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Central Bank Cryptocurrencies in a Competitive Equilibrium Environment: Can Strong Money Demand Survive in the Digital Age?

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Abstract: This paper discusses the possible macroeconomic consequences of the introduction of cryptocurrencies by central banks (so-called central bank cryptocurrencies or CBCCs) in a competitive equilibrium environment. In this setup, central banks set not only the money supply, but also the interest rate on CBCCs, whereas bond interest rates, the price level, and the exchange rates between CBCCs are determined in competitive markets. We first resolve a severe confrontation between the quantity theory of money (QTM) and the fiscal theory of the price level (FTPL) in that as long as the currency interest rate lies below the bond interest rate, the QTM is applicable in principle. However, once the bond interest rate (asymptotically) matches that of the currency, the FTPL takes the place of the QTM. We then investigate whether the introduction of CBCCs plays a role in the disappearance of strong money demand (currently present at near-zero interest rates in Japan) and its alternatives. We find that if a central bank sets the currency interest rate below a near-zero bond interest rate, then strong money demand disappears, and the massive issuance of long-term public bonds is no longer absorbed in currency markets. However, once the consolidated government succeeds in lowering the currency interest rate to be deeply negative, it can obtain immense seigniorage, allowing it to repay these public bonds. In addition, if the bond interest rate also falls, even below zero for long periods, then the government can exploit seigniorage from CBCC holders without limit.

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1. Introduction

Can strong money demand, which is currently present at near-zero interest rates in Japan, survive in the digital age? Does the emergence of central bank cryptocurrencies (CBCCs) expel strong money demand from the macroeconomy? We provide definitive responses to these questions by the end of this paper. However, for this challenging purpose, we first require a long detour. Why do cryptocurrencies attract acute interest from those in the financial world in the first place? One of the most important reasons is that it is possible to transfer cryptocurrencies promptly in any amount to whoever transacts within a cryptocurrency system. That is, the electric delivery of cash from one peer to another may be just as convenient as the physical delivery of cash from one hand to another. As suggested by the title of Nakamoto (2008), cryptocurrencies may simply be in the form of peer-to-peer electric cash, as opposed to hand-to-hand physical cash.

Nevertheless, cryptocurrencies, delivered on networks and not in person, still appear as something mysterious to many. In the case of physical cash, that one has a note in a wallet immediately guarantees one as the legitimate owner. If that owner as payer then delivers this by hand, the other receiving it as payee can rightly claim to be the new owner. This is why delivering cash from one person to another by hand immediately implies settlement between the payer and payee. However, in the case of electric cash, owners do not retain tokens, with ownership only recorded in electric ledgers. Why then among those participating in a cryptocurrency system do they believe that a certain token belongs to a particular person as the legitimate owner just by recording it? They believe so because ledgers are trusted to record this information with unrivaled accuracy, with a complete record of the changes in the ownership of tokens from the initial issue to the present.

Any cryptocurrency system requires three kinds of cryptanalytic mechanisms to guarantee the extreme accuracy of the electric ledgers distributed among the system participants. First, owing to asymmetric cryptography, only the true owner of a token can transfer it to another, while only the one receiving it from its true owner can employ it for payment. While there is a pairing of public and secret keys in asymmetric cryptography, a public key identifies every participant in a cryptocurrency system. A legitimate owner of a token can then use this by placing his own secret key, unknown to others, into the system. In remitting cash, the owner as payer transfers a token to another identifiable person with a public key, such that only the payee can unlock a token-use restriction using the secret key, paired with the key publicly posted by the payee. Utilizing asymmetric cryptography in this way, the one receiving a token from its true owner can transfer it to another as the new legitimate owner.

Second, a cryptanalytic mechanism prevents double payments in an effective manner. With physical cash such as coins and banknotes, the transfer of a token to a particular person immediately implies that it is not possible to transfer it to anyone other than this recipient. With physical cash, unless counterfeited at considerable cost, it is almost impossible to make double payments. In the case of electric cash, however, it is relatively easy to duplicate tokens on distributed ledgers. Nevertheless, while even a true owner may counterfeit a token, and then transfer false tokens to multiple peers, any cryptocurrency system will involve sophisticated devices to prevent double payments. As an example, with Bitcoin, a representative private cryptocurrency system (described in Section 2), special participants, known as miners, have an incentive to check rigorously for double payments every ten minutes.

Third, a cryptanalytic mechanism also prevents the falsification of any record in the distributed ledgers. A brief explanation of this mechanism is that a distributed ledger records for each token its transaction history from the initial issue to the present using a long sequence of binary numbers, zero or one. In most cryptocurrency systems, a hash function seals this binary sequence at some time interval, say every ten minutes. This transforms the binary sequence into a particular 256-digit binary sequence. Importantly, once a hash function seals a token's transaction history, a vestige of broken seals easily detects any record falsification; that is, the hash function yields a very different number from any falsified sequence, even if slightly altered. Combined, these three mechanisms ensure that the recorded transaction history of each token entails extreme accuracy, thereby preventing double payments and record falsification in an effective manner.

International remittances can exploit these conveniences provided by cryptocurrency systems to the maximum degree. In the existing currency system, fund transfers over national borders operate in a complicated manner, that is, via bucket brigades among multiple private banks and through the intermediation of several central banks. As shown in **Figure 1-1**, for example, a money transfer from Firm A in Country I to Firm B in Country II involves two central banks (located in Country I and Country II), and Private Banks a, b, and c. In the actual remittance system, Private Bank c, which has branches in both countries, may not necessarily intermediate over national borders, if Private Bank a and Private Bank b are connected via SWIFT (Society for Worldwide Interbank Financial Telecommunication). In any case, international money transfers are quite time-consuming and costly because of the involvement of multiple private banks and several central banks. However, as **Figure 1-1** also illustrates, it is possible in a cryptocurrency system to transfer tokens directly from Firm A in Country I to Firm B in Country II without any intermediation by private or central banks. Accordingly, cryptocurrency systems achieve quick and cheap international transfers.

Given the highly convenient functions described, cryptocurrencies are likely to substitute for or complement existing central bank (CB) currencies such as reserves and notes. In fact, not a few central banks have begun large-scale experiments to test the validity of CBCCs, and conduct practical trials. For example, the Bank of Canada (BOC) and the Monetary Authority of Singapore (MAS), both of which were among the first central banks to trial CBCCs, are jointly pursuing the Jasper–Ubin project (BOC and MAS, 2019). Elsewhere, the European Central Bank (ECB) and the Bank of Japan (BOJ) are also undertaking a joint project known as STELLA (ECB and BOJ, 2020).

In this paper, we demonstrate that CBCCs make for a possibly significant impact on the nominal pricing system, consisting of the price level and the nominal rates of interest, owing to the following two features, either of which is practically impossible using traditional banknotes or coins, but quite possible with the newly introduced CBCCs. First, it is difficult to remunerate conventional banknotes or coins regularly. In the case of a banknote, it is challenging to determine how long a legitimate owner precisely holds it before handing it to another because there is no transaction record on either its face or reverse side. Thus, according to the holding period, there may not be any positive (or negative) interest on banknotes or coins. If instead positive interest is forced on a banknote, the banknote owner needs to go to a bank's offices to collect the positive coupons, and have their receipt recorded on either side of the banknote itself. In the case of negative interest, banknote owners will have to stick stamps on notes at a fixed interval to validate their own notes. In contrast, in the case of cryptocurrencies, it is quite easy to remunerate tokens with either positive or negative interest because the electric ledgers correctly record the holding period for each owner of a particular token.

Second, it is physically inconvenient to hold different kinds of banknotes with flexible exchange rates. For example, it is much easier to pay using yen notes only than a combination of yen and dollar notes. Once selected for daily use, one also does not need to worry about the exchange rate between and among yen notes and coins as the exchange rates between different yen currency notes and coins are fixed at one-to-one rates. For example, it is always possible to exchange a ¥1,000 note for two ¥500 coins. However, if CB currencies are transacted not hand-to-hand physically but peer-to-peer electronically, flexible exchange rates among the various cryptocurrencies issued by the same central bank will not sacrifice any currency convenience. For example, it is convenient for consumers and retailers to use different CBCCs with flexible exchange rates at the same time once electric wallets have money-changing functions installed for the various kinds of electronic tokens.

In sum, unlike conventional banknotes or coins, it is easy to remunerate CBCCs with either positive or negative interest, and exchange them at flexible rates among various kinds of tokens. This is likely to change the current currency system drastically in which CB currencies such as banknotes, coins, and reserves are not in principle remunerated,[3] and with a one-to-one exchange rate legally established among the

[3] There are important exceptions in which it is possible to add positive or negative interest to excess reserves held at a central bank. However, apart from some quantity restrictions, remunerated reserves continue to be one-to-one exchanged for unremunerated reserves.

different CB currencies.[4]

In this paper, we demonstrate that this revolutionary reform in currency systems also revises conventional monetary theory fundamentally. More concretely, the introduction of CBCCs with remuneration and flexible exchange rates will change how central banks operate in a competitive equilibrium environment. In most existing monetary models, a central bank is intended to directly control short-term interest rates as in actual policy practice, despite a common theoretical setup in which all agents, including a central bank and a government, behave as price-takers in competitive markets. However, such a combination of policy practice and theoretical setup often generates serious logical inconsistencies. As one of the best-known examples, Sargent and Wallace (1975) argue that if a central bank directly controls short-term interest rates, the price level is indeterminate, and inflation rates fluctuate via self-fulfilling expectations without any exogenous shocks.

The fiscal theory of the price level (FTPL), which assumes that a government and a central bank behave as a consolidated government, is an alternative theoretical device used to determine a unique price level under interest-rate controls. However, the FTPL does not help resolve this logical inconsistency, but instead contributes to deepening it further. Under the FTPL, a consolidated government's intertemporal budget constraint, which states that the current real balance of public bonds is equal to the present value of future fiscal surpluses, needs only hold at the equilibrium price level. In other words, the budget constraint serves as an equilibrium condition for public bond markets in the FTPL. This setup of the FTPL is inconsistent with that of a competitive equilibrium environment, in which a budget constraint of any agent holds at not only on- but also at off-equilibrium prices. To overcome this logical inconsistency, Bassetto (2002) presents a new equilibrium concept as an alternative to a competitive equilibrium, and treats the consolidated government as a big player, who is no longer a price-taker.

Here is yet another logical inconsistency. In the presence of interest-free CB notes, but in the absence of exchange markets for CB currencies, the introduction of interest-bearing excess reserves creates opportunities for arbitrage. In particular, unconventional monetary policies accompanied by negative interest on excess reserves would trigger a large-scale shift from excess reserves to CB notes without any quantity control on currency holdings. The presence of arbitrage opportunities clearly jeopardizes the existence of competitive equilibria. Bassetto (2004) again proposes an alternative to a competitive equilibrium framework to justify negative interest rate policy. However, taking leave of a competitive equilibrium framework immediately implies that its analytical simplicity and lucidity are lost altogether. In Bassetto (2002, 2004), analysis

[4] As discussed in Sargent and Velde (1999), England, Continental Europe, and North America established the standard formula through which any CB currency is convertible at one-to-one exchange rates only in the nineteenth century. In this regard, the current currency system has a relatively short history.

indeed becomes extremely complicated, even for his admittedly simple setup. In the context of monetary theory, it may then be a better idea to stick to a competitive equilibrium framework instead of abandoning it completely.

As discussed, the introduction of CBCCs is likely to expand greatly the set of available monetary policy instruments. For example, a central bank can set interest rates on currencies as well as a supply plan for each currency, and create exchange markets among CB currencies. Consequently, money market rates, exchange rates among CB currencies, and the price level in a core CB currency are determined in a competitive equilibrium manner. Unlike the current currency system, a central bank never exercises direct controls over money market rates, but instead sets only the currency interest rate.[5] In this case, the currency interest rate serves only as the lower bound for market interest rates.

In this paper, we investigate in detail how to reformulate two major monetary theories, namely, the quantity theory of money (QTM) and the FTPL, in this competitive equilibrium setup. As discussed in Section 3, as long as the market interest rate lies above the currency interest rate in a core currency, the standard QTM in principle holds. That is, the price level is still proportional to the aggregate quantity of currencies after the conversion of currency exchange rates. Alternatively, if the market interest rate coincides with the currency interest rate in a core currency, and real money demand saturates at a certain level, then the standard FTPL holds. Because CB currencies do not yield any additional liquidity service, but carry the same interest rate as bonds, public bonds and CB currencies are now exactly equivalent. In this situation, the real balance of both public bonds and additional CB currencies should equal the present value of any future fiscal balances.

However, if real money demand never saturates, and the market interest rate (asymptotically) matches the currency interest rate, then the augmented FTPL is applicable, as discussed in Saito (2020a, 2020b). The real balance of public bonds in excess of future fiscal surpluses is now supported by excess money demand (Saito, 2020b), or a bubble term coexisting with strong money demand (Saito, 2020a). In the augmented FTPL, strong money demand, which we may interpret as the bubble term, generates deflationary pressures on the price level, despite the rapid expansion of currencies and public bonds.

As discussed so far, we can clearly respond to the question posed at the beginning of this paper, "Can strong money demand survive in the digital age?" If a central bank controls the currency interest rate below a near-zero market interest rate, strong money demand never emerges. That is, unless the lower bound for market interest rates, as imposed by the currency interest rate, is binding at any moment, the QTM always holds,

[5] Iwamura (2016) distinguishes between currency and bond interest rates, treating the former as policy instruments and the latter as market rates.

and the price level is accordingly proportional to the aggregate quantity of currencies. At the same time, strong money demand never absorbs massively issued public bonds. As discussed in Saito (2020a, 2020b), the price level would experience a one-off jump immediately after strong money demand disappears. In this way, the introduction of CBCCs provides an opportunity to restore both the QTM in currency markets and fiscal sustainability in public bond markets, but also triggers a large-scale adjustment in the price level and market interest rates, particularly long-term rates, during the normalization process for currency markets.

However, if the consolidated government can maintain a negative currency interest rate far below the near-zero market interest for a long period, the situation changes dramatically. In this case, it loses strong money demand as an instrument to support the massive issuance of public bonds, but instead may obtain immense seigniorage from currency holders. If not only the currency interest rate, but also the market interest rate is negative in terms of some core currency, then the consolidated government can enjoy seigniorage without any limit, thereby sustaining massively issued public bonds. However, double negative rates tax currency holders heavily, and they may forgo the use of CB currencies for everyday settlement.

This paper is organized as follows. In Section 2, we briefly explore how Bitcoin works as a representative private cryptocurrency, and then how CBCCs deliver convenience equivalent or even superior to existing CB reserves and notes. We also demonstrate that the system of CBCCs is much simpler than that of Bitcoin. A major reason for this is that with private cryptocurrencies, participants never trust each other, whereas with CBCCs, central banks and private banks authorized by central banks obtain the trust of participants in a currency system. In Section 3, we investigate how the QTM and the FTPL change in a competitive market equilibrium with multiple currency interest rates and currency exchange markets. Whether the QTM or FTPL holds then depends on the difference between market and currency interest rates, while how the consolidated government obtains seigniorage is contingent on the signs of the market and currency interest rates in some core currency. Section 4 offers the final answer to the initial question, "Can strong money demand survive in the digital age?"

2. An overview of CBCCs

2.1. How do private cryptocurrencies work?

We first describe how Bitcoin, a representative private cryptocurrency, works according to Nakamoto (2008), among others, and then compare it to CBCCs. The main reason for this is that Bitcoin is a private cryptocurrency in which system participants never trust each other, and thus exhibits a striking contrast with CBCCs in which participants place great confidence in a central bank as the system operator. Given such high confidence in a central bank, CBCCs have a much simpler structure for consensus building for transaction records in the electric ledgers distributed among system participants compared with those for private cryptocurrencies such as Bitcoin.

For example, Bitcoin implements a special mechanism, called proof-of-work, to prevent remitters from making double payments. All transactions in electric tokens stay together in the one block every ten minutes, and multiple miners, engaged in closely examining each block, must present proof-of-work for their investigation by devoting a tremendous amount of computational resources. In the presence of such proof-of-work, it also costs substantially more to balance altered records whenever there is any falsification in the distributed ledgers. With Bitcoin, as anyone can access the online ledgers and participate in investigating each block, system participants never trust each other. Without such a proof-of-work mechanism, perfect strangers in a system never believe that miners have a proper incentive to check double payments thoroughly, and little inducement to alter open ledgers.

Let us take a closer look at a distributed ledger system, referred to as a blockchain. With Bitcoin, the recording of token transfers is in chronological order, in series, and in binary sequences, with all recorded transactions organized in the one block every ten minutes. A new block, just examined for double payments by miners, is then stacked on these blocks. Thus, a distributed ledger system is a chain of blocks, or a blockchain.

Bitcoin seals this chain of blocks as follows. To start, all transactions in the one block are summarized by a Merkle tree, and time-stamped by a hash function. More concretely, a long binary sequence, summarized by a Merkle tree, is returned as a 256-digit binary value by a hash function. Once a sealed transaction record is falsified, the hash value from any altered sequence differs completely from the hash value originally computed from the authentic sequence. In Bitcoin, as shown in **Figure 2-1**, a hash value is generated from not just the transaction sequence in one block, but also from the sequence extended by the hash value of the previous block. Consequently, if falsification is made in any past block, then the hash values for all of the blocks following this falsified block differ completely from the initially time-stamped values. In this way, any ledger falsification is immediately recognized by the participants concerned.

In this blockchain system, proof-of-work works as follows. Miners, engaged in checking double payments, do not merely compute hash values from the summarized sequence plus the previous hash value, but have to satisfy demanding conditions for a newly generated hash value. As shown in **Figure 2-2**, a miner inserts a nonce, an arbitrary binary sequence, between the hash value from the previous block and the summarized sequence, and has to present a hash value with N digits initially equal to all zeros. As the Bitcoin system chooses a larger N, it takes more computational resources for a miner to find an appropriate nonce. For example, if $N = 40$, then the probability that the initial 40 digits are all zeros in the one trial is about one over 1.1 trillion ($\approx 1/2^{40}$). A miner finding an appropriate nonce before anyone else can receive a certain number of newly supplied tokens as a reward.

In the Bitcoin system, the value of newly supplied tokens is equal to the value of the entire computational resources used, as devoted by all miners participating in proof-of-work.[6] Suppose that I participants as miners have identical computational abilities. Each miner wins new tokens as a reward with probability $\frac{1}{I}$, and the expected value of this reward is equal to $(l \times P_{BC})/I$, where l is the number of new tokens, and P_{BC} is the price per token. Miners will participate in competition for proof-of-work as long as the expected reward exceeds the computation cost to find a nonce for a required N, or $C(N)$. That is, the following inequality is available for P_{BC}.

$$P_{BC} \geq \frac{I \times C(N)}{l}$$

Given that the above inequality holds, increasingly more miners continue to participate in proof-of-work. Accordingly, $P_{BC} = [I \times C(N)]/l$ is satisfied in equilibrium. That is, the value of Bitcoin is proportional to the total computational resources devoted by all participating miners ($I \times C(N)$).

The Bitcoin system controls N, such that the number of newly supplied tokens can be almost constant every ten minutes. That is, the token supply is nearly fixed per unit of time. As the demand for Bitcoin becomes stronger, more and more miners participate in proof-of-work with the anticipation of token appreciation. Then, given N, it takes less time for miners to find an appropriate nonce, and the token supply increases per unit of time. To maintain a fixed supply every ten minutes, the system raises N as a way to place tasks that are more difficult on the increasing number of miners. Conversely, with the anticipation of token depreciation accompanied by weaker demand, the system lowers N to make a decreasing number of miners find an appropriate nonce more easily. Given this fixed nature of token supply, the value of Bitcoin is quite sensitive to fluctuations in token demand; that is, the price of Bitcoin appreciates (depreciates) quickly with stronger (weaker) token demand.

This proof-of-work mechanism, which disciplines miners for the examination of double payments, also contributes to discouraging ledger alteration. Given extremely costly proof-of-work, it is next to impossible for anyone to quickly balance falsified records in a completely sealed block by recomputing hash values for the following blocks consistently given the past series of N. However, this also works to sacrifice the convenience of Bitcoin as a currency. First, the major convenience of hand-to-hand physical cash such as banknotes or coins comes from immediate settlement after handing it to someone. However, it takes a relatively long time for peer-to-peer electric cash such as Bitcoin to complete settlement. In the case of Bitcoin, it takes 6 blocks or about one hour to finalize ordinary transactions, while it takes 100 blocks or more than 16 hours to activate the newly supplied tokens as reward.

[6] Iwamura et al. (2019) provide a detailed discussion of the valuation of Bitcoin.

Second, all currency systems have evolved in such a way that operational costs may be economized, but Bitcoin runs counter to this history of currencies. For example, precious metals such as gold and silver have been common as coinage in the past, but this is mainly because these metals do not find ready use for production or consumption, other than as a means of currency. However, in undertaking proof-of-work, Bitcoin exhausts an enormous amount of computational resources, which are rather useful for production and services. The fact that scarce computational resources are so wastefully used for a private cryptocurrency system may be interpreted as not the evolution of currency, but rather its devolution.

2.2. How do CBCCs work?

We now examine how a central bank introduces cryptocurrencies as substitutes or complements to CB reserves and notes. As outlined earlier, CBCCs have simpler structures in consensus building among system participants than private cryptocurrencies in the following respects. First, the most significant difference between private cryptocurrencies and CBCCs is that an unspecified large number of agents can have access to distributed ledgers, and may be engaged in making a close examination of ledgers in the case of the former. However, only a central bank, and private banks chartered by a central bank, can share transaction records on ledgers, and are then solely responsible for ledger examination in the latter. We refer to this as a permissioned system. Given the confidence placed in central banks and chartered private banks, a permissioned system can be constructed in a much simpler manner.

Second, CBCCs never use resource-wasting and time-consuming mechanisms such as proof-of-work to build consensus among participants. For example, RS Coin, which was developed by cryptologists, employs two-phase commitment for a consensus-building mechanism, in which consensus is formed immediately after all designated participants (cohorts) as ledger checkers send an agreement to a coordinator (Danezis and Meiklejohn, 2016). An earlier version of Stella, which is the CBCC project being carried out jointly by the ECB and the BOJ, adopted Practical Byzantine Fault Tolerance (PBFT), in which if more than two-thirds of participants (permissioned banks) reach agreement in ledger examination, then settlement is finalized immediately (ECB and BOJ, 2017).

Third, another consensus-building mechanism, and one even simpler than the earlier mechanism, is in CBCC systems. In the previous mechanism, the central bank and designated private banks are on an equal footing in making a close examination of ledgers, and they share completely all transactions recorded in a blocked ledger. However, a second phase of Project Jasper, initiated by the BOC, adopted Corda[7] as a distributed ledger system. In Corda, only a central bank (the BOC in this case) administers the entire ledger system, and makes final checks of all transactions (Chapman et al. 2017).

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[7] See Brown (2018) and Hearn and Brown (2019) for detailed descriptions of Corda.

Meanwhile, private banks share ledgers and check records only for transactions carried out as an interested party. Elsewhere, e-krona, a cryptocurrency investigated as a substitute for banknotes by Sweden's central bank (the Riksbank), also employs Corda as a distributed ledger system. With e-krona, only an official third party, other than the central bank, can have access to the entire transaction information (Sveriges Riksbank, 2017, 2018, 2020). In this alternative mechanism, not all transactions made at a certain time interval have to be organized in the one block for perusal purposes, while any financial institution participating in a CBCC system does not have to share the entire ledger.

In a number of countries, serious consideration is being given to CBCC systems characterized by the above features as substitutes or complements to CB reserves and notes. Because consensus building concerning transactions and ledgers can be formed quickly among participants in CB systems, unlike with private cryptocurrencies, payments through CBCC systems can almost mimic real-time gross settlement.

2.2.1. CBCCs substituting for CB reserve accounts

Let us present some examples in which a central bank introduces cryptocurrencies as alternatives to CB reserves. In this case, system participants consist of only private banks opening current accounts at the central bank. One of the earliest experiments testing this category of CBCCs is Project Jasper by the BOC.[8] In Jasper, private banks exchange Canadian dollars (CAD) as legal tender for CAD-Coins issued as a cryptocurrency by the BOC. These CAD-Coins can then be used for large-order settlements between private banks. In the first phase of Project Jasper, proof-of-work was employed as a consensus-building mechanism, but was abandoned because of the extremely time-consuming settlement process. As noted, in response, the second phase adopted Corda as a distributed ledger system, which enabled settlement by CAD-Coin to almost mimic the finalization achieved through real-time gross settlement.

In this second phase, the BOC conducted an experiment to test a liquidity-saving mechanism, in which designated-time net settlement was made feasible in a distributed ledger system. In this mechanism, a private bank temporarily places a large remittance order in a waiting queue when its holding of CAD-Coin is short of this remittance. At the same time, other banks place similarly large orders in queues. At a certain point of time, the BOC as the central clearinghouse provides net settlement for the large orders accumulated in queues by the private banks.

The large-order CBCC system has since been extended to not only settlement between private banks, but also to delivery versus payment (DVP) for security settlement as well as international money transfers. As noted, the BOC and the MAS are undertaking joint work to develop a CBCC system in this direction (BOC and MAS,

[8] See Chapman et al. (2017) for an overview of Project Jasper.

2019), and using an early experiment in Stella, the ECB and the BOJ have successfully achieved gross settlement in a relatively short time using PBFT. Later, they also adopted Corda and applied it to a liquidity-saving mechanism as well as DVP for security settlement (ECD and BOJ, 2017, 2018).

2.2.2. CBCCs as substitutes for CB notes

While CBCCs as substitutes for or complements to CB reserves attract broad support among those in the financial world, CBCCs as substitutes for CB notes encounter fierce opposition from a number of economists and analysts. Berentsen and Schär (2018) and Bindseil (2020) oppose the introduction of CBCCs partly because CBCCs may be used for illegal transactions or money laundering given their anonymity, and partly because it is difficult for instantaneous settlement to be achieved given their time-consuming requirements for consensus building. Instead, they propose that individual households and firms should be allowed to directly open current accounts at a central bank, and recommend such simple and traditional CB current accounts as substitutes for CB notes. They believe that the rigorous and thorough administration of these accounts by the central bank effectively prevents illegal transactions and money laundering.

However, small-order CBCCs, provided through the intermediation of private banks by a central bank, can achieve almost the same instantaneous settlement as CB notes and coins. As noted, in Sweden, e-krona as a CBCC for small-order settlements by the Riksbank allows for instantaneous settlement even among individuals and firms using Corda as a distributed ledger system. In Corda, consensus building is never time-consuming given that only an official third party administers the entire transaction records (Sveriges Riksbank, 2017, 2018, 2020). In addition, with RS Coin, financial institutions called mintettes (which are not necessarily private banks) use a simple consensus-building mechanism (two-phase commitment) to avoid time-consuming settlement (Danezis and Meiklejohn, 2016). In addition, private banks, which intermediate the issuance of CBCCs between individual users and the central bank, are expected to install CBCCs for various kinds of financial services, such as useful electric wallets, thereby improving the convenience of CBCCs.

In principle, anonymity in currency transactions can be eliminated completely in a CBCC system. Because a central bank or an official third party in CBCCs administers all transaction information, it is indeed next to impossible to use CBCCs for illegal transactions in either wholesale or retail settlement. However, those in the financial world remain deeply concerned that anonymous CBCC transactions are completely absent, particularly in retail settlement. With the digital euro, which has been considered for retail settlement by the ECB, the Anti-Money Laundering (AML) Authority is responsible for the prevention of illegal transactions, but allows for anonymous currency transactions in retail settlement (ECB, 2019). Concretely, digital euro users receive anonymity vouchers from the AML Authority, and they attach the vouchers in remitting digital euros up to limited amounts to avoid inspection by the AML Authority.

Viewed in this way, it may be better to issue CBCCs as substitutes for or complements to CB notes for retail settlement not directly through a central bank, but via the intermediation of private banks to improve currency convenience for individual users, while maintaining a moderate degree of anonymity for retail transactions.

3. CBCCs in a competitive equilibrium environment

3.1. How to formulate CBCCs in macroeconomic models

As discussed in Section 1, it is possible to remunerate CBCCs with either positive or negative interest, but convert them at flexible market rates. Systems of CBCCs with these two features should have a drastic impact on the current currency system, in which CB currencies are interest-free, and any CB currency is converted in a one-to-one fashion. However, as explored in Section 2, it is difficult to imagine that CBCCs as substitutes for and complements to CB reserves and notes would sacrifice any currency convenience for private banks, firms, or individuals. On the contrary, we would expect currency convenience to improve with the financial services for CBCCs provided by private banks. In this section, we demonstrate that the introduction of such convenient CBCCs with these features indeed reformulates central theorems in conventional monetary theory, particularly, the theory of the price level.

Two research agendas concern the possible macroeconomic impact of the introduction of CBCCs. The first concerns the interest on CB currencies, with the conversion between CB currencies and private deposit currencies explored as policy instruments to control a large-scale shift between private deposits and CBCCs. In a broader macroeconomic context, Benes and Kumhof (2012), Raskin and Yermack (2016), and others discuss the introduction of CBCCs as an effective instrument to implement the Chicago Plan, in which private deposits backed by commercial loans are replaced entirely by those backed by CBCCs for financial stability.

In a narrower macroeconomic context, Barrdear and Kumhof (2016) investigate the possible macroeconomic effects of a large-scale shift from private deposit currencies to CBCCs.[9] Kumhof and Noone (2018) propose the following as controls in this currency shift: (i) negative interest is temporarily added to the CBCC; (ii) conversion between CBCCs and conventional CB reserves is forbidden; (iii) conversion from private deposits to CBCCs is not automatically guaranteed; and (iv) CBCCs should be backed not by private deposits or bonds but by public bonds only. Bindseil (2020) makes a more moderate proposal in which CBCCs are remunerated with either positive or zero interest

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[9] Barrdear and Kumhof (2016) do not analyze any substitution between traditional CB reserves/notes and CBCCs.

up to a certain limit, but the amount of CBCCs beyond this upper limit is remunerated with either negative or zero interest. According to Bindseil, such two-tier remuneration substantially reduces the attractiveness of holding large volumes of CBCCs, and works to prevent a large-scale currency shift to CBCCs.

The second research agenda theoretically addresses the exchange market between interest-free CB notes and negative-interest-bearing CBCCs in detail. As surveyed by Rogoff (2016), the idea that negative interest was added to currencies, and that these then negative-interest-bearing currencies would be exchanged with interest-free currencies at market rates was originally proposed by Eisler (1933).[10] In a modern macroeconomic context, Buiter and Panigirtzoglou (2003), Agarwal and Kimball (2015), and others rigorously investigate the exchange market between negative-interest-bearing and interest-free currencies.

In this section, we pursue this second agenda in a macroeconomic environment. That is, we investigate the possible macroeconomic consequences of the remuneration and conversion of CBCCs. At a more fundamental level, we attempt to resolve serious dilemmas that emerge in conventional monetary theories. In conventional monetary models, a central bank and a government directly control the short-term interest rate and even the price level, but this assumption seriously contradicts that of competitive equilibrium, in which any agent, including the central bank and the government, behave as price-takers. However, such a contradiction can be resolved once we can remunerate and convert CBCCs at market rates. In this case, the central bank sets the currency interest rate, but the bond interest rate is determined in money markets. Owing to this separation between the currency and bond interest rates, the severe confrontation between the QTM and the FTPL can be resolved in a consistent manner. Given macroeconomic investigation of CBCCs, we will finally tackle the question we posed at the beginning of this paper, "Can strong money demand survive in the digital age in which various kinds of CBCCs are issued?"

3.2. Basic setup

Let us present a basic setup of a monetary model with CBCCs introduced in a competitive equilibrium environment. In this model, utility at time t arises not only from consumption $c(t)$, but also from a conventional interest-free CB note (currency 0), and an interest-bearing CBCC (currency 1), whose convenience may be equivalent to, or even superior to existing CB reserves and notes, as described in Section 2.

The interest rate on currency 1 ($i_1(t)$), as well as the supply plans for currencies 0 and 1 are chosen by the central bank. In contrast, the price level ($P_1(t)$) and the short-term interest rate on public bonds ($i_B(t)$), both of which are quoted in terms of currency

[10] Ilgmann and Menner (2011) explore the historical contexts in which currencies have been remunerated with negative interest.

1, are determined in competitive markets. In addition, the exchange rate of currency 0 per unit of currency 1 ($e(t)$) is agreed to in markets. While some part of the consumption goods may be quoted in currency 0 in a more realistic setup, all consumption goods are assumed to be quoted in terms of currency 1 in the present setup.

The representative household's period utility is formulated as

$$u\big(c(t)\big) + v_0\left(\frac{M_0(t)}{P_0(t)}\right) + v_1\left(\frac{M_1(t)}{P_1(t)}\right), \qquad (6\text{-}3\text{-}1)$$

where $P_1(t)$ ($P_0(t)$) is the price level in terms of currency 1 (0), and $M_1(t)$ ($M_0(t)$) is the nominal balance of currency 1 (0) and $u(c)$ is increasing, concave, and twice differentiable. The variables $v_1(m)$ and $v_0(m)$ are also increasing and concave, but differentiability takes any of the following three forms. First, as assumed in Saito (2020a, 2020b), both functions are twice differentiable, and $\lim_{m\to\infty} v_1'(m) = 0$ and $\lim_{m\to\infty} v_0'(m) = 0$ hold. That is, the utility from real money balances asymptotically approaches its upper limit as shown in **Figure 3-1**. Second, money utility saturates at $\bar{m}$, and its marginal utility from the right is zero at $\bar{m}$ ($v_1'(\bar{m}) = 0$ and $v_0'(\bar{m}) = 0$) as in **Figure 3-2**. Third, money utility is constant for $m \geq \bar{m}$, and its marginal utility is zero for $m > \bar{m}$ as in **Figure 3-3**.

A budget constraint for the representative household is derived as follows:

$$\begin{aligned}
&B(t) + M_1(t) + \frac{1}{e(t)}M_0(t) \\
&= P_1(t)[y(t) - c(t) - tax(t)] \\
&+[1 + i_B(t)]B(t-1) + [1 + i_1(t)]M_1(t-1) + \left[1 - \frac{\Delta e(t)}{e(t-1)}\right]\frac{1}{e(t-1)}M_0(t-1),
\end{aligned}$$

where $B(t)$, $M_1(t)$, and $M_0(t)$ are the end-of-period nominal balances of public bonds, currency 0, and currency 1, respectively. $y(t)$ and $tax(t)$ denote real household income and real tax at time t, respectively. The nominal interest rates on public bonds ($i_B(t)$) and currency 1 ($i_1(t)$) are determined at the beginning of the period, but the price level ($P_1(t)$ and $P_0(t)$) and the exchange rate of currency 0 per unit of currency 1 ($e(t)$) are determined at the end of the period. $\frac{e(t-1)}{e(t)}$ is approximated by $1 - \frac{\Delta e(t)}{e(t-1)}$.

Given $\frac{B(t)}{P_1(t)} + \frac{M_1(t)}{P_1(t)} + \frac{M_0(t)}{e(t)P_1(t)}$ as a state variable at the end of the period, the above budget constraint can be rewritten as follows.

$$\begin{aligned}
&\frac{B(t-1)}{P_1(t-1)} + \frac{M_1(t-1)}{P_1(t-1)} + \frac{M_0(t-1)}{e(t-1)P_1(t-1)} \\
&= \frac{1}{1+\rho(t)}[c(t) + tax(t) - y(t)] + \frac{1}{1+i_B(t)}\left\{[i_B(t) - i_1(t)]\frac{M_1(t-1)}{P_1(t-1)} + \left[i_B(t) + \frac{\Delta e(t)}{e(t-1)}\right]\frac{M_0(t-1)}{e(t-1)P_1(t-1)}\right\} \\
&+ \frac{1}{1+\rho(t)}\left[\frac{B(t)}{P_1(t)} + \frac{M_1(t)}{P_1(t)} + \frac{M_0(t)}{e(t)P_1(t)}\right] \qquad (6\text{-}3\text{-}2)
\end{aligned}$$

The real rate of interest $\rho(t)$ is defined as

$$1+\rho(t)=\left(1+i_B(t)\right)\frac{P_1(t-1)}{P_1(t)}\approx 1+i_B(t)-\frac{\Delta P_1(t)}{P_1(t-1)}.$$

Note that $i_B(t)$ is determined at the beginning of the period, but $\rho(t)$ and $P_1(t)$ are determined at the end of the period.

$[i_B(t)-i_1(t)]\frac{M_1(t-1)}{P_1(t-1)}$ and $\left[i_B(t)+\frac{\Delta e(t)}{e(t-1)}\right]\frac{M_0(t-1)}{e(t-1)P_1(t-1)}$ in equation (6-3-2) correspond to the holding cost of currencies 0 and 1 from the viewpoint of the representative household. For interest-bearing currency 1, $i_B(t)$ in excess of $i_1(t)$ is a holding cost per real unit, while for interest-free currency 0, $i_B(t)$ adjusted by exchange rates is a real holding cost. However, from the viewpoint of the consolidated government, consisting of the government and the central bank, these currency holding costs are equivalent to seigniorage from currency holders. As shown in equation (6-3-2), we discount one-period-ahead real consumption, income, and taxes by the real interest rate, but discount the one-period-ahead real holding costs associated with the two currencies by the nominal interest rate.

Let us move to the budget constraint of the consolidated government. The government's budget constraint is written as follows.

$$B(t)+M_1(t)+\frac{M_0(t)}{e(t)}$$

$$=P_1(t)[g(t)-tax(t)]+[1+i_B(t)]B(t-1)+[1+i_1(t)]M_1(t-1)+\left[1-\frac{\Delta e(t)}{e(t-1)}\right]\frac{M_0(t-1)}{e(t-1)}$$

Because $g(t)$ denotes real government consumption, $tax(t)-g(t)$ represents a fiscal surplus or a primary fiscal balance.

Given $\frac{B(t)}{P_1(t)}+\frac{M_1(t)}{P_1(t)}+\frac{M_0(t)}{e(t)P_1(t)}$ as a state variable at the end of the period, the above budget constraint can be rewritten as follows.

$$\frac{B(t-1)}{P_1(t-1)}+\frac{M_1(t-1)}{P_1(t-1)}+\frac{M_0(t-1)}{e(t-1)P_1(t-1)}$$

$$=\frac{1}{1+\rho(t)}[tax(t)-g(t)]+\frac{1}{1+i_B(t)}\left\{[i_B(t)-i_1(t)]\frac{M_1(t-1)}{P_1(t-1)}+\left[i_B(t)+\frac{\Delta e(t)}{e(t-1)}\right]\frac{M_0(t-1)}{e(t-1)P_1(t-1)}\right\}$$

$$+\frac{1}{1+\rho(t)}\left[\frac{B(t)}{P_1(t)}+\frac{M_1(t)}{P_1(t)}+\frac{M_0(t)}{e(t)P_1(t)}\right] \qquad (6\text{-}3\text{-}3)$$

As discussed, $[i_B(t)-i_1(t)]\frac{M_1(t-1)}{P_1(t-1)}$ and $\left[i_B(t)+\frac{\Delta e(t)}{e(t-1)}\right]\frac{M_0(t-1)}{e(t-1)P_1(t-1)}$ correspond to seigniorage from the viewpoint of the consolidated government.

Solving equation (6-3-3) leads to the consolidated government's budget constraint.

$$\frac{B(t-1)}{P_1(t-1)}+\frac{M_1(t-1)}{P_1(t-1)}+\frac{M_0(t-1)}{e(t-1)P_1(t-1)}$$

$$= \sum_{\tau=t}^{\infty}\left\{\frac{1}{\prod_{k=t}^{\tau}\left(1+\rho(k)\right)}[tax(\tau)-g(\tau)]\right\}$$

$$+\sum_{\tau=t}^{\infty}\left\{\frac{1}{\prod_{k=t}^{\tau}\left(1+i_B(k)\right)}\left\{[i_B(\tau)-i_1(\tau)]\frac{M_1(\tau-1)}{P_1(\tau-1)}+\left[i_B(\tau)+\frac{\Delta e(\tau)}{e(\tau-1)}\right]\frac{M_0(\tau-1)}{e(\tau-1)P_1(\tau-1)}\right\}\right\}$$

$$+\lim_{\tau\to\infty}\left\{\frac{1}{\prod_{k=t}^{\tau}\left(1+\rho(k)\right)}\left[\frac{B(\tau)}{P_1(\tau)}+\frac{M_1(\tau)}{P_1(\tau)}+\frac{M_0(\tau)}{e(\tau)P_1(\tau)}\right]\right\} \qquad (6\text{-}3\text{-}4)$$

As shown in equation (6-3-4), the current real balances of public bonds and the two currencies are equal to the sum of the present value of future fiscal surpluses and seigniorage, and the terminal value of public bonds and the two currencies.

In this section, we carefully consider the following four possibilities in a competitive equilibrium environment where CBCCs may be interest-bearing and exchanged at flexible rates. First, we investigate whether the standard QTM holds, that is, whether stable demand is present for the two currencies, and the two currency price levels $P_1(t-1)$ and $P_0(t-1)$ are proportional to the aggregate quantity of money. For example, monetary aggregation may be expressed by $M_1(t-1)+M_0(t-1)$ or $M_1(t-1)+\frac{M_0(t-1)}{e(t-1)}$.

Second, the standard FTPL is usually where money demand is abstracted from the initial setup as in Woodford (1995, 1998) and others. Here, we explore whether there is room for the FTPL to hold even if money demand is present. As discussed in Section 3.5, if the bond and currency interest rates are equal, and money demand saturates at a certain real balance, then the standard FTPL holds. In this case, CB currencies are equivalent to public bonds in terms of interest as well as no additional liquidity service.

Third, we investigate when public bonds serve as net wealth from the viewpoint of households as shown in Saito (2020a, 2020b). That is, the real balance of public bonds in excess of the future tax burdens ($\frac{B(t-1)}{P_1(t-1)} > \sum_{\tau=t}^{\infty}\left\{\frac{1}{\prod_{k=t}^{\tau}(1+\rho(k))}[tax(\tau)-g(\tau)]\right\}$) is supported by strong money demand as in Saito (2020a, 2020b). As shown in Section 3.5, if the bond and currency interest rates are again equal, and money demand never saturates, then part of the real balance of public bonds serves as net wealth for households. In this case, the FTPL augmented by either strong money demand or the nonzero terminal condition determines the price level. More concretely, the QTM cannot determine the current price level because demand deviates from supply in currency markets, and is instead determined according to the government's budget constraint augmented by strong money demand.

Fourth, we investigate whether we can pay for the massive issuance of public bonds through immense seigniorage from currency holders by imposing negative currency interest rates or widening the spread between the bond and currency interest rates. In this case, strong money demand disappears as the bond interest rate exceeds the currency interest rate, and public bonds are no longer absorbed in currency markets. Instead, the negative interest rate set by the central bank may yield immense seigniorage. In addition, if not only the currency interest rate but also the bond interest rate falls below zero for a long period, the consolidated government can obtain seigniorage without any limit. Immense seigniorage through negative currency interest rates may substitute for strong money demand in supporting massively issued public bonds, but differs fundamentally from strong money demand because public bonds no longer serve as net wealth for households, and have to be repaid through heavy tax burdens from currency holders.

3.3. Optimality conditions, interest parity, and purchasing power parity

We now set $\frac{B(t)}{P_1(t)}+\frac{M_1(t)}{P_1(t)}+\frac{M_0(t)}{e(t)P_1(t)}$ as a state variable in the representative household's budget constraint (6-3-2), and solve the problem to maximize the following life-time utility:

$$\sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}\left[u\big(c(\tau)\big)+v_1\left(\frac{M_1(\tau)}{P_1(\tau)}\right)+v_0\left(\frac{M_0(\tau)}{P_0(\tau)}\right)\right]\right\},$$

where $\delta>0$ is the rate of time preference.

The Euler equation associated with the optimal allocation of consumption between time t and $t+1$ is derived as

$$\frac{1}{1+\delta}\frac{u'\big(c(t+1)\big)}{u'\big(c(t)\big)}\left[1+i_B(t)-\frac{\Delta P_1(t)}{P_1(t-1)}\right]=\frac{1}{1+\delta}\frac{u'\big(c(t+1)\big)}{u'\big(c(t)\big)}[1+\rho(t)]=1. \quad (6\text{-}3\text{-}5)$$

The optimal allocation between consumption and currency 1 or 2 is determined by the equality between marginal currency utility and currency holding costs in terms of marginal utility of consumption.

$$v_1'\left(\frac{M_1(t)}{P_1(t)}\right)=[i_B(t)-i_1(t)]u'\big(c(t)\big) \quad (6\text{-}3\text{-}6)$$

$$v_0'\left(\frac{M_0(t)}{P_0(t)}\right)=\left[i_B(t)+\frac{\Delta e(t)}{e(t-1)}\right]u'\big(c(t)\big) \quad (6\text{-}3\text{-}7)$$

Because the currency interest rate serves as the lower bound for the bond interest rate, the following inequalities hold.

$$i_B(t)\geq i_1(t) \quad (6\text{-}3\text{-}8)$$

$$i_B(t)\geq -\frac{\Delta e(t)}{e(t-1)} \quad (6\text{-}3\text{-}9)$$

As to the interest parity between the two CB currencies, the marginal currency utility from investment in currency 1, and that from investment in currency 0 first, and conversion to currency 1 later, should be equal.

$$i_1(t)v_1'\left(\frac{M_1(t)}{P_1(t)}\right)=-\frac{\Delta e(t)}{e(t-1)}v_1'\left(\frac{M_1(t)}{P_1(t)}\right)$$

Thus, the following interest rate parity relationship holds.

$$i_1(t) = -\frac{\Delta e(t)}{e(t-1)} \tag{6-3-10}$$

Alternatively, the purchasing power from holding currency 1 should match that from currency 0 and converting it to currency 1 in terms of marginal consumption utility yields:

$$\frac{1}{P_1(t)}u'\big(c(t)\big) = \frac{e(t)}{P_0(t)}u'\big(c(t)\big).$$

Thus, purchasing power parity holds.

$$\frac{P_1(t)}{e(t)P_0(t)} = 1 \tag{6-3-11}$$

As policy instruments, the central bank sets interest rates on currency 1 ($i_1(t)$), and determines the money supply plans for the two currencies ($\frac{\Delta M_1(t)}{M_1(t-1)}$ and $\frac{\Delta M_0(t)}{M_0(t-1)}$). In this paper, we consider the simplest case to illuminate the possible impacts of introducing CBCCs in a competitive equilibrium setup. First, we assume that consumption is constant over time.

$$c(t) = c$$

Accordingly, the following relation is obtained from equation (6-3-5).

$$i_B(t) - \frac{\Delta P_1(t)}{P_1(t-1)} = \rho(t) = \delta$$

That is, the real interest rate on public bonds is always equal to the rate of time preference.

The central bank then sets a constant currency growth rate as follows.

$$\frac{\Delta M_1(t)}{M_1(t-1)} = \mu_1$$

$$\frac{\Delta M_0(t)}{M_0(t-1)} = \mu_0$$

In this policy environment, the central bank never faces the choice between its nominal interest rate and money supply plans, but it is able to choose the currency interest rate and money supply plans simultaneously.

In the current competitive equilibrium setup, given c, μ_1, μ_0, $\{i_1(t-1), i_1(t), i_1(t+1), i_1(t+2) \ldots\}$, real money balances ($\frac{M_1(t)}{P_1(t)}$ and $\frac{M_0(t)}{P_0(t)}$), the price levels ($P_1(t)$ and $P_0(t)$), inflation rates ($\frac{\Delta P_1(t)}{P_1(t-1)}$ and $\frac{\Delta P_0(t)}{P_0(t-1)}$), and interest on public bonds ($i_B(t)$) are determined. Note that the exchange rate between the two currencies is determined as the relative price levels from purchasing power parity relationship in equation (6-3-11), while its rate of change is determined by interest on currency 1 ($i_1(t)$) from the interest parity relationship in equation (6-3-10).

3.4. Extended QTM under fixed interest on currencies

Let us first demonstrate how to reformulate the QTM in the presence of stable demand for the two CB currencies. For the moment, assume that the interest rate on currency 1 is fixed at some nonnegative value, while that on public bonds is above the currency interest rate, that is, $i_B(t) > i_1 \geq 0$. If $i_1 = 0$, then currency 1 is equivalent to currency 0 in terms of the currency interest rate, but they still differ in terms of currency convenience.

Suppose that the real demand for currency 1 (0) is stable at $\frac{M_1(t)}{P_1(t)} = m_1$ ($\frac{M_0(t)}{P_0(t)} = m_0$). Then, $v_1'(m_1) = [i_B - i_1]u'(c)$ and $v_0'(m_0) = \left[i_B + \frac{\Delta e}{e}\right]u'(c) = [i_B - i_1]u'(c)$ hold. In this case, inflation rates are equal to the monetary growth rates, $\frac{\Delta P_1}{P_1} = \mu_1$ and $\frac{\Delta P_0}{P_0} = \mu_0$. The nominal rate of interest on public bonds is then equal to the real rate of interest plus the inflation rate of currency 1 ($i_B = \delta + \mu_1$). Given $i_B > i_1$, μ_1 needs to be above $i_1 - \delta$. Note that a low currency interest rate is compatible with low inflation but not with monetary expansion, but rather with monetary contraction in the current setup.

Let us establish the proportionality between the price level and the quantity of money. We average the two currency prices with the weights of the real money balances.

$$P(t) = \frac{m_1}{m_1+m_0}P_1(t) + \frac{m_0}{m_1+m_0}P_0(t)$$

Substituting $P_1(t) = \frac{M_1(t)}{m_1}$ and $P_0(t) = \frac{M_0(t)}{m_0}$ into the above equation leads to the following version of the QTM.

$$P(t) = \frac{1}{m_1+m_0}[M_1(t) + M_0(t)]$$

Two additional versions of the QTM are also available.

$$P_1(t) = \frac{1}{m_1+m_0}\left[M_1(t) + \frac{M_0(t)}{e(t)}\right]$$

$$P_0(t) = \frac{1}{m_1+m_0}[e(t)M_1(t) + M_0(t)]$$

As these three versions of the QTM imply, the price level is proportional to not only the simple sum of money supply ($M_1(t) + M_0(t)$), but also to the exchange rate-weighted sum ($M_1(t) + \frac{M_0(t)}{e(t)}$ and $e(t)M_1(t) + M_0(t)$). We refer to these versions of the price–money relationship as the extended QTM.

Given that the extended QTM holds under $i_B(t) > i_1 \geq 0$, the consolidated government's budget constraint (6-3-4) is rewritten as follows.

$$\frac{B(t-1)}{P_1(t-1)} + m_1 + m_0$$

$$= \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\} + \sum_{\tau=t}^{\infty}\frac{i_B-i_1}{(1+i_B)^{\tau-t+1}}(m_1+m_0) + \lim_{\tau\to\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}\left[\frac{B(\tau)}{P_1(\tau)}+m_1+m_0\right]\right\}$$

$$= \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\} + \frac{i_B-i_1}{i_B}(m_1+m_0) + \lim_{\tau\to\infty}\frac{1}{(1+\delta)^{\tau-t+1}}(m_1+m_0) + \lim_{\tau\to\infty}\frac{1}{(1+\delta)^{\tau-t+1}}\frac{B(\tau)}{P_1(\tau)}$$

For the two terminal conditions on the right-hand side of the above equation, the first condition $\lim_{\tau\to\infty}\frac{1}{(1+\delta)^{\tau-t+1}}(m_1+m_0)$ definitely converges to zero, while the second condition $\lim_{\tau\to\infty}\frac{1}{(1+\delta)^{\tau-t+1}}\frac{B(\tau)}{P_1(\tau)}$ also converges to zero under the assumption that the consolidated government adopts a Ricardian fiscal policy. Then, the above government's budget constraint is simplified as

$$\frac{B(t-1)}{P_1(t-1)} + \frac{i_1}{i_B}(m_1+m_0) = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\}. \qquad (6\text{-}3\text{-}12)$$

As the above equation implies, if the consolidated government sets a positive currency interest rate ($i_1 > 0$), not only the outstanding public bonds ($\frac{B(t-1)}{P_1(t-1)}$), but also the payment of future currency interest ($\frac{i_1}{i_B}(m_1+m_0)$) has to be covered by future fiscal surpluses. Accordingly, the consolidated government faces a much tighter budget constraint.

Note that the current price level is determined according to the extended QTM, not by the consolidated government's budget constraint (6-3-12) as in the FTPL. Equation (6-3-12) merely implies that the obligation of redeeming public bonds and paying currency interest requires financing by future fiscal surpluses at any price level determined by the extended QTM.

3.5. Standard FTPL and augmented FTPL given equivalence between currencies and public bonds

Let us move to the case where the bond interest rate exactly matches the currency case, consumers no longer pay currency-holding costs, while the government never receives seigniorage from currency holders. In terms of optimality conditions, the marginal utility from holding the two currencies is equal to zero.

$$v_1'\left(\frac{M_1(t)}{P_1(t)}\right) = 0$$

$$v_0'\left(\frac{M_0(t)}{P_0(t)}\right) = 0$$

Accordingly, the currencies are equivalent to public bonds in terms of not only interest, but also in that they provide no additional liquidity service.

Given that the bond interest rate equals the currency interest rate, the following three cases are considered, depending on the functional forms of the utility from the real balance of currency. First, if currency utility saturates at $\bar{m}_1$ and $\bar{m}_0$, as shown in **Figure 3-2**, then the standard FTPL takes the place of the QTM in determining the initial price level. That is, the current price level is determined by not only the aggregate money supply, but also by future fiscal surpluses. The reasoning is as follows. For the moment, suppose that the consolidated government matches the real money supply with the saturation level ($\frac{M_1^s(t-1)}{P_1(t-1)} = \bar{m}_1$ and $\frac{M_0^s(t-1)}{P_0(t-1)} = \bar{m}_0$), and adopts a Ricardian fiscal policy ($\frac{B(t-1)}{P_1(t-1)} = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau) - g(\tau)]\right\}$). In this case, the price levels, $P_1(t-1)$ and $P_0(t-1)$, are still determined by the aggregate quantity of $M_1^s(t-1)$ and $M_0^s(t-1)$.

However, the situation changes if there is an additional increase in the supply of currency 1 ($\frac{M_1^s(t-1)+\Delta M_1}{P_1(t-1)} > \bar{m}_1$). Money supply beyond saturation earns the same return as the bond interest rate, but never yields any additional liquidity service. Thus, the additional currency supply (ΔM_1) is identical to public bonds, demanded as public bonds by consumers, and repaid by future fiscal surpluses. As to currency 0, its nominal supply is adjusted to make its real balance equal to $\bar{m}_0$. Then, given the newly determined price level $P_1'(t-1)$, the government's budget constraint is rewritten as follows.

$$\frac{B(t-1)}{P_1'(t-1)} + \left[\frac{M_1^s(t-1)+\Delta M_1}{P_1'(t-1)} - \bar{m}_1\right] = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau) - g(\tau)]\right\} \quad (6\text{-}3\text{-}13)$$

In the above, $P_1'(t-1)$ cannot be determined according to the QTM, because the real money supply temporarily exceeds demand in the currency 1 market ($\frac{M_1^s(t-1)+\Delta M_1}{P_1(t-1)} > \bar{m}_1$). Instead, $P_1'(t-1)$ is governed by the government's budget constraint, or equation (6-3-13), as in the standard FTPL. Accordingly, the change in the price level is less than proportional to monetary growth $\frac{P_1'(t-1)-P_1(t-1)}{P_1(t-1)} = \frac{\Delta M_1}{B(t-1)+M_1^s(t-1)} < \frac{\Delta M_1}{M_1^s(t-1)}$. Then, the government resets the nominal money supply $M_1^{s\prime}(t-1)$ at $P_1'(t-1)\bar{m}_1$, and raises it at the rate of μ_1. Consequently, the real money balance remains $\bar{m}_1$ from then on. Given $\frac{B(t-1)+[M_1^s(t-1)+\Delta M_1 - P_1'(t-1)\bar{m}_1]}{P_1'(t-1)} = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau) - g(\tau)]\right\}$, a Ricardian fiscal policy is recovered at the newly determined price level $P_1'(t-1)$. The monetary rule proposed by Friedman (1969), or $i_B = i_1 = 0$, is included in this case in which real money demand saturates. However, what we emphasize here is that there is room for the standard FTPL but not the QTM, to hold even under Friedman's rule.

Second, as shown in **Figure 3-3**, if the utility from currencies never saturates, but ceases to increase as soon as the real money balance reaches $\bar{m}_1$ ($\bar{m}_0$), then the augmented FTPL takes over the QTM in determining the initial price level. Again, suppose that $\frac{M_1^s(t-1)}{P_1(t-1)} = \bar{m}_1$, $\frac{M_0^s(t-1)}{P_0(t-1)} = \bar{m}_0$, and $\frac{B(t-1)}{P_1(t-1)} = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau) - g(\tau)]\right\}$. So far, the QTM determines the price level, while a Ricardian fiscal policy is in place. Then, the real demand for currency 1 can increase to m_1' without any impact on utility. An additional increase in money demand can absorb public bonds, which are now equivalent to currencies. Again, the nominal supply of currency 0 is adjusted to make its real balance equal to $\overline{m}_0$.

In contrast to the first case, the supply of public bonds in excess of the future fiscal surplus is now demanded as currencies. Thus, the government's budget constraint is rewritten at the new price level $P_1'(t-1)$ as follows.

$$\frac{B(t-1)}{P_1'(t-1)} = \sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\} + \left[m_1' - \frac{M_1^s(t-1)}{P_1'(t-1)}\right] \qquad (6\text{-}3\text{-}14)$$

Given $m_1' > \frac{M_1^s(t-1)}{P_1(t-1)}$, $M_1^s(t-1)$ never determines $P_1'(t-1)$ according to the QTM. Instead, the price level is given by equation (6-3-14), or the present value of future fiscal surpluses, augmented by the current excess demand in the currency 1 market. From then on, both currency 1 supply and the price level grow at the rate of μ_1, and real money demand $m_1(s-1)$ is arbitrarily chosen such that $\frac{B(s-1)}{P_1(s-1)} - \sum_{\tau=s}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\} = m_1(s-1) - \frac{M_1^s(s-1)}{P_1(s-1)}$ may hold at the end of time $s-1$. Accordingly, the government can ignore a Ricardian fiscal policy as long as the bond and currency interest rates are equal. We may refer to the second case as the augmented FTPL as in Saito (2020b). However, note that once the bond interest rate $i_B(t)$ slightly exceeds the currency interest rate i_1, excess currency demand disappears, and the QTM again takes the place of the augmented FTPL.

Third, as shown in **Figure 3-1**, currency utility asymptotically approaches its upper limit. In this case, money demand expands quickly as the bond interest rate $i_B(t)$ converges to the currency interest rate i_1. As in Saito (2020a), we assume that seigniorage $[i_B(\tau)-i_1(\tau)]\left[\frac{M_1(\tau-1)}{P_1(\tau-1)}+\frac{M_0(\tau-1)}{P_0(\tau-1)}\right]$ is reimbursed to households, and that the consolidated government adopts a non-Ricardian fiscal policy. Then, the terminal condition or the bubble term $\left(\lim_{\tau\to\infty}\frac{1}{\prod_{k=t}^{\tau}(1+\rho(k))}\left[\frac{B(\tau)}{P_1(\tau)}\right]\right)$ in the following government's budget constraint neither degenerates to zero nor explodes, but rather converges to a positive finite.

$$\frac{B(t-1)}{P_1(t-1)} - \sum_{\tau=t}^{\infty}\left\{\frac{1}{\prod_{k=t}^{\tau}(1+\rho(k))}[tax(\tau)-g(\tau)]\right\} = \lim_{\tau\to\infty}\frac{1}{\prod_{k=t}^{\tau}(1+\rho(k))}\left[\frac{B(\tau)}{P_1(\tau)}\right] < \infty \qquad (6\text{-}3\text{-}15)$$

Accordingly, the supply of public bonds in excess of future fiscal surpluses is now supported by the positive bubble term. This case may be also termed the augmented FTPL in the sense that it is augmented by the bubble term coexisting with strong money demand. As in the second case, the bond interest rate above the currency interest rate coincides with the bursting of the bubble term, and the government is then forced to adopt a Ricardian fiscal policy under a one-off price jump.

As discussed in the second and third cases, if the bond interest rate (asymptotically) matches the currency interest rate, the augmented FTPL takes over the QTM in determining the initial price level. In this situation, public bonds, now equivalent to currencies, are absorbed by excess money demand or the bubble term coexisting with strong money demand. It is worthwhile emphasizing here that in the two cases, the massive issuance of public bonds is absorbed not by seigniorage, which is basically zero given the equality between the bond and currency interest rates, but by strong money demand, which is triggered by that equality. Conversely, such strong money demand disappears immediately after the bond interest rate exceeds the currency interest rate. Then, the QTM again takes the place of the FTPL as fiscal sustainability is restored according to a Ricardian fiscal policy.

3.6. Money demand and seigniorage under fixed spreads

Let us now fix not currency interest rates at i_1, but rather the spread between the bond and currency interest rates at $s_{B1} = i_B(t) - i_1(t) > 0$. That is, the central bank adjusts the currency interest rate $i_1(t)$ with the bond interest rate $i_B(t)$. In this case, real demand for the two currencies (m_1 and m_2) is stabilized according to $v_1'(m_1) = v_2'(m_2) = s_{B1}u'(c)$. Thus, the extended QTM holds as in Section 3.4. Given fixed monetary growth at μ_1, $i_B(t)$ is constant at $\delta + \mu_1$. In other words, the bond interest rate is controllable through monetary growth. Because the government adopts a Ricardian fiscal policy, $\lim_{\tau\to\infty} \frac{1}{(1+\delta)^{\tau-t+1}} \frac{B(\tau)}{P_1(\tau)} = 0$ always holds.

Given $s_{B1} = i_B(t) - i_1(t) > 0$, there are four cases: (1) $i_B > i_1 > 0$, (2) $i_B > i_1 = 0$, (3) $i_B > 0 > i_1$, and (4) $0 \geq i_B > i_1$. The first case with $i_B > i_1 > 0$ is identical to that of Section 3.4. Only part of the interest burden $i_B(t)$ associated with the currency issue is covered by the interest spread s_{B1} ($< i_B(t)$), and the remainder $i_1(t)$ (> 0) needs to be financed by future fiscal surpluses.

The second case with $i_B > i_1 = 0$ is identical to the current currency system where CB currencies are never remunerated. The interest burden associated with the currency issue is fully covered by seigniorage ($i_B = s_{B1}$), and the consolidated government does not have to pay the burden associated with the currency issue. Thus, its budget constraint (6-3-12) reduces to

$$\frac{B(t-1)}{P_1(t-1)} = \sum_{\tau=t}^{\infty} \left\{ \frac{1}{(1+\delta)^{\tau-t+1}} [tax(\tau) - g(\tau)] \right\}.$$

The third case with $i_B > 0 > i_1$ is quite interesting. Owing to negative currency interest rates, the government can raise more seigniorage not through i_B as in the second case, but through $s_{B1} > i_B$. Thus, the present value of the additional seigniorage amounts to $\left(\frac{s_{B1}-i_B}{i_B}-1\right)(m_1+m_0)=-\frac{i_1}{i_B}(m_1+m_0)>0$. With the additional seigniorage, the government's budget constraint is relaxed as follows.

$$\frac{B(t-1)}{P_1(t-1)}=-\frac{i_1}{i_B}(m_1+m_0)+\sum_{\tau=t}^{\infty}\left\{\frac{1}{(1+\delta)^{\tau-t+1}}[tax(\tau)-g(\tau)]\right\}$$

With negative currency interest rates, however, households as currency holders are forced to pay additional taxes, proportional to their real currency holdings.

The final case with $0 \geq i_B > i_1$ is even more interesting. In this case, borrowing benefits not costs arise when the consolidated government issues currencies at $i_B \leq 0$. Consequently, the government enjoys triple seigniorage, that is, negative currency interest ($i_1 < 0$), positive currency spreads ($s_{B1} = i_B - i_1 > 0$), and borrowing benefits ($i_B \leq 0$). If both bond and currency interest rates are controlled below zero for a long period, the government can obtain seigniorage without any limit. However, households are forced to pay triple costs for currency holding. Note that the issuance of public bonds is always discounted by not the nominal rate of interest, but by the real rate of interest, which equals the rate of time preference, and that negative interest on public bonds never helps to reduce the burden associated with the public bond issue.

4. Conclusion: Strong money demand or immense seigniorage in the digital age?

Let us now return to our original policy question, "Can strong money demand survive in the digital age in which various kinds of CBCCs are issued?" If our response to this question is "No", then we have to ask ourselves whether a certain macroeconomic mechanism can be substituted for strong money demand in supporting massively issued public bonds.

As discussed in Section 3.5, strong money demand disappears immediately when the bond interest rate ($i_B(t)$) exceeds the positive or zero currency interest rate ($i_B(t) > i_1 \geq 0$). Accordingly, the massive issuance of public bonds in excess of future fiscal surpluses can no longer be absorbed by excess money demand, or the bubble term coexisting with strong money demand. In addition, the consolidated government faces a more serious budget constraint because of fiscal expenditures on positive currency interest. As discussed in Saito (2020a, 2020b) where the currency interest rate is set at zero, the price level and bond interest rates, including the long-term rates, immediately jump after the bond interest rates deviate upward from the zero currency interest rate.

However, the situation changes dramatically when the consolidated government can set a negative interest rate on currency ($i_1 < 0$). Given a positive spread between the bond and currency interest rates ($i_B(t) > 0 > i_1$), strong money demand disappears, but the government can obtain immense seigniorage from both negative currency interest and positive interest spreads. If the bond interest rate is moved even below zero for a long period ($0 > i_B(t) > i_1$), then the government can obtain seigniorage without any limit. Thus, immense seigniorage resulting from deeply negative currency interest can be substituted for strong money demand in repaying the massive issuance of public bonds, whose value is currently far above the present value of future fiscal surpluses.

We should emphasize here that strong money demand and immense seigniorage can serve as firm support for the massive issuance of public bonds, but that they differ fundamentally from the viewpoint of households. For consumers as investors, public bonds in excess of tax burdens can serve as net assets as long as strong money demand is present. For households as taxpayers, however, immense seigniorage in itself is equivalent to heavy tax burdens.

In a deflationary environment where bond interest has already been close to zero, the currency interest rate needs to be deeply negative to maintain a large interest spread $(i_B(t) - i_1)$. However, given deeply negative interest rates on currencies, CBCCs have to be extremely convenient relative to private deposits or currencies. If the marginal utility from holding CBCCs ($v_1'(m_1)$ and $v_2'(m_2)$) fails to be high at a moderate level of real money balances, then currency demand shifts dramatically from CBCCs to private deposit currencies. Consequently, not only strong money demand for CB currencies, but also ordinary money demand, may rapidly disappear. A common concern is that financial and economic crises could trigger the large-scale shift from private deposit currencies to CB currencies. However, if the consolidated government attempts to finance enormous debt through immense seigniorage by imposing deeply negative interest rates on CBCCs, then a reverse shift from CB currencies to private deposit currencies may emerge as a new form of currency crisis.

References

Agarwal, Ruchir, and Miles Kimball, 2015, "Breaking through the zero lower bound," International Monetary Fund Working Paper No. 15/224.

Bank of Canada (BOC) and Monetary Authority of Singapore (MAS), 2019, "Jasper–Ubin design paper: Enabling cross-border high value transfer using distributed ledger technologies," Accenture.

Barrdear, John, and Michael Kumhof, 2016, "The macroeconomics of central bank issued digital currencies," Bank of England, Staff Working Paper No. 605.

Bassetto, Marco, 2002, "A game-theoretic view of the fiscal theory of the price level," *Econometrica*, 70 (6), 2167-2195.

Bassetto, Marco, 2004, "Negative nominal interest rates," *American Economic Review*, 94 (2), 104-108.

Bassetto, Marco, 2018, "Fiscal theory of the price level" in *The New Palgrave Dictionary of Economics,* 3rd edition, volume 3, London: Palgrave Macmillan.

Benes, Jaromir, and Michael Kumhof, 2012, "The Chicago Plan Revisited," International Monetary Fund Working Paper No. 12/202.

Berentsen, Aleksander, and Fabian Schär, 2018, "The case for central bank electronic money and the non-case for central bank cryptocurrencies," *Federal Reserve Bank of St. Louis Review*, 100 (2), 97-106.

Bindseil, Ulrich, 2020, "Tiered CBDC and the financial system," Working Paper Series No. 2351, European Central Bank.

Brown, Richard Gendal, 2018, "The Corda Platform: An Introduction," R3.

Buiter, Willem H., 2002, "The fiscal theory of the price level: A critique," *Economic Journal*, 112 (481), 459-480.

Buiter, Willem, H., 2014, "The simple analytics of helicopter money: Why it works – always," *Economics: The Open-Access, Open-Assessment E-Journal*, 8 (28), 51 pages.

Buiter, Willem, H., and Nikolaos Panigirtzoglou, 2003, "Overcoming the zero bound on nominal interest rates with negative interest on currency: Gesell's solution," *Economic Journal* 113 (490), 723-746.

Chapman, James, Rodney Garratt, Scott Hendry, Andrew McCormack, and Wade McMahon, 2017, "Project Jasper: Are distributed wholesale payment systems feasible yet?" *Bank of Canada, Financial System Review*, June 2017.

Danezis, George, and Sarah Meiklejohn, 2016, "Centrally banked cryptocurrencies," http://dx.doi.org/10.14722/ndss.2016.23187.

Eisler, Robert, 1933, *Stable Money: The Remedy for the Economic World Crisis*, London: Search Publishing.

European Central Bank (ECB), 2019, "Exploring anonymity in central bank digital currencies," *In Focus*, 4, 10 pages.

European Central Bank (ECB) and Bank of Japan (BOJ), 2017, "STELLA: Payment systems: liquidity saving mechanisms in a distributed ledger environment."

European Central Bank (ECB) and Bank of Japan (BOJ), 2018, "STELLA: Securities settlement systems: Delivery-versus-payment in a distributed ledger system."

Friedman, Milton, 1969, *The Optimum Quantity of Money*, London: Macmillan.

Hayek, Friedrich A., 1978 (1990), *Denationalization of Money: The Argument Refined*, London: The Institute of Economic Affairs.

Hearn, Mike, and Richard Gendal Brown, 2019, "Corda: A distributed ledger," R3.

Ilgmann, Cordelius, and Martin Menner, 2011, "Negative nominal interest rates: History and current proposals," *International Economics and Economic Policy*, 8, 383-405.

Iwamura, Mitsuru, 2016, *The Day When a Central Bank is Terminated: The Future of Bitcoin and Currencies*, (*Chuo ginko ga owaru hi: Bitcoin to tsuuka no shorai*, in Japanese), Tokyo: Shin-cho sha.

Iwamura, Mitsuru, Yukinobu Kitamura, Tsutomu Matsumoto, and Kenji Saito, 2019, "Can we stabilize the price of a cryptocurrency?: Understanding the design of bitcoin and its potential to compete with central bank money," *Hitotsubashi Journal of Economics*, 60 (1), 41-60.

Kumhof, Michael, and Clare Noone, 2018, "Central bank digital currencies: Design principles and balance sheet implications," Bank of England Staff Working Paper No. 725.

Nakamoto, Satoshi, 2008, "Bitcoin: A peer-to-peer electric cash system," mimeographed.

Raskin, Max, and David Yermack, 2016, "Digital currencies, decentralized ledgers, and the future of central banking," National Bureau of Economic Research Working Paper Series, No. 22238.

Rogoff, Kenneth S., 2016, *The Curse of Cash*, Princeton: Princeton University Press.

Saito, Makoto, 2020a, "Long-run mild deflation under fiscal unsustainability in Japan," Hitotsubashi University Institute of Economic Research Discussion Paper No. 703.

Saito, Makoto, 2020b, "Public bonds as money substitutes at near-zero rates: Disequilibrium analysis of the current and future Japanese economy," Hitotsubashi University Institute of Economic Research Discussion Paper No. 714.

Sargent, Thomas J., and François R. Velde, 1999, "The big problem of small change," *Journal of Money, Credit and Banking*, 31 (2), 137-161.

Sargent, Thomas J., and Neil Wallace, 1975, ""Rational" expectations, the optimal monetary instrument, and the optimal money supply rule," *Journal of Political Economy*, 83 (2), 241-254.

Sveriges Riksbank, 2017, "The Riksbank's e-krona project: Report 1," Sveriges Riksbank.

Sveriges Riksbank, 2018, "The Riksbank's e-krona project: Report 2," Sveriges Riksbank.

Sveriges Riksbank, 2020, "The Riksbank's e-krona pilot," Reg. No. 2019-00291, Sveriges Riksbank.

Woodford, Michael, 1995, "Price-level determinacy without control of a monetary aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43 (1), 1-46.

Woodford, Michael, 1998, "Doing without money: Controlling inflation in a post-monetary world," *Review of Economic Dynamics*, 1 (1), 173-219.

Figure 1-1: Comparison of central bank reserves and cryptocurrencies

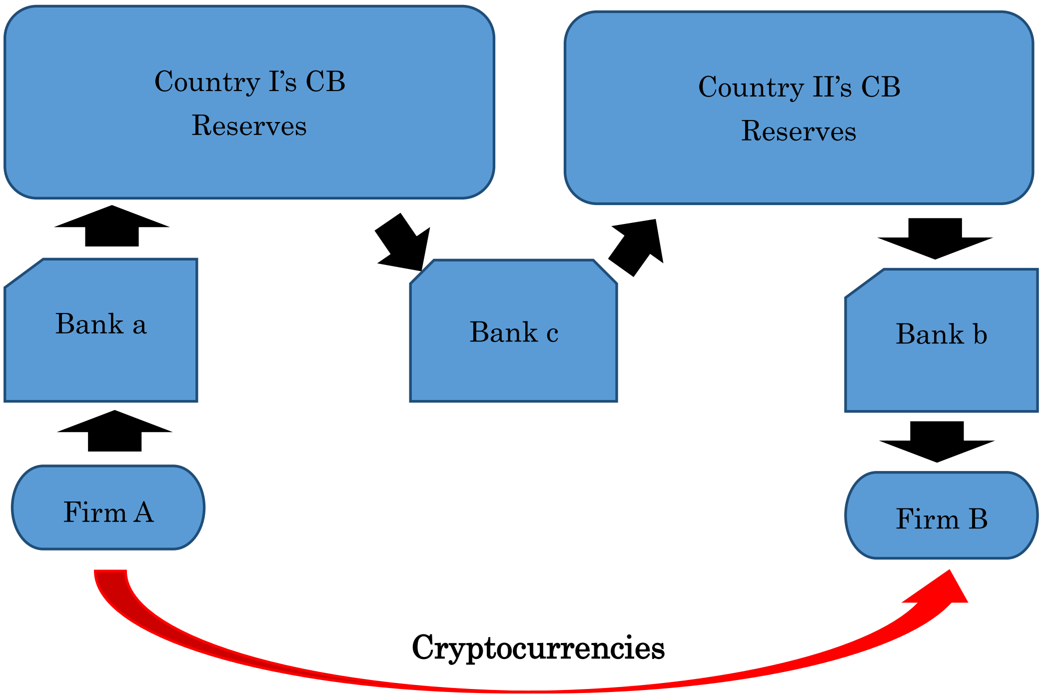

Figure 2-1 : Blocks time-stamped by hash functions

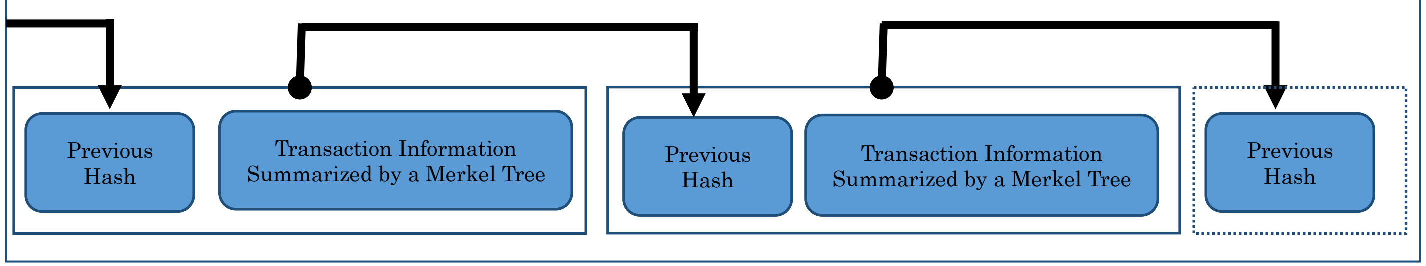

Figure 2-2 : Blocks verified by proof-of-work

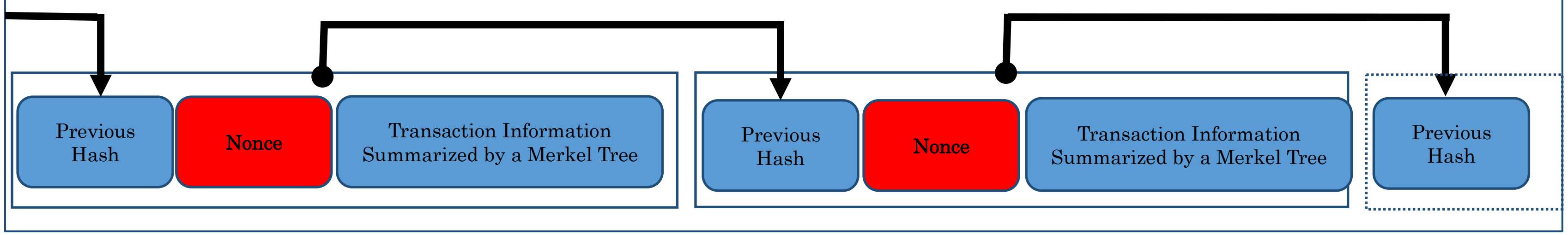

Figure 3-1: Function of utility from money holding (1)

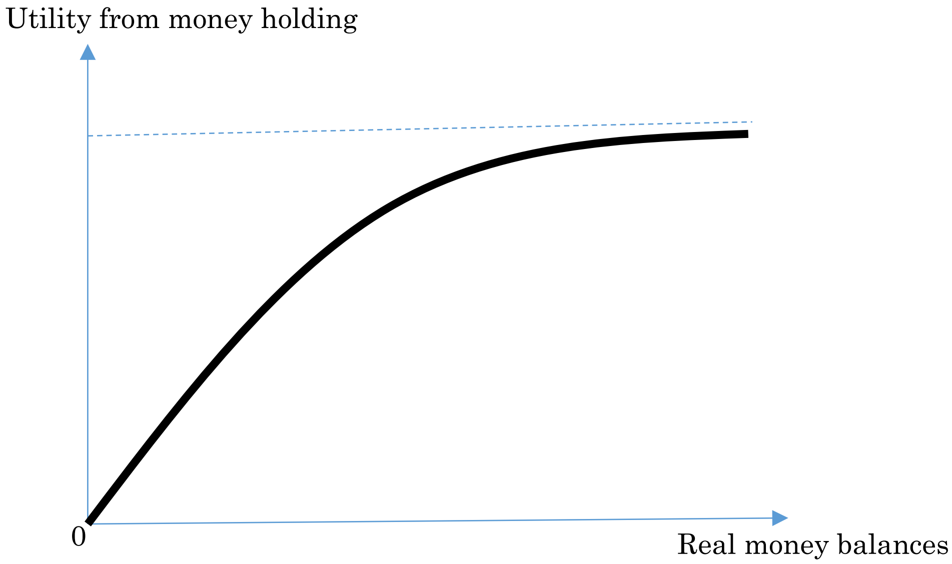

Figure 3-2: Function of utility from money holding (2)

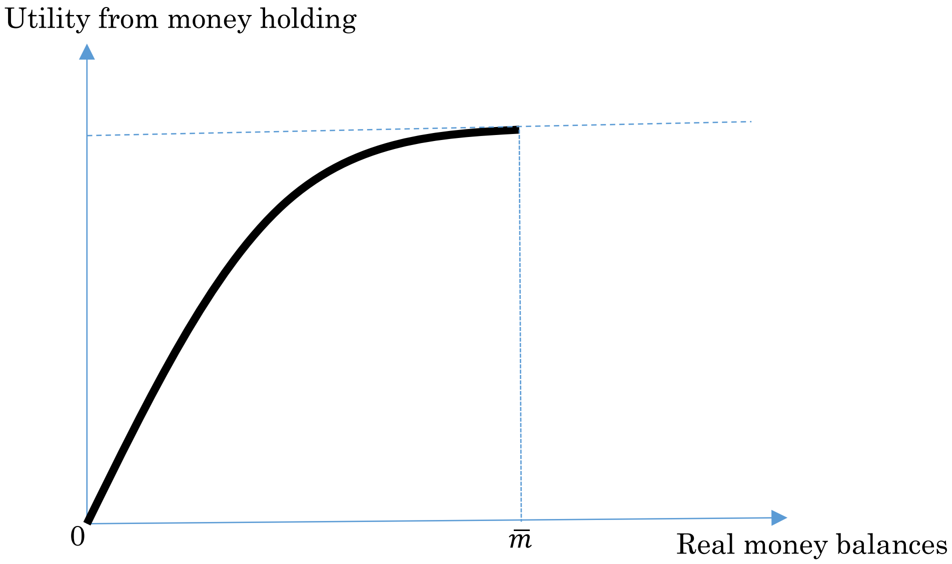

Figure 3-3: Function of utility from money holding (3)