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**The Theory of Exploitation as the Unequal Exchange of
Labour**

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The theory of exploitation as the unequal exchange of labour*

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Abstract

This paper explores the foundations of the theory of exploitation as the unequal exchange of labour (UEL). The key intuitions behind all of the main approaches to UEL exploitation are explicitly analysed as a series of formal axioms in a general economic environment. Then, a single domain condition called *Labour Exploitation* is formulated, which summarises the foundations of UEL exploitation theory, defines the basic domain of all UEL exploitation forms, and identifies the formal and theoretical framework for the analysis of the appropriate definition of exploitation.

Keywords: Exploitation, Unequal Exchange of Labour, axiomatic analysis.

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1 Introduction

What is exploitation? In political philosophy, the most general definition affirms that A exploits B if and only if A takes unfair advantage of B . Despite its intuitive appeal, this definition leaves two major issues in need of a precise specification, namely the kind of unfairness involved and the structure of the relationship between A and B that allows A to take advantage of B . There is considerable debate in the economic and philosophical literature on both issues. Although both aspects of exploitative relations are arguably crucial, and a full-fledged theory should include an account of the structural dimension of exploitation,¹ the analytical focus of this paper is on the unfairness, or more precisely, on the economic inequalities involved in the concept of exploitation.²

To be specific, this paper analyses the concept of *exploitation as the unequal exchange of labour* (hereafter, UEL). At the most general level, according to UEL theory, exploitative relations are characterised by systematic differences between the amount of labour that individuals ‘give’ to the economy, in some relevant sense, and the amount of labour that they ‘receive’, in some relevant sense. Yet, there are many conceivable ways of defining the labour given and received by agents. In his seminal book on exploitation theory alone, John Roemer examines no fewer than six distinct UEL definitions (see Roemer [29], Part I and pp.121, 132-133, 168), and many other approaches have been proposed in the literature.³

Alternative UEL definitions may seem to differ for relatively minor, and merely technical details. At a closer look, however, some deep theoretical cleavages emerge and different UEL approaches incorporate such distinct normative and positive intuitions that it is legitimate to wonder whether they actually bear any family resemblance. In some approaches, exploitation is defined as a *property* of individuals and “refers to the relationship between a person and society as a whole as measured by the transfer of the person’s labor to the society, and the reverse transfer of society’s labor to the person” (Roemer [31], p.31). In others,

¹For a thorough discussion of the limits of purely distributive accounts, and a defence of the importance of the notions of power, force and coercion in exploitation theory, see Veneziani [43].

²Alternatively, one may interpret our analysis as focusing on the *measurement* of exploitation, which is relevant even in approaches that reject the idea that exploitation is (primarily) an injustice (e.g., Holmstrom [17]; Reiman [26]; Wood [49], fn.15). For a discussion of the structural aspects of exploitation, see Arneson [1]; Warren [48]. Domination is central in the approach developed by Vrousalis [47].

³The literature is too vast for a comprehensive list of references. Classic contributions include Morishima [22]; Duménil [5]; Foley [14]; Roemer [28]; and Flaschel [10, 11]. For a discussion, see Yoshihara [50, 51].

exploitation is primarily a *relation* between individuals (e.g., Holmstrom [17]; Fleurbaey [13]). The normative content of the notion of UEL exploitation is also contested. According to Elster ([7], p.167), “Being exploited means, fundamentally, working more hours than are needed to produce the goods one consumes,” and thus exploitative relations are affected by saving/investment decisions. Other authors, instead, emphasise purchasing power and the idea that “workers give more labor to their employers than they receive through the goods their wages can afford” (Fleurbaey [13], p.653), and consumption decisions are irrelevant to define exploitation status. And so on.

This paper explores the foundations of UEL exploitation in an abstract economic environment. The main purpose of the analysis is descriptive: we aim to identify the core of UEL theory that is shared by *all* of the main approaches in the literature. The (often implicit) intuitions incorporated in the various definitions are rigorously formalised as separate axioms, and thoroughly examined. Then, a single domain condition called *Labour Exploitation* is formulated, which summarises the foundations of UEL exploitation theory and identifies the set of admissible definitions.

Domain axioms are routinely formulated and analysed in social choice theory and axiomatic bargaining theory.⁴ They do not represent full-fledged theories or definitions. Rather, they can be interpreted as meta-properties which usually identify the main object of research (e.g., a social welfare functional, a social welfare ordering, or an allocation mechanism), the space in which such object is analysed (e.g. welfare allocations, or economic environments with certain properties), and some foundational properties defining the set of admissible solutions (e.g. completeness, transitivity, or single-valuedness). Domain axioms thus delineate the basic perimeter of the theoretical exercise.⁵

⁴See, for example, the thorough discussion in chapters 2 and 3 of Roemer [33].

⁵For example, in the standard Nash approach to axiomatic bargaining, the *object* of analysis is a mechanism F that identifies a solution to any possible two-agent bargaining problem and the *space* of analysis is given by a pair (S, d) where S is a compact, convex set in \mathbb{R}^2 and $d \in S$. Then, the *Axiom of Unrestricted Domain* states that F is a mapping from the set of all pairs (S, d) such that S contains a point $s > d$ into S . According to Roemer ([33], section 2.5), however, the proper object of analysis should be a correspondence φ that maps a given economic environment into a lottery. His *Axiom of Domain* requires that the mechanism φ be defined on the class of economic environments comprising two agents endowed with von Neumann-Morgenstern utilities defined on a compact, convex set of lotteries, and such that the status quo point is not on the weak Pareto frontier. In his analysis of axiomatic mechanism theory, Roemer ([33], chapter 3) also

No such axiom exists in exploitation theory: it is not clear a priori what the object and space of the analysis are, and the information relevant to define exploitation has hardly been explicitly and systematically discussed. This paper fills the gap in the literature: it identifies a domain property that defines the object and the space of analysis of UEL exploitation theory, and some normatively relevant restrictions on the information that can be brought to bear in identifying the appropriate definition of exploitation.

Labour Exploitation is rigorously stated in section 6, however it expresses the following idea: at any allocation, given any definition of exploitation, the exploitation status of every agent is unambiguously determined by comparing the labour that she ‘gives’ to the economy, and the labour she ‘receives’. The former quantity is a scalar and it captures the labour contributed, or spent in productive activities. The labour agents ‘receive’ is a (possibly degenerate) interval and it is determined by identifying two (possibly equal) commodity bundles, which should be affordable and technically feasible, *and* the labour associated with, or contained in them, which is (a linear transformation of) the labour necessary to produce them as net output. If the labour contributed is more (*resp.*, less) than the maximum (*resp.*, minimum) amount of labour received, then the agent is regarded as exploited (*resp.*, an exploiter). *Labour Exploitation* sets weak restrictions on the way in which the set of exploiters and the set of exploited agents are identified and it is shown that indeed all of the main approaches satisfy it.

It is important to stress at the outset that, although we believe there are good reasons to focus on labour as a variable of normative interest, and in many economic contexts the notion of exploitation is inextricably linked with some form of labour exchange,⁶ our aim is *not* to defend the normative relevance of the distributive aspects of UEL exploitation theory. Nor does the paper propose a specific UEL definition. Rather, our main objective is to clarify a

considers objects of analysis with more structure (e.g. by postulating that “the allocation mechanism F is a full correspondence which is essentially a function” Roemer [33], p.98) and informationally richer spaces of analysis (e.g. economic environments including information concerning what utility represents and what the resources in the economy are, certain restrictions on admissible utility functions, and so on).

⁶Tellingly, after criticising the UEL approach and suggesting to replace it with a definition based on property relations (PR), Roemer has later acknowledged the importance of labour in exploitation theory. He has proposed that “an agent is ... capitalistically exploited, *if and only if PR holds and the exploiter gains by virtue of the labor of the exploited*” (Roemer [32], p.96). For, “the expenditure of effort is characteristically associated with exploitation” (Roemer [32], fn.11).

number of views and intuitions that are widely, albeit implicitly held in the literature, and to provide a rigorous formal and conceptual framework to analyse UEL exploitation. Although this leaves the questions of the injustice of the allocation of labour and of the appropriate UEL definition open, a clear statement of the foundations of UEL theory is necessary in order to tackle both issues.

Furthermore, our theoretical exercise does have some relevant normative implications. Because domain axioms identify the object and space of analysis, and the information that can be brought to bear in the investigation, they normally impose minimal normative restrictions (compared, for example, to axioms explicitly embodying ethical properties such as Hammond Equity, the Pigou-Dalton condition, Anonymity, and so on). Nonetheless, the definition of a domain axiom is not a merely technical exercise. As is well known in social choice theory, certain domain restrictions – most notably, welfarism but also transitivity and completeness – have significant ethical implications by restricting the normatively relevant information in certain ways, or by implicitly or explicitly ruling out certain approaches. Furthermore, certain ethical properties may be plausible in certain domains but not in others.⁷ *Labour Exploitation* is no exception, as it incorporates the basic normative intuitions of UEL theory that are shared by all of the main approaches in the literature.

Methodologically, this paper represents a first important step in the development of a novel axiomatic framework to analyse exploitation using the standard tools of social choice and axiomatic bargaining theory.⁸ For, if *Labour Exploitation* is a domain condition identifying the class of admissible exploitation forms, then it provides the conceptual and formal framework to identify the appropriate definition, which can be characterised by imposing additional, ethical properties. We return to this issue in sections 7 and 8 below.

⁷For example, Roemer ([33], chapter 2) argues that in axiomatic bargaining theory the standard Nash axioms are plausible in a welfaristic setting but not on informationally richer economic environments. He also shows that the axiom of welfarism plays a key role in the characterisation of the Nash mechanism and “when Nash bargaining theory is restated on the domain of economic environments that motivates the Nash axioms, those axioms are weak in the sense that many mechanisms exist that satisfy them” (Roemer [33], p.87).

⁸An axiomatic approach was long overdue in exploitation theory. Exceptions include Hollander [16] and, more recently, Yoshihara and Veneziani [52]; Veneziani and Yoshihara [44, 45, 46]; and Yoshihara [50].

2 The structure of UEL theory

In order to illustrate the key conceptual issues, consider the baseline case of a two-class economy in which one good, say corn, is produced by means of itself and labour. To be specific, $a < 1$ units of corn seed and $L > 0$ units of (a single type of homogeneous) labour are necessary to produce one unit of corn. Assume that all agents have the same skills. Suppose further that capitalists save and accumulate, while all workers earn a subsistence wage and consume the same (subsistence) amount of corn c . Under these rather stringent assumptions, the definition of UEL exploitation is relatively uncontroversial. Let v denote the amount of labour necessary to produce one unit of corn: this comprises both the amount of labour L directly necessary in production and the amount of labour contained in the corn seed, va . Formally, $v = va + L$ and so $v = L(1 - a)^{-1}$. Then agent ν is exploited (resp., an exploiter) if and only if the labour she expends in production, λ^ν , is greater (resp., lower) than the labour contained in c , vc .⁹

In this paper, we argue that the definition of an agent's exploitation status requires the identification of *three* separate normative benchmarks. The first is the amount of labour that the agent gives, or contributes - which, in this setting, is simply equated to the labour time performed in economic activities, λ^ν . The definition of the amount of labour received, instead, requires the definition *both* of a relevant bundle (or, more generally, bundles) received by the agent, which can be called the *exploitation reference bundle(s)* (henceforth, ERB(s)), *and* of the labour associated with, or contained in it - here, respectively, c and vc .

In the simple economy considered, all three benchmarks are relatively uncontroversial.¹⁰ As soon as the restrictive assumptions on technology, behaviour and endowments are relaxed, however, the definition of the labour 'given' and 'received' by agents is not obvious, and several approaches have indeed been proposed. For example, if agents have heterogeneous skills, then perhaps one should focus on *effective* (i.e. skill-adjusted) labour contributed, rather than *time* expended in production. But maybe not.

Similarly, if more general technologies are considered (allowing, for example, for joint products), the naïve generalisation of the standard definition of labour received can yield

⁹In this paper, we follow the literature and consider exploitation within a given time period. For an analysis of the complications arising in a dynamic setting, see Elster [7] and Veneziani [42, 43].

¹⁰These arguments can be easily generalised to n -good economies.

paradoxical results - such as bundles containing a negative amount of labour, as famously shown by Steedman [40] - and so various definitions of the labour contained in a *given* bundle have been proposed. As Roemer ([30], p.283) puts it, “If the production set is complicated in various ways, then it is not possible to give an unambiguous definition of what ‘labor-embodied’ means”.

Finally, if agents do not consume a given, equal subsistence bundle, then the choice of the ERB(s) is not obvious: it could be either the bundle actually consumed by an agent, or some normatively relevant bundle(s) unrelated to individual consumption decisions.

Different choices reflect different normative and positive intuitions. In the next section, we illustrate the range of possible approaches to UEL exploitation in an abstract economic environment by focusing on some of the classic definitions proposed in the literature.

3 The framework

We analyse UEL exploitation theory at an abstract level. Hence, in what follows, we impose only minimal restrictions on endowments, technology and preferences, and make no assumptions concerning agents’ behaviour - agents may be standard utility maximisers, or they may be characterised by some form of bounded rationality.

Let $\mathcal{N} = \{1, \dots, N\}$ be the set of *agents*, with generic element $\nu \in \mathcal{N}$, and let $\mathcal{M} = \{1, \dots, M\}$ be the set of produced *commodities* in the economy. Let \mathbb{R} (resp., \mathbb{R}_+) denote the (resp., nonnegative) real numbers. Let P be the production set, describing the technologically feasible production techniques. Elements of P , *activities*, are vectors denoted as α , which describe the amount of *effective* labour α_l and the vector of produced goods $\underline{\alpha}$ used as inputs in order to produce a vector of outputs $\bar{\alpha}$.¹¹ We assume that technology displays constant returns to scale.¹² Denote the vector of net outputs (outputs minus inputs) arising from α as $\hat{\alpha} \equiv \bar{\alpha} - \underline{\alpha}$.

For each agent $\nu \in \mathcal{N}$, let $s^\nu > 0$ be ν ’s *skill* level. If $\bar{L} > 0$ is the amount of *time* each agent is endowed with, then ν ’s *effective* labour endowment is $l^\nu = s^\nu \bar{L}$.¹³ Similarly, if

¹¹Formally, $\alpha = (-\alpha_l, -\underline{\alpha}, \bar{\alpha})$ with $\alpha_l \in \mathbb{R}_+$, $\underline{\alpha} \in \mathbb{R}_+^M$, and $\bar{\alpha} \in \mathbb{R}_+^M$.

¹²Formally, P is a closed, convex cone with $\mathbf{0} = (0, \dots, 0) \in P$. This assumption is standard in the literature and it is appropriate if all factors of production are accounted for.

¹³Although we allow for different *skills*, we are implicitly ruling out different *types* of labour used in production. This is only for simplicity and all of the key insights of the paper can be extended to economies

λ^ν is the amount of time spent by ν in production, then $\Lambda^\nu = s^\nu \lambda^\nu$ is the effective labour she performed by ν . Let $\omega^\nu \in \mathbb{R}_+^M$ denote ν 's endowment of productive assets, and let $\omega = \sum_{\nu \in \mathcal{N}} \omega^\nu$ be the vector of social endowments. Let p , w , and r , denote, respectively, the vector of the prices of produced commodities, the wage rate, and the rate of return on capital.

Each economy has a list $(\mathcal{N}, \mathcal{M}, P, (\omega^\nu)_{\nu \in \mathcal{N}}, (l^\nu)_{\nu \in \mathcal{N}})$. Given this list, a vector $\mathbf{z} \equiv (\mathbf{x}, \mathbf{\Lambda}) \in \mathbb{R}_+^{NM+N}$ represents an allocation $\mathbf{x} = (x^\nu)_{\nu \in \mathcal{N}}$ of the M commodities to the N agents in the economy and a profile $\mathbf{\Lambda} = (\Lambda^\nu)_{\nu \in \mathcal{N}}$ of (effective) labour supplied by each agent. This vector is called an *economically feasible allocation at prices* (p, w, r) if and only if there exists a production activity $\alpha \in P$ satisfying $\underline{\alpha} \leq \omega$, $\hat{\alpha} = \sum_{\nu \in \mathcal{N}} x^\nu$, $\alpha_l = \sum_{\nu \in \mathcal{N}} \Lambda^\nu$, $0 \leq \Lambda^\nu \leq l^\nu$, and $px^\nu = w\Lambda^\nu + rp\omega^\nu$ for each agent $\nu \in \mathcal{N}$.¹⁴ In other words, an allocation is economically feasible if the aggregate amount of each commodity allocated to agents can be produced given the aggregate endowments of physical and human capital in the economy, and each agent's budget constraint is satisfied at the given prices.¹⁵ Denote the set of *economically feasible allocations at prices* (p, w, r) by $\mathcal{Z}^{(p,w,r)} \subseteq \mathbb{R}_+^{NM+N}$. If x^ν is the commodity bundle actually purchased by agent $\nu \in \mathcal{N}$ at an allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$ then we shall denote it as c^ν .

In this general economic environment, there is no obvious or widely shared definition of UEL exploitation. For any bundle $c \in \mathbb{R}_+^M$, let $l.v.(c)$ denote the *minimum* amount of (effective) labour necessary to produce c as net output.¹⁶ In his classic definition, Morishima [22] focuses on the bundle actually purchased by an agent, c^ν , and defines its labour content as $l.v.(c^\nu)$:

with different labour inputs. For a discussion see Veneziani and Yoshihara [45].

¹⁴The notation for vector inequalities is: for all $y, y' \in \mathbb{R}^p$, $y \geq y'$ if and only if $y_i \geq y'_i$ for all i ; $y \geq y'$ if and only if $y \geq y'$ and $y \neq y'$; $y > y'$ if and only if $y_i > y'_i$ for all i .

¹⁵As noted by an anonymous referee, the model is not very realistic in that it does not explicitly account for public goods. This might be a shortcoming, if we intended to provide a comprehensive description of exploitative relations in actual capitalist economies. Our aim here is different, though: ours is a normative analysis of the concept of UEL exploitation that aims to identify the intuitions shared by all of the main approaches. For this purpose, and as a first step towards a more complete analysis, it is appropriate to adopt the same model used in the literature and neglect the complications arising from public goods. (For a pioneering discussion of exploitation in economies with public goods, see Fleurbaey [13].)

¹⁶Formally, $l.v.(c) \equiv \min \{\alpha_l \mid \alpha \in P, \hat{\alpha} \geq c\}$.

Definition 1 (Morishima [22]): At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, agent $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν , is exploited if and only if $\Lambda^\nu > l.v.(c^\nu)$ and an exploiter if and only if $\Lambda^\nu < l.v.(c^\nu)$.

Definition 1 has some desirable characteristics, according to Morishima ([22], pp.616-618): the notion of exploitation is well-defined because, under mild assumptions on technology, $l.v.(c)$ is unique, well-defined and positive whenever $c \neq (0, \dots, 0)$ and exploitation status is determined prior to and independent of price information, as in the standard Marxian approach, focusing only on production data.

According to Roemer ([29], p.152), however, Definition 1 is conceptually flawed because it embodies a merely technological concept of value and is independent of the social relations of production, as it identifies exploitation status based on production techniques that may never be used by profit-maximising capitalists. Like Morishima [22], Roemer [29] focuses on the bundle actually bought by agents, c^ν , but argues that its labour content should be determined focusing on profit-rate-maximising activities, for only the latter production processes will be activated in a capitalist economy.

Let an allocation $\mathbf{z} = (\mathbf{\Lambda}, \mathbf{x}) \in \mathcal{Z}^{(p,w,r)}$ with a price vector (p, w, r) be an *equilibrium allocation* whenever for any $\nu \in \mathcal{N}$, (x^ν, Λ^ν) is preferred by ν to any other (x'^ν, Λ'^ν) satisfying $x'^\nu \in \mathbb{R}_+^M$, $0 \leq \Lambda'^\nu \leq l^\nu$, and $px'^\nu = w\Lambda'^\nu + rp\omega^\nu$.¹⁷ For any bundle $c \in \mathbb{R}_+^M$, let $l.v.(c; p, w, r)$ be the *minimum* amount of (effective) labour necessary to produce c as net output with a profit-rate-maximising activity at given equilibrium prices.¹⁸ Then:

Definition 2 (Roemer [29], chapter 5): Consider an equilibrium allocation \mathbf{z} with prices (p, w, r) . Agent $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν , is exploited if and only if $\Lambda^\nu > l.v.(c^\nu; p, w, r)$ and an exploiter if and only if $\Lambda^\nu < l.v.(c^\nu; p, w, r)$.

While individual consumption choices are central in Definition 2, Roemer [29] has also proposed an alternative approach in which agents' exploitation status is independent of their

¹⁷An equilibrium allocation is essentially a *Reproducible solution* as defined by Roemer ([29], pp.64, 114). A thorough discussion of this notion goes beyond the boundaries of this paper. Intuitively, a reproducible solution involves individual optimisation, nonnegative aggregate excess demand in all markets of produced goods, and market clearing in the labour and/or credit market.

¹⁸Formally, let $P^\pi(p, w, r) \equiv \left\{ \alpha \in P \mid \frac{p\hat{\alpha} - rp\alpha - w\alpha_l}{p\alpha} = \max_{\alpha' \in P} \frac{p\hat{\alpha}' - rp\alpha' - w\alpha'_l}{p\alpha'} \right\}$ be the set of activities that maximise the rate of return on capital at (p, w, r) . Then, $l.v.(c; p, w, r) \equiv \min \{ \alpha_l \mid \alpha \in P^\pi(p, w, r), \hat{\alpha} \geq c \}$.

preferences over bundles of produced goods, focusing on the *maximum* and the *minimum* amounts of labour embodied in bundles that they can purchase. For all $\nu \in \mathcal{N}$ and all $\Lambda^\nu \leq l^\nu$, let $B(\omega^\nu, \Lambda^\nu; p, w, r)$ be the set of consumption bundles that agent ν can purchase at prices (p, w, r) , if she supplies Λ^ν units of labour, given her endowment ω^ν .¹⁹ Then:

Definition 3 (Roemer [29], chapter 5): Consider an equilibrium allocation \mathbf{z} with prices (p, w, r) . Agent $\nu \in \mathcal{N}$, who supplies Λ^ν and possesses ω^ν is exploited if and only if $\Lambda^\nu > \max_{c^\nu \in B(\omega^\nu, \Lambda^\nu; p, w, r)} l.v.(c^\nu; p, w, r)$ and an exploiter if and only if $\Lambda^\nu < \min_{c^\nu \in B(\omega^\nu, \Lambda^\nu; p, w, r)} l.v.(c^\nu; p, w, r)$.

Although they preserve many standard insights of exploitation theory, Definitions 1-3 have been criticised because exploitation status depends on counterfactual amounts of labour, as the production activities yielding $l.v.(c^\nu)$ or $l.v.(c^\nu; p, w, r)$ may be different from those actually used (Flaschel [10, 11]). According to critics, this use of counterfactuals is theoretically undesirable and makes exploitation an empirically vacuous notion, since the computation of $l.v.(c^\nu)$ and $l.v.(c^\nu; p, w, r)$ requires information that is not available, including, in Morishima's ([22], p.617) own words, "information about all the available techniques of production, actually chosen or potentially usable".

The last definition considered here is an extension of the so-called 'New Interpretation' (Duménil [5]; Foley [14]).²⁰ Let $\alpha^{p,w,r} \in P$ be the *aggregate (profit-rate-maximising) production activity* at an equilibrium allocation \mathbf{z} with prices (p, w, r) . Definition 4 identifies the labour associated with, or contained in any nonnegative bundle of goods.

Definition 4 : Consider any equilibrium allocation \mathbf{z} with prices (p, w, r) and associated $\alpha^{p,w,r}$. For each $c \in \mathbb{R}_+^M$ with $pc \leq p\hat{\alpha}^{p,w,r}$, let $\tau^c \in [0, 1]$ be such that $\tau^c p\hat{\alpha}^{p,w,r} = pc$. The labour content of c at \mathbf{z} is $\tau^c \alpha_l^{p,w,r}$.

By Definition 4, the labour content of aggregate net output, $\hat{\alpha}^{p,w,r}$, is equal to total social labour, $\alpha_l^{p,w,r}$, and for any bundle c whose value does not exceed national income $p\hat{\alpha}^{p,w,r}$, the labour contained in c is equal to the fraction τ^c of social labour necessary to produce a fraction of aggregate net output, $\tau^c \hat{\alpha}^{p,w,r}$, that has the same value as c .²¹ As in Roemer's [29] approach, in Definition 4 the labour content of a bundle can be identified only if the

¹⁹Formally: $B(\omega^\nu, \Lambda^\nu; p, w, r) \equiv \{c^\nu \in \mathbb{R}_+^M \mid pc^\nu \leq w\Lambda^\nu + rp\omega^\nu\}$.

²⁰See Yoshihara and Veneziani [52]; Yoshihara [50]; Veneziani and Yoshihara [44, 45]

²¹If $p\hat{\alpha}^{p,w,r} = 0$, then $\tau^c = 0$ by definition.

price vector is known. Yet social relations play a more central role, because the definition of labour content requires a prior knowledge of social production $\alpha^{p,w,r}$ and labour content is explicitly linked to the redistribution of total social labour. Then:

Definition 5 (*New Interpretation*): Consider any equilibrium allocation \mathbf{z} with prices (p, w, r) and associated $\alpha^{p,w,r}$. For any $\nu \in \mathcal{N}$, who supplies Λ^ν and purchases c^ν , let τ^{c^ν} be defined as in Definition 4. Agent ν is exploited if and only if $\Lambda^\nu > \tau^{c^\nu} \alpha_i^{p,w,r}$ and an exploiter if and only if $\Lambda^\nu < \tau^{c^\nu} \alpha_i^{p,w,r}$.

In other words, for any agent $\nu \in \mathcal{N}$, τ^{c^ν} represents ν 's share of national income, and so $\tau^{c^\nu} \alpha_i^{p,w,r}$ is the share of social labour that ν receives by earning income barely sufficient to buy c^ν . Thus, in the New Interpretation, the notion of exploitation is related to the production and distribution of national income and social labour, and depends on empirically observable magnitudes. Yet, Definition 5 has been criticised because, unlike Definitions 1 and 2, the actual bundles purchased by agents are only indirectly relevant to determine exploitation status, and unlike Definition 1, the notion of exploitation depends on price information.

In summary, our brief, and admittedly partial review shows that various definitions can be, and have in fact been, proposed which incorporate very different normative and positive intuitions. In the next sections, we show that in spite of these differences there exists a common core defining UEL theory.

4 Unequal exchange of labour: foundations

In this section, and the next, we identify the foundations of UEL exploitation theory that are shared by all of the main approaches. Our purpose is not to discriminate among competing views, but rather to define the theoretical boundaries of UEL theory. Formally, we aim to identify a general *domain* for all admissible UEL exploitation forms.

Let \mathcal{N}^{ter} , \mathcal{N}^{ted} , \mathcal{N}^n be three subsets of the set of agents \mathcal{N} denoting, respectively, the set of exploiters, the set of exploited agents, and the set of agents that are neither exploited nor exploiters. At the most general level, a *definition of exploitation* can be interpreted as a rule that identifies the sets \mathcal{N}^{ter} , \mathcal{N}^{ted} , \mathcal{N}^n at each feasible allocation - that is, as a mapping from $\mathcal{Z}^{(p,w,r)}$ to the power set of \mathcal{N} .

The first axiom of UEL theory requires that a definition of exploitation unambiguously identifies the exploitation status of every agent at any feasible economic allocation.

Axiom 1 (*The exploitation partition*) *At any feasible allocation, any definition of exploitation identifies a partition of the set of agents. Formally, at any $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, the sets \mathcal{N}^{ter} , \mathcal{N}^{ted} , \mathcal{N}^n are pairwise disjoint and $\mathcal{N}^{ter} \cup \mathcal{N}^{ted} \cup \mathcal{N}^n = \mathcal{N}$.*

According to Axiom 1, even though exploitation may be a property of aggregates, such as social classes (or nations, see Roemer [29], chapter 1), such aggregates are ultimately made of individuals with a well-defined exploitation status. Axiom 1 thus excludes radically holistic approaches, which identify the existence of exploitation *only* at the aggregate level, without any reference to (coalitions of) individuals. It does not rule out approaches viewing “exploitation as a social phenomenon, and the existence of exploitation need not imply, in principle, the existence of individual exploiters or exploited” (Roemer [29], p.136). But it does require the exploitation status of each agent to be clearly defined: at any feasible allocation, for every agent $\nu \in \mathcal{N}$, either $\nu \in \mathcal{N}^{ter}$ and ν is an exploiter; or $\nu \in \mathcal{N}^{ted}$ and ν is exploited; or $\nu \in \mathcal{N}^n$ and ν is exploitation-neutral.²²

Axiom 1 defines exploitation as a *property*, not primarily as a *relation* (Elster [7], p.174). It requires exploitation status to be defined at an overall allocation, and is silent about exploitation in individual transactions (unlike in approaches focusing on just acquisition, such as Steiner [41], or disadvantage, such as Goodin [15] and Sample [36]). This does not exclude the possibility that exploitation be diagnosed relationally, at the level of individual transactions, and agents may be exploited in some transactions, and exploiters in others (e.g., Fleurbaey [13], p.661). According to Axiom 1, however, the agents’ overall exploitation status should also be identified taking into account all of their economic activities.²³

In principle, Axiom 1 may apply to any definitions of exploitation, including those focusing on fair pricing (e.g., Robinson [27] and Reiff [25]) or property relations (e.g. Roemer [29], chapter 7). In UEL theory, however, at any allocation, the exploitation status of an

²²Axiom 1 rules out the possibility of incommensurable dimensions, or ‘spheres’, of exploitation: agents can be exploiters in one dimension and exploited in a different dimension, but their overall exploitation status must be well-defined.

²³The fact that, from a positive viewpoint, the overall exploitation status depends on the outcome of many transactions does not imply that exploitation status can, or should be defined at the level of individual transactions in a normatively relevant way.

agent ν is identified by comparing the *labour* that ν ‘gives’, in some relevant sense, and the *labour* that ν ‘receives’, in some relevant sense. Thus, in any UEL approach, two variables must be specified for each agent: the labour given *and* the labour received. As a shorthand notation, let L_G^ν and L_R^ν denote, respectively, the labour ‘given’ and the labour ‘received’ by agent $\nu \in \mathcal{N}$ at allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$. Our task is to identify the restrictions on L_G^ν and L_R^ν common to all UEL approaches.

In *all* UEL approaches, the labour ‘given’ by agent ν is just a scalar. In simple economies with homogeneous labour, it is equal to the labour *time* expended by ν in production.²⁴ In more general settings, however, various definitions can be adopted, which incorporate different normative intuitions. In order to focus on the core of UEL exploitation theory shared by all approaches, we leave L_G^ν unspecified while assuming that it is a scalar for all $\nu \in \mathcal{N}$. We return to this issue in section 7 below.

The definition of the labour that each agent ‘receives’ is more controversial, and many different proposals can be (and have been) made, as shown in section 3. The labour ‘received’ by an agent at an allocation is not necessarily captured by a single number (see, for example, Definition 3) and each L_R^ν could be a (possibly infinite) set of labour amounts related, for example, to an agent’s budget set. In all UEL approaches, however, exploitation status is monotonic in the sense that for an agent to be classified as exploited, she must ‘give’ more labour - have a higher L_G^ν - than it would be necessary to classify her as an exploiter, *everything else equal* (that is, for given consumption, income, preferences, and so on). Therefore we submit that in UEL theory, at any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, the exploitation status of each agent ν can be defined by identifying *at most* two cut-off values $L_{\min}^\nu, L_{\max}^\nu$ which may be interpreted as the upper and lower bounds of the labour ‘received’ by ν , and can be called the *reference labour amounts*.²⁵

The next axiom defines the core of the theory of exploitation as the unequal exchange of *labour*.

²⁴Some UEL approaches - such as Emmanuel’s [9] classic analysis of international inequalities - focus exclusively on the sphere of exchange and define L_G^ν and L_R^ν as the labour embodied in the bundles that agents give up and receive in exchange, respectively. Although our axiomatic framework also encompasses these approaches, we shall not analyse them in detail because they focus mostly on international relations and, more importantly, their normative foundations are unclear (see Roemer [29] and Schweickart [37]).

²⁵That is, for any $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, either $L_R^\nu \in [L_{\min}^\nu, L_{\max}^\nu]$ or $L_R^\nu \subseteq [L_{\min}^\nu, L_{\max}^\nu]$ holds.

Axiom 2 (*UEL exploitation*) *At any allocation $z \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, there exist $L_G^\nu \in \mathbb{R}$ and $L_{\min}^\nu, L_{\max}^\nu \in \mathbb{R}$ with $L_{\min}^\nu \leq L_{\max}^\nu$ such that agent ν is an exploiter if and only if $L_G^\nu < L_{\min}^\nu$; and is exploited if and only if $L_G^\nu > L_{\max}^\nu$.*

Axiom 2 incorporates an “idea of exploitation, as a certain kind of lack of reciprocity” (Cohen [3], p.343), whereby an agent is exploited whenever “the labor he contributes in one form does not return to him in another form” (Husami [18], p.44). The lack of reciprocity may be normatively relevant per se,²⁶ or because differences in labour ‘given’ and ‘received’ reflect normatively significant inequalities. Axiom 2 holds in approaches that define exploitation as the (forced) extraction of surplus, or unpaid labour (Holmstrom [17]; Elster [8]; Reiman [26]), but it is more general than that and it does not imply a commitment to an objective theory of value. For the labour ‘given’ and ‘received’ by agents need not be determined objectively, and may reflect agents’ preferences and beliefs. Indeed, Axiom 2 does not imply a commitment to *any* theory of value, or to the claim that labour is the only thing that produces value.²⁷ It only identifies labour as the key normative variable of interest - the main unit of account of exploitation theory - and thus rules out approaches based on wealth, income, utility, and so on. In exploitation theory, labour accounting is the “way of characterizing what it is that people give one another ... (where ‘give’ is understood very broadly to refer to any way in which some person undergoes a loss that ends up a gain to another)” (Reiman [26], p.9).²⁸

Axiom 2 provides a general statement of the basic principle of UEL theory. In order to have an operational definition, however, and setting aside for the moment the definition of L_G^ν , it is necessary to identify the labour ‘received’ by agents. This is by no means trivial. For there are potentially up to $2N$ reference labour amounts at any allocation. So the question

²⁶ “The capitalist gets something for nothing, or much for little, at the expense of others” (Elster [8], p.39). More generally, UEL exploitation may reflect a deviation from some fair reference norm in exchange (Fleurbaey [13], p.666). According to Arneson ([1], p.213), “Exploitation is one important form of mistreatment. To be exploited is roughly to be forced to perform drudgery to an unfairly great extent, and to receive in return an unfairly small share of goods” (as measured in the labour numéraire).

²⁷ See, for example, Buchanan ([2], p.46); Roemer ([31], p.62); Cohen ([3], ch.11); Fleurbaey ([12], p.181; [13], p.654); Vrousalis ([47], p.138).

²⁸ According to Sen ([38], p.177), labour accounting is meant to provide “a description of production that focus[es] on human beings” and the foundations of a normative approach whereby “personal participation can be seen as the basis of entitlement of fruits of production.” For an alternative justification of labour accounting in exploitation theory, see Roemer ([29], pp.283-288).

is; how are $(L_{\min}^{\nu}, L_{\max}^{\nu})_{\nu \in \mathcal{N}}$ determined? Or, equivalently, how do agents ‘receive’ labour?

As Roemer ([31], p.31) puts it, “exploitation theory views goods as vessels of labor, and calculates labor accounts for people by comparing the ‘live’ labor they expend in production with the ‘dead’ labor they get back in the vessels.” Axiom 3 formalises this view and states that agents receive labour via certain commodity bundles.

Axiom 3 (*Exploitation reference bundles, ERBs*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for each $\nu \in \mathcal{N}$, there exist $c_{\min}^{\nu}, c_{\max}^{\nu} \in \mathbb{R}_+^M$ and a function f^{ν} such that $L_{\min}^{\nu} = f^{\nu}(c_{\min}^{\nu})$ and $L_{\max}^{\nu} = f^{\nu}(c_{\max}^{\nu})$.*

Axiom 3 implies that, for each agent ν , the reference labour amounts, $L_{\min}^{\nu}, L_{\max}^{\nu}$, can be interpreted as the amounts of labour associated with, or contained in some theoretically relevant reference commodity bundles, the ERBs. In the standard approach outlined in section 2, the labour received by workers is equal to the labour embodied in the goods required for their subsistence: $c_{\min}^{\nu}, c_{\max}^{\nu}$ are equal and coincide with a given subsistence basket, and the labour associated with them, f^{ν} , is computed by identifying employment multipliers which uniquely identify the *unit* labour embodied in each commodity, and a fortiori in *any* bundles of goods.²⁹ Axiom 3 is more general, as it allows $c_{\min}^{\nu}, c_{\max}^{\nu}$ to be different and unrelated to any subsistence notion (or indeed to agents’ consumption choices) and it does not require the adoption of the standard notion of labour embodied. Indeed, it does not require a *general* theory for the determination of the labour contained in any bundles of goods: it only states that any definition of UEL exploitation should have a well-defined (albeit possibly implicit) notion of labour associated with, or contained in, the ERBs. Thus, Axiom 3 also captures the intuitions of approaches that do not focus on the labour embodied in given consumption bundles, such as the ‘New Interpretation’ (Duménil [5]; Foley [14]), according to which the labour ‘received’ by agents is equivalent to the amount of social labour agents receive a claim to via their *income*.

Axiom 3 says nothing about the relation between $(c_{\min}^{\nu}, c_{\max}^{\nu})$ and $(L_{\min}^{\nu}, L_{\max}^{\nu})$. For each agent ν and each $(c_{\min}^{\nu}, c_{\max}^{\nu})$, f^{ν} is a purely normative (and possibly agent-specific) construct that identifies the labour ‘received’ by ν based on some reference bundles. Axiom 3 does not specify how f^{ν} translates the ERBs into amounts of labour, as its properties are left

²⁹For a discussion in general n -good economies, see Morishima [21, 22]; Flaschel [10, 11].

unspecified. Some theoretically relevant restrictions on f^ν , and on the choice of the ERBs $(c_{\min}^\nu, c_{\max}^\nu)$, are identified in the next section.

5 The exploitation reference bundles

Axioms 1-3 are not a complete theory of exploitation. They provide a rigorous framework to conceptualise UEL exploitation, and the theoretical choices of different approaches, in terms of restrictions on $(L_G^\nu, L_R^\nu)_{\nu \in \mathcal{N}}$. Concerning L_R^ν , Axiom 3 implies that in order to define the exploitation status of each agent, it is necessary both to select the relevant ERBs, $c_{\min}^\nu, c_{\max}^\nu$, and to identify their labour content - that is, the (possibly implicit) function f^ν that associates a reference labour amount with each ERB. In this section, we explore some restrictions on $c_{\min}^\nu, c_{\max}^\nu$ and f^ν that aim to incorporate formally weak, theoretically robust, and widely shared intuitions in UEL theory.

Axiom 4 requires $c_{\min}^\nu, c_{\max}^\nu$ to be in the set of economically feasible choices for agent ν .

Axiom 4 (*Economic feasibility*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$ for all $\nu \in \mathcal{N}$.*

Axiom 4 stipulates that the ERBs be at least *potentially* affordable (recall that l^ν is agent ν 's labour *endowment*). By focusing on potential, rather than actual income, it allows for many different views: the ERBs may coincide with the bundle actually purchased by agents, or they may be some normatively relevant commodity bundles that are actually or only potentially affordable.³⁰ The former approach is relevant if one believes that the ERBs should be related to agents' actual consumption choices, because the way in which revenues are actually allocated (e.g. between consumption and investment) may be relevant to the moral status of their distribution (e.g. Elster [7], p.177). The latter approach is important, for example, in *modal* definitions in which exploitation status cannot be established just by looking at actual behaviour (Elster [7], pp.173ff)

Thus, Axiom 4 holds in UEL approaches that take the ERB to be unique and equal to the workers' subsistence basket.³¹ But it also holds in approaches that abandon a subsistence

³⁰Yet, Axiom 4 incorporates an emphasis on current economic data - and specifically current endowments - that contradicts approaches focusing on *historical* injustice, such as Steiner's [41].

³¹Indeed, the weak notion of economic feasibility formalised in Axiom 4 is (implicitly) endorsed even

view and define the labour received by worker ν focusing on “commodities whose value is equivalent to the wages $[\nu]$ receives” (Buchanan [2], p.37; see also Morishima [21], p.46); or even more generally, in those approaches that determine the labour received by any agent by looking at the amount of labour contained “in the goods he can purchase with his revenues from production (which may come from wages, profits, or the sale of commodities)” (Roemer [32], p.90).

Axiom 4 imposes an *economic* feasibility condition on the ERBs. Following classic UEL theory, Axiom 5 requires that $c_{\min}^{\nu}, c_{\max}^{\nu}$ be also *technologically* feasible as net output of some production process.

Axiom 5 (*Technical feasibility*) *At any feasible allocation, for all $\nu \in \mathcal{N}$ there exist technically feasible activities that can produce $c_{\min}^{\nu}, c_{\max}^{\nu}$ as net output. Formally, at any $z \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, there exist $\alpha^{\nu}, \alpha''^{\nu} \in P$ such that $\hat{\alpha}^{\nu} \geq c_{\min}^{\nu}$ and $\hat{\alpha}''^{\nu} \geq c_{\max}^{\nu}$.*

The emphasis on production conditions is standard in the traditional Marxist view according to which the amount of labour received by workers is equal to the *value of labour power*. Because labour power is a produced commodity, its value is given by the amount of labour necessary to produce it, which is identical with the amount of time required to produce the means of subsistence of the worker. More generally, in UEL theory, exploitation is a specific wrong rooted in the sphere of production (Cohen [3], p.345; Elster [7], p.167; Fleurbaey [12], p.184), and formal UEL definitions “map from the fundamental relations of production into the labour market and from this to the market of commodities, and then invert the mapping so as to go from the market of the final goods back to the production relationship” (Morishima and Catephores [23], p.43).

Axiom 5 requires the ERBs to be technically feasible as *net* output of some production process.³² This is because the direct labour used to produce a bundle c as net output allows one to capture the *total* amount of labour - direct and indirect - contained in producing c from scratch. A focus on net output is also natural in those UEL approaches that define embodied labour such that total labour performed in the economy in a given period is equal

in approaches that deny the relevance of income distribution in exploitation theory, while focusing on the concept of necessary labour (e.g., Holmstrom [17], pp.360ff).

³²Axiom 5 may be considered as a strong requirement if each agent has different bundles $c_{\min}^{\nu}, c_{\max}^{\nu}$. Yet it is easily satisfied for realistic choices of $c_{\min}^{\nu}, c_{\max}^{\nu}$, or if the set P possesses some standard properties (such as free disposal; see Veneziani and Yoshihara [44]).

to the labour contained in net national product, and conceive of exploitation as measuring (using a labour metric) how net national product is parcelled out to individuals.³³

While Axioms 4 and 5 impose restrictions on the ERBs, the next two properties focus on the function f^ν that associates a reference amount of labour to each ERB. Let $\mathbf{s} = (s^1, \dots, s^N)$. Axiom 6 is a natural complement to Axiom 5 in that it requires that the reference labour amounts also be related to production conditions.

Axiom 6 (*Reference labour amounts*) *At any feasible allocation, the reference labour amounts $L_{\min}^\nu = f^\nu(c_{\min}^\nu)$ and $L_{\max}^\nu = f^\nu(c_{\max}^\nu)$ depend on the amount of labour necessary to produce $c_{\min}^\nu, c_{\max}^\nu$. Formally, for any $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$ and each $\nu \in \mathcal{N}$, there exist $\alpha_{\min}^\nu, \alpha_{\max}^\nu \in P$ and a positive scalar $\kappa^\nu(P, \mathbf{s})$ such that $\hat{\alpha}_{\min}^\nu \geq c_{\min}^\nu, \hat{\alpha}_{\max}^\nu \geq c_{\max}^\nu, L_{\min}^\nu = \kappa^\nu(P, \mathbf{s})\alpha_{\min}^\nu$, and $L_{\max}^\nu = \kappa^\nu(P, \mathbf{s})\alpha_{\max}^\nu$.*

The notion of labour contained adopted in Axiom 6 is by no means metaphysical and bears no conceptual relation with the disputes on the labour theory of value. The labour contained in the ERBs is a well-defined amount related to current technical conditions,³⁴ and to the direct amount of (effective) labour used in production. Contra Steedman [40], Axiom 6 rules out negative reference labour amounts, because the labour contained in the ERBs, and thus the labour received by agents should be non-negative by definition (Morishima and Catephores [23], p.32; Flaschel [11], p.18).

According to Axiom 6, the reference labour amounts may be quantities of effective labour, but this is not a requirement: for each $\nu \in \mathcal{N}$, the positive scalar $\kappa^\nu(P, \mathbf{s})$ is meant to linearly transform effective labour units into units of labour, and different specifications of the linear transformations $(\kappa^\nu(P, \mathbf{s}))_{\nu \in \mathcal{N}}$ correspond to different notions of the labour ‘received’ (as well as ‘given’) by agents. That is, the appropriate choice of $(\kappa^\nu(P, \mathbf{s}))_{\nu \in \mathcal{N}}$ may depend on the normative use of the concept of exploitation (Fleurbaey [12], pp.176-177). We return to this issue in section 7 below.

A focus on production and on the notion of labour *contained* in the ERBs may seem restrictive in that it rules out, for example, approaches focusing on exchange whereby the reference labour amounts are determined based on labour *commanded* - that is, on the

³³For example, Flaschel [10, 11]; Duménil [5]; Foley [14]; and Fleurbaey ([12], p.171). See also the discussion in Morishima and Catephores ([23], pp.42-43) and Roemer ([31], pp.30-31).

³⁴Rather than labour actually used in the past to produce a certain bundle of goods (Cohen [3], pp.346ff).

amount of labour power that an agent may purchase with her income. This should not be a major concern. Considerations of purchasing power enter our framework explicitly in Axiom 4. Further, Axiom 6 allows for the possibility that price magnitudes and market relations enter the definition of labour contained in the ERBs, and although it does impose some restrictions on f^ν , they are extremely weak and the choice of reference labour amounts remains significantly open, as there may be very many ways of producing the ERBs.³⁵

Given that our aim is to identify the foundations of UEL theory shared by all of the main approaches, there is no reason to impose any strong restrictions on f^ν . Different restrictions may appropriately reflect alternative views, or may be desirable in different economic environments.³⁶ Yet, a common assumption in the UEL literature is that, however determined, the reference labour amounts be well-defined and unique.

Axiom 7 (*Production objectivism*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for any $\nu, \mu \in \mathcal{N}$, if $c_{\min}^\nu = c_{\min}^\mu$ and $c_{\max}^\nu = c_{\max}^\mu$, then $L_{\min}^\nu = L_{\min}^\mu$ and $L_{\max}^\nu = L_{\max}^\mu$.*

According to Axiom 7, *if* the ERBs of two agents are equal, then the associated reference labour amounts should be the same: the labour contained in the ERBs should be uniquely identified based on production conditions and not on idiosyncratic subjective factors. Although there are many conceivable ways of determining L_{\min}^ν and L_{\max}^ν , Axiom 7 states that such differences should be at the level of the theory, and not of arbitrary individual characteristics. This does not imply that subjective preferences and individual choices are irrelevant in determining exploitation status. It only means that the labour contained in the ERBs should not depend on the identity of the agents' receiving them.³⁷

Axiom 7 holds in the standard Marxian approach, whereby the labour content of a bundle, however defined, is *uniquely* determined and exploitation status is independent of

³⁵If the production set P satisfies a standard free disposal condition, then for any $c \in \mathbb{R}_+^M$, there may be an uncountably infinite number of production activities that produce c as net output.

³⁶As discussed in section 3, for example, some authors insist that reference labour amounts be determined based on the production techniques actually used, while others allow for counterfactual techniques.

³⁷Thus, Axiom 7 does *not* imply, for example, that agents consuming the same bundle must receive the same amount of labour regardless of any other potentially relevant information (concerning, for example, the source of their income). Rather, it says that normatively relevant differences between agents at a given allocation should be reflected in the choice of the ERBs (which, as we have noted, do not necessarily coincide with actual consumption bundles), and not in the determination of the labour associated with them.

agents' subjective characteristics. According to Morishima ([21], p.181), for example, a fundamental property of a definition of exploitation is that “each worker has to be shown to be equally exploited by the capitalists.” Yet, Axiom 7 seems reasonable also more generally and it is widely shared in UEL approaches: as argued by Yoshihara and Veneziani [53], one of the essential, if not defining characteristics of exploitation as a normative construct in political philosophy is its (weakly) objectivist thrust. “If an agent could change from being exploited into being an exploiter simply as a result of a change of tastes, some of the moral connotations of exploitation would be lost” (Elster [7], p.174; see also Roemer [29], p.110). Axiom 7 seems the weakest, and most easily defensible, part of a general objectivist theory of UEL exploitation. For if labour is ‘received’ via some commodity bundles, then it seems natural to require that a single reference labour amount be associated with any ERB.

Axioms 4-7 complete the analysis of the basic structure of UEL approaches. In UEL theory, the exploitation status of an agent ν is characterised by the labour ‘given’ and ‘received’ by her. The labour received by ν is determined by identifying some relevant commodity bundles, the ERBs, which are potentially affordable (Axiom 4) and technically feasible (Axiom 5), and the labour contained in them, which is (a linear transformation of) the amount of labour necessary to produce them (Axiom 6), and is uniquely determined independently of subjective factors (Axiom 7).

6 Labour exploitation

Axioms 1-7 define the common structure underlying different approaches to UEL theory. In this section, we formalise a *domain condition*, called *Labour Exploitation*: it summarises Axioms 1-7 and so, we argue, identifies the domain of admissible exploitation forms.³⁸

Labour Exploitation (LE): *Given any definition of exploitation, the sets \mathcal{N}^{ted} , \mathcal{N}^{ter} should have the following property. At any allocation $z \in \mathcal{Z}^{(p,w,r)}$ with a price system (p, w, r) , there exist a nonnegative vector $(L_G^\nu)_{\nu \in \mathcal{N}}$, a positive vector $(\kappa^\nu(P, \mathbf{s}))_{\nu \in \mathcal{N}}$, a profile of commodity bundles $(c_{\min}^\nu, c_{\max}^\nu)_{\nu \in \mathcal{N}}$ and a profile of activities $(\alpha_{\min}^\nu, \alpha_{\max}^\nu)_{\nu \in \mathcal{N}}$ that satisfy production objectivism such that for each agent $\nu \in \mathcal{N}$: $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$, $\alpha_{\min}^\nu, \alpha_{\max}^\nu \in P$*

³⁸LE generalises similar axioms analysed by Yoshihara and Veneziani [52], Yoshihara [50], and Veneziani and Yoshihara [44, 45, 46].

with $\hat{\alpha}_{\min}^{\nu} \geq c_{\min}^{\nu}$, $\hat{\alpha}_{\max}^{\nu} \geq c_{\max}^{\nu}$, and $\alpha_{l_{\max}}^{\nu} \geq \alpha_{l_{\min}}^{\nu}$, and

$$\begin{aligned} \nu &\in \mathcal{N}^{ted} \text{ if and only if } \kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\max}}^{\nu} < L_G^{\nu}, \\ \nu &\in \mathcal{N}^{ter} \text{ if and only if } \kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\min}}^{\nu} > L_G^{\nu}. \end{aligned}$$

According to **LE**, at any feasible allocation, the exploitation status of every agent ν is determined by the difference between the labour that ν ‘gives’ to the economy, and the labour she ‘receives’. Whereas the former quantity is left unspecified, the labour received by ν is determined by identifying at most two (potentially) affordable and technically feasible bundles, c_{\min}^{ν} , c_{\max}^{ν} , and their labour content which is equal to (a linear transformation of) the labour necessary to produce them as net output, $\kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\min}}^{\nu}$, $\kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\max}}^{\nu}$. The amount of labour that ν receives is the (possibly degenerate) interval $[\kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\min}}^{\nu}, \kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\max}}^{\nu}]$, and so, for any $\nu \in \mathcal{N}$, if L_G^{ν} is more (*resp.*, less) than $\kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\max}}^{\nu}$ (*resp.*, $\kappa^{\nu}(P, \mathbf{s}) \alpha_{l_{\min}}^{\nu}$) then ν is regarded as ‘giving’ more (*resp.*, less) labour than ν ‘receives’ and therefore a member of \mathcal{N}^{ted} (*resp.*, \mathcal{N}^{ter}).

The domain condition **LE** summarises the content of Axioms 1-7. Indeed, the next result proves that **LE** is logically equivalent to them.³⁹

Theorem 1. *At any allocation $z \in \mathcal{Z}^{(p,w,r)}$, a definition of exploitation satisfies **LE** if and only if it satisfies Axioms 1-7.*

In the previous sections, we have argued that Axioms 1-7 describe the structure and the fundamental intuitions common to all of the main approaches in UEL theory. Then, by Theorem 1, **LE** also captures the key intuitions of UEL theory. In order to substantiate this claim, we now show that in spite of all the differences highlighted in section 3, all of the main definitions in the literature satisfy **LE**, and thus Axioms 1-7.

To see this, note that in all of the approaches examined in section 3, at all $z \in \mathcal{Z}^{(p,w,r)}$, $L_G^{\nu} = \Lambda^{\nu}$ and $\kappa^{\nu}(P, \mathbf{s}) = 1$, for all $\nu \in \mathcal{N}$. In Definitions 1 and 2, at all $z \in \mathcal{Z}^{(p,w,r)}$, the ERB is unique and corresponds to the bundle actually purchased by agents, $c_{\min}^{\nu} = c_{\max}^{\nu} \equiv c^{\nu} \in B(\omega^{\nu}, l^{\nu}; p, w, r)$, for all $\nu \in \mathcal{N}$. However, in Definition 1, the labour content of the ERB is the minimum amount of labour necessary to produce it among *all* conceivable production

³⁹We state the equivalence between **LE** and Axioms 1-7 as a formal Theorem, because this precisely defines the relationship between the domain axiom and our characterisation of UE theory and it clearly decomposes the elements of **LE**. Because **LE** summarises Axioms 1-7, however, the proof of Theorem 1 is straightforward. We enclose it in the Appendix for completeness.

activities, whereas in Definition 2 only profit rate maximising processes are considered.⁴⁰ In Definition 3, given an equilibrium allocation \mathbf{z} with prices (p, w, r) , using the same notion of labour content as in Definition 2, c_{\max}^{ν} is the bundle whose labour content is maximal among all the bundles that the agent can purchase at (p, w, r) , while c_{\min}^{ν} is the bundle whose corresponding labour content is minimal.⁴¹ Note that, in general, $c_{\min}^{\nu} \neq c_{\max}^{\nu} \neq c^{\nu}$ under Definition 3. Finally, in Definition 5, given an equilibrium allocation \mathbf{z} with prices (p, w, r) and aggregate profit-rate-maximising production activity $\alpha^{p,w,r}$, the ERB is unique but, for all $\nu \in \mathcal{N}$, it is defined counterfactually as the proportion of net aggregate output that the agent may purchase with her income $c_{\min}^{\nu} = c_{\max}^{\nu} = c_e^{\nu} \equiv \tau^{c^{\nu}} \widehat{\alpha}^{p,w,r} \in B(\omega^{\nu}, l^{\nu}; p, w, r)$, where $\tau^{c^{\nu}} = \frac{pc^{\nu}}{p\widehat{\alpha}^{p,w,r}}$, and its labour content is given by the corresponding share of aggregate social labour, $\alpha_l^{c_e^{\nu}} \equiv \tau^{c^{\nu}} \alpha_l^{p,w,r}$.

Similarly, it is possible - albeit notationally intensive - to show that other definitions satisfy **LE**, including the input-output approach proposed by Flaschel [10, 11]; the variant of Definition 2 that focuses on *any* activities in $P^{\pi}(p, w, r)$ such that $\widehat{\alpha} \geq c$, and not only labour-minimising ones (Roemer [29], pp.164-168); and even the subjectivist approach developed by Matsuo [20], according to which the labour received by an agent who consumes a bundle c corresponds to the minimum amount of labour necessary to produce another bundle c' as net output, which gives at least as much utility as c .

This confirms that **LE** incorporates the main insights of UEL exploitation theory shared by *all* of the main approaches.

7 Ethical restrictions

Despite the seemingly irreconcilable differences between definitions focusing on physical or monetary magnitudes, actual choices or potential consumption, objective data or subjective preferences, actual or counterfactual production processes, and so on, Axioms 1-7, and therefore **LE**, identify a fundamental structure that defines UEL exploitation theory in *all* of

⁴⁰Formally: in Definition 1, $\alpha_{\min}^{\nu} = \alpha_{\max}^{\nu} = \alpha^{c^{\nu}} \in \arg \min \{\alpha_l \mid \alpha \in P, \widehat{\alpha} \geq c^{\nu}\}$; in Definition 2, $\alpha_{\min}^{\nu} = \alpha_{\max}^{\nu} = \alpha^{c^{\nu}} \in \arg \min \{\alpha_l \mid \alpha \in P^{\pi}(p, w, r), \widehat{\alpha} \geq c^{\nu}\}$. Correspondingly, the labour content specified in Definition 1 is $\alpha_l^{c^{\nu}} = l.v.(c^{\nu})$, while in Definition 2 it is $\alpha_l^{c^{\nu}} = l.v.(c^{\nu}; p, w, r)$.

⁴¹Formally: in Definition 3, the ERBs are $c_{\max}^{\nu} \equiv \arg \max_{c^{\nu} \in B(\omega^{\nu}, \Lambda^{\nu}; p, w, r)} l.v.(c^{\nu}; p, w, r)$ and $c_{\min}^{\nu} \equiv \arg \min_{c^{\nu} \in B(\omega^{\nu}, \Lambda^{\nu}; p, w, r)} l.v.(c^{\nu}; p, w, r)$, and their labour contents are $l.v.(c_{\max}^{\nu}; p, w, r)$ and $l.v.(c_{\min}^{\nu}; p, w, r)$, respectively.

its variants - even though this structure, and the notions of ERBs and their labour content are usually left implicit. Hence they can be interpreted as defining the boundaries of the *admissible class* of UEL exploitation forms.

Of course, the boundaries of UEL theory delineated by Axioms 1-7 are extremely wide and potentially include an infinite number of conceivable definitions - i.e. an infinite number of ways of specifying the ERBs, the reference labour amounts, and L'_G . It is therefore natural to ask how our framework could be extended to determine exploitation status within a (possibly core) class of definitions. To be precise, can one identify additional restrictions on the definitions of labour given and labour received that characterise a core class of approaches within the admissible set defined by **LE**?

In this section, we show that, by identifying the common foundations of UEL theory, **LE** also provides the formal and conceptual framework to answer this question and to examine the normative and positive implications of different approaches. We illustrate this point by showing how the differences between alternative definitions can be conceived of as different additional ethical restrictions imposed on **LE**. We return to the more general issue of the possible extensions of our approach in the Conclusions.

A first important issue concerns the relevance of individual choices in determining exploitation status. In **LE**, the labour received by agent ν is determined by some reference bundles that ν can in principle purchase. In Definitions 1 and 2, however, the ERB should be the bundle c^ν actually chosen by agent ν at allocation \mathbf{z} . Formally:

Axiom 8 (*Consumer subjectivism*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, $c^\nu_{\min} = c^\nu_{\max} = c^\nu$ for all agents $\nu \in \mathcal{N}$.*

The theoretical justification for Axiom 8 rests on the idea that disregarding agents' consumption choices implies identifying their exploitation status based on information that is potentially at odds with their actual situation and well-being.

This subjectivist view is not uncontroversial. Following the standard Marxian approach, for example, one may insist that exploitation status depend on *production* decisions, and not on possibly idiosyncratic *consumer* choices. This intuition can be formalised as follows:

Axiom 9 (*Consumer objectivism*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for any two agents $\nu, \mu \in \mathcal{N}$, if $B(\omega^\nu, l^\nu; p, w, r) = B(\omega^\mu, l^\mu; p, w, r)$ then $c^\nu_{\min} = c^\mu_{\min}$ and $c^\nu_{\max} = c^\mu_{\max}$.*

Axiom 9 is called ‘Consumer objectivism’ because it does not rule out the possibility that subjective factors matter in other aspects of UEL theory - for example, by defining L_G^ν as the agents’ actual labour supply or even by introducing subjective elements in the determination of the labour ‘content’ of the ERBs.

A second question concerns the kind of production processes that are normatively relevant to identify exploitation status. **LE** requires that both the ERBs and their labour content be determined with reference to production, but otherwise imposes no restrictions. In UEL approaches, however, it is often required that only *efficient* production processes be considered in order to determine exploitation status. Let ∂P denote the set of efficient production activities.⁴² The next axiom states that the ERBs should be producible as net output of efficient production activities.

Axiom 10 (*The efficiency view*) *At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, $\alpha^{\nu}_{\min} \in \partial P$ and $\alpha^{\nu}_{\max} \in \partial P$ with $\hat{\alpha}^{\nu}_{\min} \not\geq c^{\nu}_{\min}$, $\hat{\alpha}^{\nu}_{\max} \not\geq c^{\nu}_{\max}$.*

The efficiency view is consistent with Marx’s emphasis on the notion of *socially necessary labour time* and it embodies the idea that exploitative relations do not emerge from productive inefficiencies and mistakes. According to Morishima ([21], p.180), for example, in the computation of labour amounts one should focus on “those techniques by the use of which the amount of labour needed to produce given amounts of commodities can be minimized.” From a normative viewpoint, these are the techniques that maximise the productivity of human labour (ibid., p.184).⁴³ From a positive viewpoint, one may argue that inefficient techniques are at best a transient epiphenomenon in a competitive economy.

Yet, one may object that if the normative emphasis of UEL theory is on participation in productive activities, then it is misleading to focus on possibly counterfactual processes, and exploitation status should be identified based on production processes that are *actually* activated in the economy (Flaschel [10, 11]). Let $\alpha^{\mathbf{z}} \in P$ be the actual aggregate production activity at a feasible allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$. One way to formalise this view is as follows.

⁴²Formally: $\partial P \equiv \{\alpha \in P \mid \nexists \alpha' \in P \text{ such that } \alpha' > \alpha\}$.

⁴³ “[I]f the worker owned all the necessary means of production himself, all he would have to do would be to work only the minimum amount of hours [necessary to produce his wage-goods]” (Morishima and Catephores [23], p.41). See also Roemer ([28], p.38; [29], pp.168-173).

Axiom 11 (*The participation view*) At any allocation $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, there exist two real numbers ρ_{\min}, ρ_{\max} such that $0 \leq \rho_{\min}, \rho_{\max} \leq 1$ and $\alpha^{\nu_{\min}} = \rho_{\min} \alpha^{\mathbf{z}}$ and $\alpha^{\nu_{\max}} = \rho_{\max} \alpha^{\mathbf{z}}$.

Axiom 11 states that the reference labour amounts should be identified based on the production processes actually activated in the economy – more precisely, they are equal to some fraction of aggregate social labour performed. Thus, it incorporates the idea that *actual* labour spent in production is what matters in defining exploitative relations because it describes actual participation (Sen [38]).

So far, we have left L_G^ν unspecified because there is no widely shared definition of the labour ‘given’ by agents in economies with heterogeneous labour. In particular, two main approaches can be distinguished which focus either on the *effective* labour contributed, or on the labour *time* expended by agents. The former approach can be formalised as follows:

Axiom 12 (*The contribution view*) At any feasible allocation, the amount of labour that each agent ‘gives’ is the effective labour that she contributes in economically relevant activities. Formally, at any $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, $L_G^\nu = \Lambda^\nu$ and, correspondingly, $\kappa^\nu(P, \mathbf{s}) = 1$.

By Axiom 12, both labour ‘given’ and labour ‘received’ are measured in terms of effective labour. As noted in section 3, this is the natural extension of all of the classic definitions of exploitation, and it is the approach adopted in much of the literature on exploitation in economies with heterogeneous labour (see, e.g., Krause [19]; Duménil *et al* [6]). Moreover, the ‘*contribution view*’ incorporates an important normative intuition: an efficient and UEL exploitation-free allocation coincides with the *proportional solution*, a well-known fair allocation rule whereby every agent’s income is proportional to her contribution to the economy (Roemer and Silvestre [35]). Proportionality is a strongly justified normative principle, whose philosophical foundations can be traced back to Aristotle, and it can be justified in terms of the Kantian categorical imperative (Roemer [34]).⁴⁴

Alternatively, one may argue that UEL theory captures some inequalities in the distribution of material well-being and free hours that are – at least *prima facie* – of normative

⁴⁴The contribution principle (‘To each according to his contribution’) is also one of the principles of justice analysed by Marx in the *Critique of the Gotha programme* (Husami [18]; Cohen [4]; Warren [48]).

relevance (Fleurbaey [12, 13]).⁴⁵ For example, they may be deemed relevant because material well-being and free hours are key determinants of *individual well-being freedom* (Rawls [24]; Sen [39]). But they are also relevant in approaches that link exploitation and the Marxian notion of alienation in production (Buchanan [2]). From this perspective, the key variable of normative interest is labour *time*.⁴⁶

Axiom 13 (*The well-being view*) *At any feasible allocation, the amount of labour that each agent ‘gives’ is the labour time that she spends in economically relevant activities. Formally, at any $\mathbf{z} \in \mathcal{Z}^{(p,w,r)}$, for all $\nu \in \mathcal{N}$, $L_G^\nu = \lambda^\nu$.*

Axiom 13 does not require that $\kappa^\nu(P, \mathbf{s}) = 1$ for all agents ν , because the positive vector $(\kappa^\nu(P, \mathbf{s}))_{\nu \in \mathcal{N}}$ is necessary to transform units of effective labour into labour *time*, and various theoretically relevant specifications are possible. For example, if $(L_R^\nu)_{\nu \in \mathcal{N}}$ captures the distribution of social labour and $\alpha_{\min}^\nu, \alpha_{\max}^\nu$ are the outcomes of *social* production activity, then, for all $\nu \in \mathcal{N}$, $\kappa^\nu(P, \mathbf{s})$ can be defined as the average skill level at allocation \mathbf{z} : $\kappa^\nu(P, \mathbf{s}) \equiv \frac{\sum_{\nu \in \mathcal{N}} \lambda^\nu}{\sum_{\nu \in \mathcal{N}} s^\nu \lambda^\nu}$. Alternatively, one may argue that L_R^ν should be equal to the labour time that would be necessary for each agent to produce the ERBs *autarkically* and set $\kappa^\nu(P, \mathbf{s}) \equiv \frac{1}{s^\nu}$, for all $\nu \in \mathcal{N}$.

Axioms 8-9, 10-11, and 12-13 incorporate different normative intuitions and have rather different implications, and in this paper we do not adjudicate them. The previous discussion shows, however, that our domain axiom provides the formal and conceptual framework to rigorously examine alternative approaches to exploitation theory and to identify a core class of definitions of exploitation within the admissible set identified by **LE**.

8 Conclusions

This paper has laid out the foundations of UEL exploitation theory. We have identified its basic conceptual structure, including the notions of exploitation reference bundles and the

⁴⁵Cohen ([4], p.203) aptly refers to differences in the “leisure-and-income sets” available to agents. Roemer ([29], p.75 and passim) talks about the “exploitation-welfare criterion”.

⁴⁶Axiom 13 holds also in the approach proposed by Arneson ([1], pp.212-3) whereby exploitative relations violate the principle that “Those who make equal productive sacrifices are equally deserving of economic remuneration”. Desert is measured against the *time* (and effort) spent in production, while discounting natural abilities and skills as morally arbitrary. See also Elster ([7], p.202) and Reiman ([26], pp.9ff).

labour associated with them, and formulated its core intuitions as a series of axioms. We have shown that our analysis is descriptively accurate: *all* of the main approaches indeed satisfy our axioms, or equivalently the domain condition **LE**. Finally, we have argued that our analysis provides the framework to analyse the differences between alternative UEL definitions, as well as the differences between UEL exploitation theory and other approaches.

To be sure, this descriptive exercise does not provide an answer to the issue of the normative relevance of UEL exploitation. Moreover, as a domain condition, **LE** does not represent a full-fledged theory but a meta-property which identifies the object and space of research, and some foundational properties defining the *set* of admissible definitions. Therefore, **LE** alone cannot characterise a unique definition of exploitation, nor can it unambiguously identify agents' exploitation status at a given allocation. Yet, a clear statement of the foundations of UEL theory represents a key step to tackle these issues.

On the one hand, Axioms 1-7, or equivalently **LE**, rigorously lay out the core normative intuitions of UEL theory. According to **LE**, UEL exploitation is fundamentally a wrong involving a kind of lack of reciprocity. As discussed in the explanation of Axiom 1, exploitation is primarily a property, rather than a relation, which is diagnosed at the individual level and at an overall economic allocation, rather than in individual transactions. Labour is the key variable of normative interest and the main unit of account of UEL exploitation theory, and the lack of reciprocity in exploitative relations is measured in terms of a mismatch between the labour given and the labour received by agents. The former magnitude is measured by the amount of labour spent, or contributed in productive activities, while agents receive labour via bundles of goods that act as 'vessels' of labor. To be precise, agents receive labour via bundles of goods that they can in principle purchase, given their productive endowments, and that can be produced, given current technological knowledge. Thus, UEL exploitation theory is different from standard liberal egalitarian approaches which focus on the distribution of opportunities, utility, or resources, but it also differs from entitlement approaches in that labour accounts are based on current, rather than historical, data.

On the other hand, this paper provides a novel axiomatic framework to examine the normative relevance of UEL exploitation theory, *and* to identify the appropriate definition – or, at least, class of definitions – of exploitation. For, if **LE** defines the domain of admissible UEL exploitation forms, then one may impose further restrictions on **LE** in order to identify a core class of UEL approaches *within the admissible set*. Some examples of such restrictions

are examined in section 7, but many others can certainly be identified. For example, as suggested by an anonymous referee, one may argue that whenever an agent ν spends no labour time at all in productive activities but her consumption expenditure can be placed, say, in the top one per cent of the population, then any definition of exploitation should be such that $L_G^\nu = 0$ and $\alpha_{l_{\min}}^\nu > 0$, thus making the agent an exploiter.⁴⁷ Conversely, it may be argued that whenever an agent ν spends her entire endowment of labour time in productive activities and obtains an income barely sufficient to afford a subsistence bundle, then any definition of exploitation should be such that $\kappa^\nu(P, \mathbf{s}) \alpha_{l_{\max}}^\nu < L_G^\nu$, – thus implying that the agent is exploited – provided that others in the economy are well off.

Indeed, once the domain of admissible UEL exploitation forms is defined, one may try to identify the normatively relevant properties that uniquely characterise one definition within the admissible set. From this perspective, some central insights of exploitation theory, such as the relation between class and exploitation status (Roemer’s [29] celebrated *Class-Exploitation Correspondence Principle*) or the correspondence between positive profits and the exploitation of at least the poorest segments of the working class (see, for example, the *Profit-Exploitation Correspondence Principle* recently proposed by Veneziani and Yoshihara [44, 45]) may be reformulated as axiomatic requirements.

Similarly, observe that **LE** allows for the possibility that *all* agents in the economy be either exploited or exploiters. This is appropriate given the nature of **LE** as a *minimum* domain condition, because even some of the classic definitions of exploitation – such as Morishima’s [22] – do not exclude such possibility. Nonetheless, one may argue that the appropriate definition of exploitation should rule it out. One way to do so is by explicitly formalising the relational aspect inherent in exploitative relations, such that if an agent is exploited, she must be exploited *by someone*, and vice versa, as suggested by Yoshihara and Veneziani [52]:

Relational Exploitation (RE): *At any equilibrium allocation \mathbf{z} with prices (p, w, r) , $\mathcal{N}^{ter} \neq \emptyset$ if and only if $\mathcal{N}^{ted} \neq \emptyset$.*

Within the domain identified by **LE**, are there any definitions that capture important normative intuitions of UEL exploitation theory, such as the correspondence between class

⁴⁷Observe that by **LE** $\kappa^\nu(P, \mathbf{s})$ is strictly positive for all $\nu \in \mathcal{N}$. Further note that if $\lambda^\nu = 0$, then $L_G^\nu = 0$ according to both Axiom 12 and Axiom 13.

and exploitation status, the relation between exploitation and profits, and **RE**? If not, and an impossibility result followed from the imposition of these properties, this would arguably raise serious questions about the UEL approach.

Some results by Yoshihara and Veneziani [52, 50, 44, 45, 46] suggest that there *are* definitions that satisfy **LE** and these additional properties and, among all of the main approaches, the only one satisfying all properties is Definition 5. Although these results are preliminary and hold in specific economic settings, they do suggest that the axiomatic framework laid out in this paper is both insightful and promising.

References

- [1] Arneson, R.J., 1981. What's Wrong with exploitation? *Ethics* 91, 202-227.
- [2] Buchanan, A.E., 1982. *Marx and Justice*. Rowman and Littlefield, Totowa.
- [3] Cohen, G.A., 1979. The Labor Theory of Value and the Concept of Exploitation. *Philosophy & Public Affairs* 8, 338-360.
- [4] Cohen, G.A., 1995. *Self-Ownership, Freedom and Equality*. Cambridge University Press, Cambridge.
- [5] Duménil, G., 1980. *De la Valeur aux Prix de Production*. Economica, Paris.
- [6] Duménil, G., Foley, D.K., Lévy, D., 2009. A Note on the Formal Treatment of Exploitation in a Model with Heterogeneous Labor. *Metroeconomica* 60, 560-567.
- [7] Elster, J., 1985. *Making Sense of Marx*. Cambridge University Press, Cambridge.
- [8] Elster, J., 1997. Exploitation, Freedom, and Justice. In: Nielsen, K., Ware, R., (eds.) *Exploitation*. Humanities press, New Jersey.
- [9] Emmanuel, A., 1972. *Unequal Exchange*. Monthly Review Press, New York.
- [10] Flaschel, P., 1983. Actual Labor Values in a General Model of Production. *Econometrica* 51, 435-454.
- [11] Flaschel, P., 2010. *Topics in Classical Micro- and Macroeconomics*. Springer, New York.

- [12] Fleurbaey, M., 1996. *Théories Économiques de la Justice*. Economica, Paris.
- [13] Fleurbaey, M., 2014. The Facets of Exploitation. *Journal of Theoretical Politics* 26, 653-676.
- [14] Foley, D.K., 1982. The Value of Money, the Value of labour Power, and the Marxian Transformation Problem. *Review of Radical Political Economics* 14, 37-47.
- [15] Goodin, R., 1987. Exploiting a situation and exploiting a person. In: Reeve, A. (ed.) *Modern theories of exploitation*. Sage, London.
- [16] Holländer, H., 1982. Class Antagonism, Exploitation and the Labour Theory of Value. *Economic Journal* 92, 868-885.
- [17] Holmstrom, N., 1977. Exploitation. *Canadian Journal of Philosophy* 7, 353-369.
- [18] Husami, Z.I., 1978. Marx on Distributive Justice. *Philosophy & Public Affairs* 1, 27-64.
- [19] Krause, U., 1982. Heterogeneous Labour and the Fundamental Marxian Theorem. *Review of Economic Studies* 48, 173-178.
- [20] Matsuo, T., 2008. Profit, Surplus Product, Exploitation and Less than Maximized Utility. *Metroeconomica* 59, 249-265.
- [21] Morishima, M., 1973. *Marx's economics*. Cambridge University Press, Cambridge.
- [22] Morishima, M., 1974. Marx in the Light of Modern Economic Theory. *Econometrica* 42, 611-632.
- [23] Morishima, M., Catephores, G., 1978. *Value, Exploitation and Growth*. McGraw Hill, London.
- [24] Rawls, J., 1971. *A Theory of Justice*. Harvard University Press, Cambridge, MA.
- [25] Reiff, M.R., 2013. *Exploitation and Economic Justice in the Liberal Capitalist State*. Oxford University Press, Oxford.
- [26] Reiman, J., 1987. Exploitation, Force and the Moral Assessment of Capitalism: Thoughts on Roemer and Cohen. *Philosophy & Public Affairs* 16, 3-41.

- [27] Robinson, J., 1933. *Economics of Imperfect Competition*. London, Macmillan.
- [28] Roemer, J.E., 1981. *Analytical foundations of Marxian economic theory*. Harvard University Press, Cambridge, MA.
- [29] Roemer, J.E., 1982. *A General Theory of Exploitation and Class*. Harvard University Press, Cambridge, MA.
- [30] Roemer, J.E., 1982. Property relations vs. in Marxian exploitation. *Philosophy & Public Affairs* 11, 281-313.
- [31] Roemer, J.E., 1985. Should Marxists be interested in exploitation? *Philosophy & Public Affairs* 14, 30-65.
- [32] Roemer, J.E., 1989. What is exploitation? Reply to Jeffrey Reiman. *Philosophy & Public Affairs* 18, 90-97.
- [33] Roemer, J.E., 1996. *Theories of Distributive Justice*. Harvard University Press, Cambridge, MA.
- [34] Roemer, J.E., 2010. Kantian equilibrium. *Scandinavian Journal of Economics* 112, 1-24.
- [35] Roemer, J.E., Silvestre, J.E., 1993. The proportional solution for economies with both private and public ownership. *Journal of Economic Theory* 59, 426-444.
- [36] Sample, R., 2003. *Exploitation: What it is and why it's wrong*. Rowman and Littlefield, Boulder.
- [37] Schweickart, D., 1991. The Politics and Morality of Unequal Exchange. *Economics and Philosophy* 7, 13-36.
- [38] Sen, A.K., 1978. On the labour theory of value: some methodological issues. *Cambridge Journal of Economics* 2, 175-190.
- [39] Sen, A.K., 1985. *Commodities and Capabilities*. North-Holland, Amsterdam.
- [40] Steedman, I., 1977. *Marx after Sraffa*. New Left Books, London.
- [41] Steiner, H., 1984. A liberal theory of exploitation. *Ethics* 94, 225-241.

- [42] Veneziani, R., 2007. Exploitation and Time. *Journal of Economic Theory* 132, 189-207.
- [43] Veneziani, R., 2013. Exploitation, Inequality, and Power. *Journal of Theoretical Politics* 25, 526-545.
- [44] Veneziani, R., Yoshihara, N., 2015. Exploitation in economies with heterogeneous preferences, skills and assets: An axiomatic approach. *Journal of Theoretical Politics* 27, 8-33.
- [45] Veneziani, R., Yoshihara, N., 2017. One million miles to go: taking the axiomatic road to defining exploitation. *Cambridge Journal of Economics* 41, 1607-1626.
- [46] Veneziani, R., Yoshihara, N., 2017. Globalisation and inequality in a dynamic economy: an axiomatic analysis of unequal exchange. *Social Choice and Welfare* 49, 445-468.
- [47] Vrousalis, N., 2013. Exploitation, Vulnerability, and Social Domination. *Philosophy and Public Affairs* 41, 131-157.
- [48] Warren, P., 1997. Should Marxists Be Liberal Egalitarians? *Journal of Political Philosophy* 5, 47-68.
- [49] Wood, A.W., 1995. Exploitation. *Social Philosophy and Policy* 12, 135-158.
- [50] Yoshihara, N., 2010. Class and Exploitation in General Convex Cone Economies. *Journal of Economic Behavior & Organization* 75, 281-296.
- [51] Yoshihara, N., 2017. A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963). *Journal of Economic Surveys* 31, 632-659.
- [52] Yoshihara, N., Veneziani, R., 2009. Exploitation as the Unequal Exchange of Labour: An Axiomatic Approach. Working paper N.655, Queen Mary University of London.
- [53] Yoshihara, N., Veneziani, R., 2011. Strong Subjectivism in the Theory of Exploitation: A Critique. *Metroeconomica* 62, 53-68.

9 Appendix: Proof of Theorem 1

Consider any economy with a list $(\mathcal{N}, \mathcal{M}, P, (\omega^\nu)_{\nu \in \mathcal{N}}, (l^\nu)_{\nu \in \mathcal{N}})$.

1. We show that if a definition of exploitation satisfies **LE**, then it satisfies Axioms 1-7.

By **LE**, at any $z \in \mathcal{Z}^{(p,w,r)}$, there exist a profile of commodity bundles $(c_{\min}^\nu, c_{\max}^\nu)_{\nu \in \mathcal{N}}$ and a profile of activities $(\alpha_{\min}^\nu, \alpha_{\max}^\nu)_{\nu \in \mathcal{N}}$ satisfying production objectivism such that for each agent $\nu \in \mathcal{N}$, $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$ and $\alpha_{\min}^\nu, \alpha_{\max}^\nu \in P$ with $\hat{\alpha}_{\min}^\nu \geq c_{\min}^\nu$, $\hat{\alpha}_{\max}^\nu \geq c_{\max}^\nu$, and $\alpha_{l_{\max}}^\nu \geq \alpha_{l_{\min}}^\nu$. Therefore, Axioms 4, 5, and 7 are satisfied.

LE also implies that there exist a nonnegative vector $(L_G^\nu)_{\nu \in \mathcal{N}}$ and a positive vector $(\kappa^\nu(P, \mathbf{s}))_{\nu \in \mathcal{N}}$ such that

$$\begin{aligned} \nu \in \mathcal{N}^{ted} &\Leftrightarrow \kappa^\nu(P, \mathbf{s}) \alpha_{l_{\max}}^\nu < L_G^\nu, \\ \nu \in \mathcal{N}^{ter} &\Leftrightarrow \kappa^\nu(P, \mathbf{s}) \alpha_{l_{\min}}^\nu > L_G^\nu. \end{aligned}$$

Then, for each agent $\nu \in \mathcal{N}$, let us define a function f^ν such that $f^\nu(c_{\min}^\nu) = \kappa^\nu(P, \mathbf{s}) \alpha_{l_{\min}}^\nu$ and $f^\nu(c_{\max}^\nu) = \kappa^\nu(P, \mathbf{s}) \alpha_{l_{\max}}^\nu$ hold. Denote $L_{\min}^\nu \equiv f^\nu(c_{\min}^\nu)$, $L_{\max}^\nu \equiv f^\nu(c_{\max}^\nu)$. Then, as $\alpha_{l_{\max}}^\nu \geq \alpha_{l_{\min}}^\nu$, $L_{\min}^\nu \leq L_{\max}^\nu$ holds. Thus, Axioms 3 and 6 are satisfied.

Moreover, by $\kappa^\nu(P, \mathbf{s}) \alpha_{l_{\max}}^\nu = L_{\max}^\nu$ and $\kappa^\nu(P, \mathbf{s}) \alpha_{l_{\min}}^\nu = L_{\min}^\nu$, we have $\nu \in \mathcal{N}^{ted} \Leftrightarrow L_{\max}^\nu < L_G^\nu$ and $\nu \in \mathcal{N}^{ter} \Leftrightarrow L_{\min}^\nu > L_G^\nu$. Therefore, Axiom 2 is satisfied.

Finally, for each agent $\nu \in \mathcal{N}$, only one of the following three holds: $L_G^\nu < L_{\min}^\nu$; $L_{\min}^\nu \leq L_G^\nu \leq L_{\max}^\nu$; and $L_{\max}^\nu < L_G^\nu$. According to the above discussion, the set of agents who are neither exploiters nor exploited is given by

$$\mathcal{N}^n = \{\nu \in \mathcal{N} \mid L_{\min}^\nu \leq L_G^\nu \leq L_{\max}^\nu\}.$$

Then, it follows that \mathcal{N}^{ter} , \mathcal{N}^{ted} , and \mathcal{N}^n are pairwise disjoint and $\mathcal{N}^{ter} \cup \mathcal{N}^{ted} \cup \mathcal{N}^n = \mathcal{N}$ holds. Thus, Axiom 1 is satisfied.

In summary, if a definition of exploitation satisfies **LE**, then it also satisfies Axioms 1-7.

2. We show that if a definition of exploitation satisfies Axioms 1-7, then it also satisfies **LE**.

Let a definition of exploitation satisfy Axioms 1-7. Consider any $z \in \mathcal{Z}^{(p,w,r)}$. Axiom 1 implies that \mathcal{N}^{ter} , \mathcal{N}^{ted} , and \mathcal{N}^n are pairwise disjoint and $\mathcal{N}^{ter} \cup \mathcal{N}^{ted} \cup \mathcal{N}^n = \mathcal{N}$ holds. Axiom 2 implies that for each agent $\nu \in \mathcal{N}$, there exist $L_G^\nu \geq 0$, $L_{\min}^\nu \geq 0$, and $L_{\max}^\nu \geq 0$ with $L_{\min}^\nu \leq L_{\max}^\nu$ such that $\nu \in \mathcal{N}^{ted} \Leftrightarrow L_{\max}^\nu < L_G^\nu$ and $\nu \in \mathcal{N}^{ter} \Leftrightarrow L_{\min}^\nu > L_G^\nu$. Moreover,

Axiom 3 implies that for each $\nu \in \mathcal{N}$, there exist $c_{\min}^\nu, c_{\max}^\nu \in \mathbb{R}_+^M$ and a function f^ν such that $L_{\min}^\nu = f^\nu(c_{\min}^\nu)$ and $L_{\max}^\nu = f^\nu(c_{\max}^\nu)$. By Axioms 4-5, for each $\nu \in \mathcal{N}$, $c_{\min}^\nu, c_{\max}^\nu \in B(\omega^\nu, l^\nu; p, w, r)$, and $c_{\min}^\nu, c_{\max}^\nu$ can be produced as net outputs. Moreover, by Axiom 6, for each $\nu \in \mathcal{N}$, there exist production processes $\alpha_{\min}^\nu, \alpha_{\max}^\nu \in P$ and a positive scalar $\kappa^\nu(P, \mathbf{s})$ such that $\hat{\alpha}_{\min}^\nu \geq c_{\min}^\nu$, $\hat{\alpha}_{\max}^\nu \geq c_{\max}^\nu$, $L_{\min}^\nu = \kappa^\nu(P, \mathbf{s}) \alpha_{\min}^\nu$, and $L_{\max}^\nu = \kappa^\nu(P, \mathbf{s}) \alpha_{\max}^\nu$. By Axiom 2, $\kappa^\nu(P, \mathbf{s}) \alpha_{\min}^\nu \leq \kappa^\nu(P, \mathbf{s}) \alpha_{\max}^\nu$, and so $\alpha_{\min}^\nu \leq \alpha_{\max}^\nu$ holds. Again, by Axiom 2, $\nu \in \mathcal{N}^{ted} \Leftrightarrow \kappa^\nu(P, \mathbf{s}) \alpha_{\max}^\nu < L_G^\nu$ and $\nu \in \mathcal{N}^{ter} \Leftrightarrow \kappa^\nu(P, \mathbf{s}) \alpha_{\min}^\nu > L_G^\nu$. Thus, noting that by Axiom 7 for any $\nu, \mu \in \mathcal{N}$, if $c_{\min}^\nu = c_{\min}^\mu$ and $c_{\max}^\nu = c_{\max}^\mu$, then $L_{\min}^\nu = L_{\min}^\mu$ and $L_{\max}^\nu = L_{\max}^\mu$, it follows that the definition of exploitation also satisfies **LE**, as desired. ■