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Power law in market capitalization during Dot-com and Shanghai bubble periods

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Abstract The distributions of market capitalization in the NASDAQ and Shanghai stock exchanges from 1990 to 2015 have a fat upper tail that follows a power law function. The power law index, which fluctuates around one depending on the economic conditions, became small during the dot-com and Shanghai bubble periods. By using the regression coefficient of random forests for market capitalization and income statement items, we found that net assets are most reflected in market capitalization for companies listed in NASDAQ. The distribution of the price book-value ratio (PBR), which is defined as market capitalization divided by net assets, also got fat during the bubble periods. These results suggest that speculative money was excessively concentrated on specific stocks during such periods.

Keywords Power law · Zipf's law · Market capitalization · Asset bubble · Stock market · Econophysics

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1 Introduction

After B. Mandelbrot identified the fractal structure of price fluctuations in asset markets in 1963 [1], statistical physicists have studied the economic mechanism that produces a fractal structure. Power law is an important characteristic in the fractal structure. Some research found that the size distribution of asset price fluctuations follows a power law function [2,3]. The distribution of firm size, based on the income statement items in real economies, also follows a power law function [4–8]. The power law index of firm size is almost 1 over the last 30 years in many capitalistic countries [9,10].

In previous studies, stock price fluctuations attracted more attention than firm size distribution in stock markets, and several models for price dynamics were proposed [11,12]. The PACK and LPPL models simulated the price fluctuations of a stock during bubble periods [13,14]. However, the firm size distribution in stock markets must also be investigated to identify stock market bubbles. The firm size in stock markets is measured by market capitalization: = stock price \times number of outstanding shares. Speculative funds flow into the stock market during stock bubble periods. Investors estimate the fluctuations of firms based on annually/quarterly reports and such information as business news/rumors and the quoted stock prices in the market. Investors often follow the market trends that reflect the majority of investors. By only reason that stock price is rising, during bubble periods speculative money is often concentrated on specific stocks with which price is rising. Then their stocks become still more expensive, and acquire more speculative money again. Such mechanism is called herding phenomena for speculative money. As a result, the market capitalization gap among the listed firms in the stock market widens, and the firm sizes in the stock market deviate from those in real economies.

In this paper, we use a dataset compiled by the Thomson Reuters Corporation that covers daily market capitalization and annual income statements of all the listed firms in NASDAQ and SSE from 1990 to 2015. This period includes the 2000 dot-com and 2007 Shanghai bubbles. We focus on the distribution of market capitalization in NASDAQ and the Shanghai stock exchange (SSE) for these bubbles. Kaizoji et al. showed that the upper tail of stock price distribution in the Tokyo stock exchange grew fat during the dot-com bubble period [15,16]. However, if we accurately investigate the firm size in stock markets, not only the price but also the outstanding shares must be taken into consideration because share consolidation and splitting often occurs in the market.

The rest of the paper is organized as follows. Section 2 examines the power law of market capitalization distribution using an expansion of the Castillo and Puig test [10,17–19]. In Section 3 we observe that the power law index fluctuates around one, depending on economic conditions, and tends to become smaller during bubble periods. In Section 4, we find that net assets

are most reflected in market capitalization for firms listed in NASDAQ during non-bubble periods. The price book-value ratio (PBR) distribution, which is defined as market capitalization divided by net assets, got fat during the bubble periods. These results suggest that speculative money is excessively concentrated on specific stocks during bubble periods. Section 5 concludes the paper.

2 Distribution of market capitalization

First, we show the cumulative distribution of market capitalization for the listed firms in a stock market. The distribution on December 31, 1997 in NASDAQ is shown in log-log plots in Fig. 1. The distribution is on a straight line that indicates a power law function. However, in this case, the distribution deviates from a straight line and is close to a log-normal distribution when the market capitalization is smaller than about 2.7×10^8 dollars:

$$P_{>}(x) \propto x^{-\mu} \text{ for } x \geq x_0, \quad (1)$$

where x is the market capitalization, μ is a power law index, and x_0 is a threshold. The index estimated with the maximum likelihood method is $\mu = 1.0$ (Fig. 1). Such a power law with $\mu = 1.0$ is called Zipf's law.

Malevergne et al. [17], who expanded the likelihood ratio test between exponential distribution and the truncated normal distribution introduced by Castillo and Puig [18], tested the null hypothesis where, beyond a threshold, a distribution's upper tail is characterized by a power law distribution against the alternative where the upper tail follows a log-normal distribution beyond the same threshold. This is known as the uniformly most powerful unbiased test. They identified the upper tail that follows a log-normal distribution or a power law distribution to detect a threshold by conducting this test.

The dashed line in Fig. 1 displays the threshold between a power law distribution and a log-normal distribution when the p -value of the significance level is set as 0.1 for Malevergne's test. Therefore, the upper tail of market capitalization distribution can be well approximated by the power law function. On the other hand, a small range of market capitalization follows the log-normal distribution, as shown in the dashed curve in Fig. 1.

3 Power law index of market capitalization distribution

The upper tail of market capitalization distribution gets fat if the speculative money is concentrated on specific stocks. Such concentration of money tends to occur during bubble periods. The power law index became smaller during the 2000 dot-com and 2007 Shanghai bubbles.

The means of market capitalization for all the listed firms in NASDAQ and SSE were respectively about 1.94×10^9 dollars on March 9, 2000 and 3.92×10^9 dollars on December 4, 2007 during their bubble periods. On the other hand,

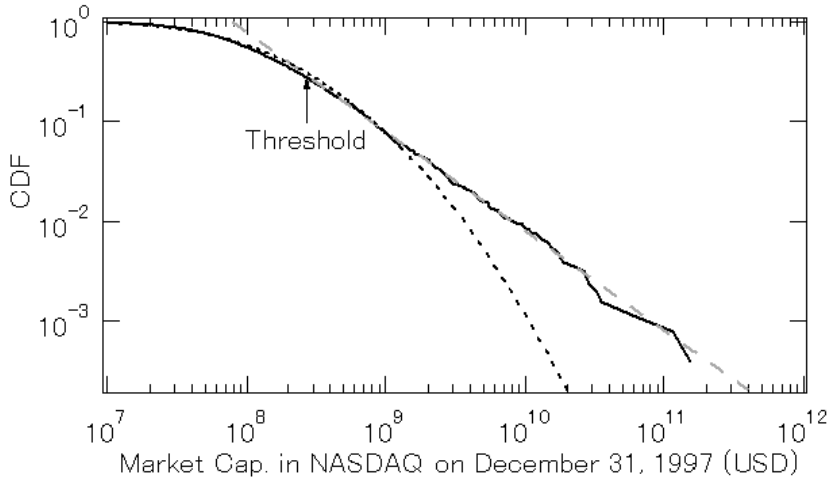


Fig. 1 Cumulative distribution of market capitalization for listed firms in NASDAQ on December 31, 1997. Dashed straight line and curve respectively express a power law distribution as $P_{>}(x) \propto x^{-1}$ and log-normal distribution with standard deviation of market capitalizations. Arrow indicates $x_0 = 2.7 \times 10^8$ dollars in Eq. (1).

the means were respectively about 1.44×10^9 and 3.04×10^9 dollars on March 14, 2011 on the non-bubble periods. The means increased during the bubble periods.

In these bubble cases, only a few firms increased the market capitalization drastically and raised the whole mean of market capitalization for all the listed firms. The black lines in Fig. 2 respectively show the market capitalization distributions for firms listed on the NASDAQ during the 2000 dot-com bubble and on the SSE during the 2007 Shanghai bubble. The gray lines express the distributions on March 14, 2011 during non-bubble periods. The cumulative probability on the vertical axis at which the black line intersects with the gray line is $P_{>}(x = 2 \times 10^9) \approx 0.1$ in the dot-com bubble case. The top 10% market capitalization in 2000 was higher than that in 2011, although the bottom 90% market capitalization in 2000 was lower than that in 2011. Such a characteristic was also observed in the Shanghai bubble case (Fig. 2(b)). The cumulative probability of its crossing point is $P_{>}(x = 1.2 \times 10^{10}) \approx 0.04$. These results suggest that speculative money flowed into a handful of stocks from a majority of stocks in each stock market during the bubble periods.

The concentration of speculative money changes the distribution slope. Fig. 3 shows the NASDAQ composite index, the SSE composite index, and the time series of the power law index in NASDAQ and SSE. The power law index, which fluctuated around one depending on the economic conditions, fell significantly when the increase of each composite index started during the dot-com and 2007 Shanghai bubble periods and remained small until each composite index steadied after the bubble burst.

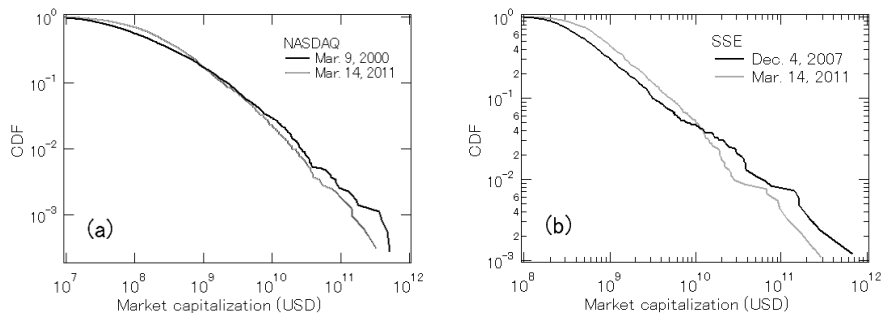


Fig. 2 Cumulative distributions of market capitalization for listed firms: (a) in NASDAQ on March 9, 2000 and on March 14, 2011 and (b) in SSE on December 4, 2007 and on March 14, 2011.

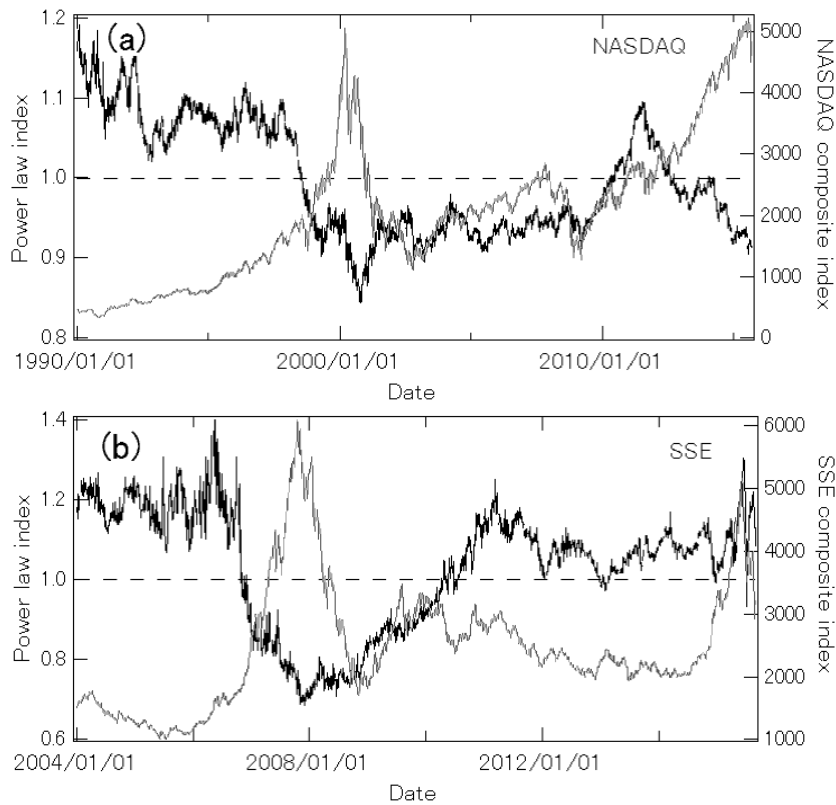


Fig. 3 (a) Power law index in NASDAQ and NASDAQ composite index and (b) Power law index in SSE and SSE composite index. Black and gray lines respectively express power law and composite indexes. Dashed lines indicate the power law index is one.

4 Fundamental-adjusted market capitalization in NASDAQ

In economics, a financial bubble is defined by the gap between market and fundamental prices. We show that the gap expanded during the 2000 dot-com bubble period.

First, we look for a firm's fundamentals that are mainly reflected in its market capitalization during non-bubble periods. We chose the following key items of income statements, total assets, net assets, total revenue, operating income, net income, operating cash flow, and number of employees, as candidates of firm fundamentals and investigated the correlation between market capitalization and each key item of income statements during non-bubble periods. Table 1 expresses the Kendall and Pearson correlation coefficients for firms listed on NASDAQ in 1997 and 2004. The correlation coefficients between market capitalization and net assets are the largest. However, other correlation coefficients also increased due to the effect of multicollinearity among income statement items.

We provide the regression coefficients of random forests, which are hardly affected by multicollinearity even though we used the data that caused it. Random forests can be used to rank the importance of the explanatory variables in a regression by estimating the explained variable using regression trees without part of the explanatory variables. We set market capitalization and the income statement items to the explained variable and the explanatory variables. Fig. 4(a) displays the time series of the importance of the income statement items from 1995 to 2013 in NASDAQ. Net assets are the most important in all years except 2011. Other items are not important for most years. On the other hand, in SSE, both net assets and net income are critical. Fig. 4(b) also expresses the importance of income statement items from 2000 to 2013 in SSE.

Table 1 Kendall and Pearson correlation coefficients between market capitalization and each key item of income statements in 1997 and 2004 in NASDAQ. Pearson correlation coefficients are calculated using logarithmic values of market capitalization and items without negative values.

Item	1997		2004	
	Kendall	Pearson (> 0)	Kendall	Pearson (> 0)
Total assets	0.461	0.684	0.526	0.755
Net assets	0.622	0.824	0.681	0.880
Total revenue	0.457	0.632	0.523	0.680
Operating income	0.473	0.820	0.452	0.841
Net income	0.410	0.805	0.420	0.851
Operating cash flow	0.351	0.745	0.449	0.829
Number of employees	0.396	0.582	0.466	0.658

Next, we introduce fundamental-adjusted market capitalization. In NASDAQ, since net assets are the main fundamental that is reflected in market capitalization, we propose market capitalization adjusted by net assets. In finance, net asset-adjusted market capitalization is called the PBR:

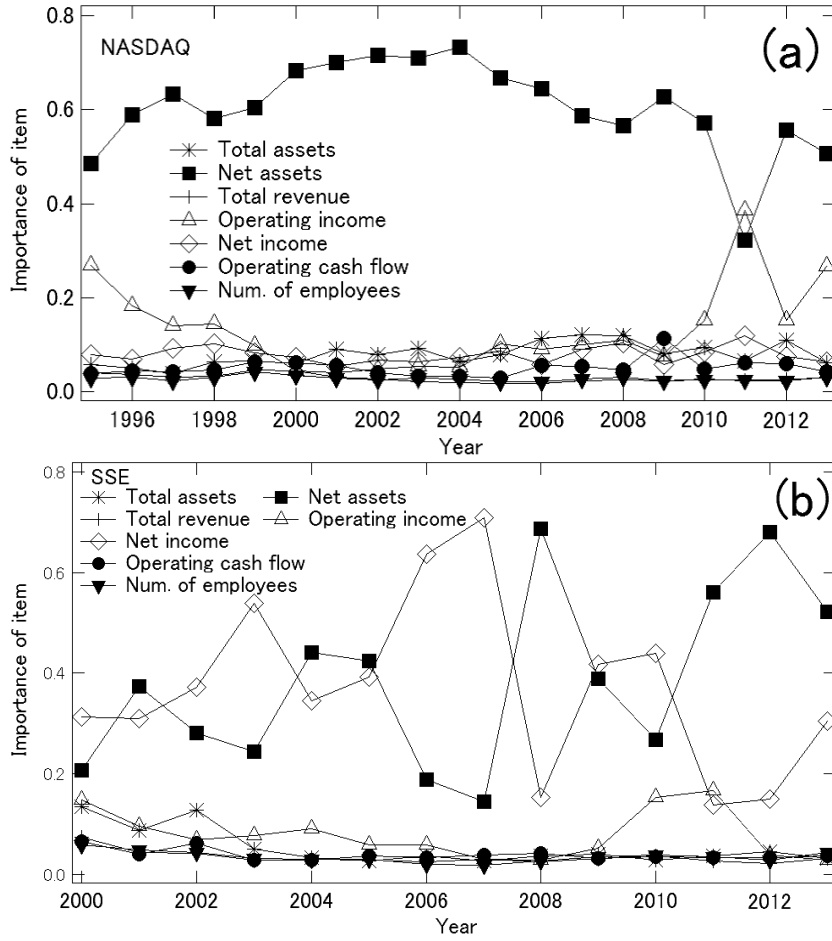


Fig. 4 Importance of income statement items: (a) from 1995 to 2013 in NASDAQ and (b) from 2000 to 2013 in SSE. *, ■, +, △, ◇, ●, and ▼ are respectively total assets, net assets, total revenue, operating income, net income, operating cash flow, and number of employees.

$$PBR_i(t) = \frac{x_i(t)}{A_i(t)}, \quad (2)$$

where $x_i(t)$ is the market capitalization of firm i on the settlement day in year t and A_i is its net assets in year t . Fig. 5 shows the distributions of $PBR(1997)$ in the pre-bubble period, of $PBR(1997)$ in the bubble period, and of $PBR(2004)$ in the post-bubble period in NASDAQ. The distribution became fat during the bubble period and returned to its former position after the bubble burst.

Next, we focus on SSE. Unlike NASDAQ (Fig. 4(b)), its market capitalizations reflect two income statement items: net assets and net income. Because net income is rarely around zero, the market capitalization divided by the net income becomes extremely big at that time. The upper tail of the distribution of divided market capitalization sensitively responds to the fluctuation of net income. Future work will propose market capitalization adjusted by both net assets and net income.

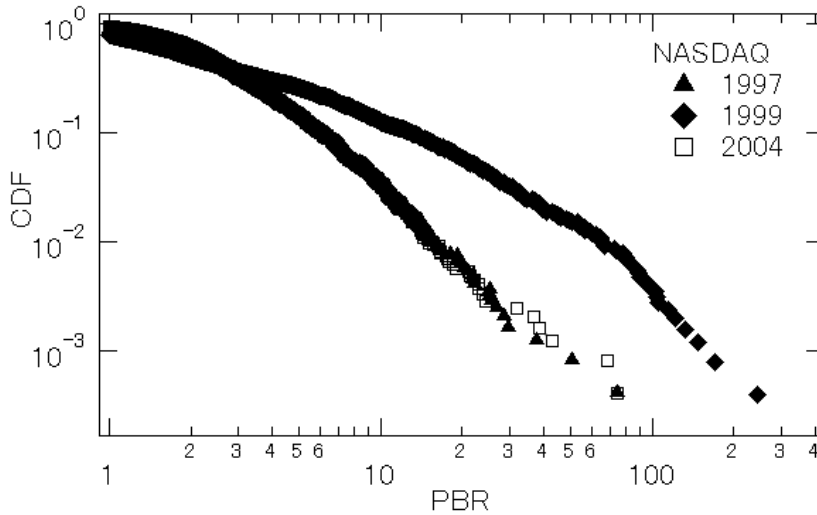


Fig. 5 Cumulative distributions of PBR in NASDAQ in (▲) 1997, (◆) 1999, and (□) 2004.

5 Conclusion

We showed the distributions of market capitalization in NASDAQ and SSE. The upper tails of the distributions follow a power law. The power law index, which fluctuates around one depending on the economic conditions, became small during the 2000 dot-com and 2007 Shanghai bubble periods, suggesting that speculative money was excessively concentrated on specific stocks during such periods.

In economics, a stock bubble is defined by the gap between firm sizes in the stock market and in real economies. We used market capitalization and income statements to estimate the firm sizes in stock markets and real economies. By using the regression coefficient of random forests for market capitalization and income statement items, we found that net assets are most reflected in the market capitalization for NASDAQ firms. For such firms, PBR is defined as market capitalization divided by net assets. The PBR distribution also got fat

during the dot-com bubble period. This result means that the gap between firm sizes in asset markets and in real economies widened during the bubble period.

Both net assets and net income are greatly reflected in market capitalization for SSE firms. Market capitalization, divided by net income, becomes extremely big when the net income is close to zero. Therefore, the upper tail of the distribution of divided market capitalization sensitively responds to the fluctuation of net income. One future work will propose market capitalization that is adjusted by net assets and net income to investigate the 2007 Shanghai bubble.

Although market capitalization is made public every day, income statements are usually announced only quarterly and annually. This difference in time scale complicates estimating daily gaps between firm sizes in stock markets and real economies. Another future work will nowcast the key items of income statements every day.

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