0.9190	0.9278	0.8951	0.8967	0.9066
21 : Q1-12	0.9104	0.8885	0.8479	0.8172	0.8831	0.9111	0.9201	0.8855	0.8869	0.8968
22 : Q2-12	0.9027	0.8798	0.8382	0.8066	0.8746	0.9016	0.9108	0.8752	0.8742	0.8841

Table. 6 Estimated Coefficients for Model 5

Name	Est Coef	T Stat	Name	Est Coef	T Stat	Name	Est Coef	T Stat
β	0.2963	3.889	ω_{22}	2.680	8.955	ω_{47}	3.179	12.167
ϕ_1	−0.00328	−2.559	ω_{23}	1.222	9.428	ω_{48}	7.608	40.656
ϕ_2	−0.00705	−3.773	ω_{24}	3.499	13.005	ω_{49}	2.205	7.893
ϕ_3	−0.03623	−3.923	ω_{25}	18.17	43.141	ω_{50}	3.910	13.640
ω_1	1.708	3.851	ω_{26}	3.520	13.513	α_2	1.0273	94.363
ω_2	4.791	22.480	ω_{27}	3.571	14.261	α_3	1.0473	95.380
ω_3	2.387	7.200	ω_{28}	7.422	35.534	α_4	1.0711	93.637
ω_4	1.934	9.490	ω_{29}	3.424	12.843	α_5	1.0937	90.191
ω_5	3.029	10.706	ω_{30}	1.395	5.775	α_6	1.1045	89.152
ω_6	3.511	8.808	ω_{31}	1.682	9.379	α_7	1.0908	94.379
ω_7	4.245	10.747	ω_{32}	3.514	9.509	α_8	1.0775	95.514
ω_8	5.710	31.715	ω_{33}	1.914	6.677	α_9	1.0484	94.111
ω_9	4.261	21.745	ω_{34}	3.873	9.657	α_{10}	1.0103	92.033
ω_{10}	2.173	32.331	ω_{35}	3.830	12.783	α_{11}	0.9792	90.516
ω_{11}	2.825	13.940	ω_{36}	1.628	6.578	α_{12}	0.9545	90.552
ω_{12}	1.650	5.024	ω_{37}	2.736	9.251	α_{13}	0.9415	89.175
ω_{13}	0.3942	1.832	ω_{38}	5.724	17.738	α_{14}	0.9348	86.293
ω_{14}	4.878	19.939	ω_{39}	0.7083	2.836	α_{15}	0.9238	86.733
ω_{15}	3.150	12.429	ω_{40}	3.258	14.320	α_{16}	0.9199	85.759
ω_{16}	1.747	13.458	ω_{41}	6.468	13.977	α_{17}	0.9177	83.972
ω_{17}	0.928	6.371	ω_{42}	3.561	20.078	α_{18}	0.9111	83.358
ω_{18}	1.993	7.877	ω_{43}	0.9705	5.984	α_{19}	0.9076	82.737
ω_{19}	10.316	18.279	ω_{44}	1.594	6.319	α_{20}	0.9066	80.694
ω_{20}	4.281	9.534	ω_{45}	4.539	15.388	α_{21}	0.8968	79.997
ω_{21}	15.410	40.295	ω_{46}	7.338	29.390	α_{22}	0.8841	79.780

Table. 7 Unconditional Probabilities of Building Survival π_s and Conditional Probabilities of Demolition ρ_s as Functions of Age s

Age s	π_s	ρ_s	Age s	π_s	ρ_s	Age s	π_s	ρ_s	Age s	π_s	ρ_s
0	1.0000	0	19	0.91	0.0109	38	0.4400	0.0435	57	0.1158	0.0417
1	0.9990	0.0010	20	0.8971	0.0142	39	0.4199	0.0455	58	0.1107	0.0435
2	0.9980	0.0010	21	0.8799	0.0192	40	0.3939	0.0620	59	0.1057	0.0455
3	0.9970	0.0010	22	0.8599	0.0227	41	0.3592	0.0882	60	0.0992	0.0620
4	0.9960	0.0010	23	0.8399	0.0233	42	0.3190	0.1119	61	0.0904	0.0882
5	0.9947	0.0013	24	0.8199	0.0238	43	0.2787	0.1261	62	0.0803	0.1119
6	0.9930	0.0017	25	0.7999	0.0244	44	0.2384	0.1446	63	0.0702	0.1261
7	0.9910	0.0020	26	0.7799	0.0250	45	0.2103	0.1179	64	0.0600	0.1446
8	0.9890	0.0020	27	0.7599	0.0257	46	0.1973	0.0621	65	0.0516	0.1408
9	0.9870	0.0020	28	0.7399	0.0263	47	0.1933	0.0204	66	0.0453	0.1223
10	0.9836	0.0035	29	0.7199	0.0270	48	0.1892	0.0208	67	0.0402	0.1119
11	0.9780	0.0057	30	0.6940	0.0360	49	0.1852	0.0213	68	0.0351	0.1261
12	0.9710	0.0072	31	0.6595	0.0497	50	0.1791	0.0329	69	0.0301	0.1446
13	0.9640	0.0072	32	0.6194	0.0607	51	0.1700	0.0510	70	0.0250	0.1694
14	0.9570	0.0073	33	0.5794	0.0647	52	0.1589	0.0652	71	0.0199	0.2048
15	0.9491	0.0082	34	0.5393	0.0692	53	0.1478	0.0698	72	0.0147	0.2595
16	0.9400	0.0096	35	0.5054	0.0629	54	0.1367	0.0751	73	0.0094	0.3572
17	0.9300	0.0106	36	0.4800	0.0502	55	0.1275	0.0674	74	0.0038	0.5954
18	0.9200	0.0108	37	0.4600	0.0417	56	0.1208	0.0522	75	0.0002	0.9601