Source of Finance for Social Security Reform with Redistribution

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Abstract

This study investigates the welfare implications of social security reforms in Japan. Based on the overlapping generations model with idiosyncratic income risk, we consider four social security reform plans: (1) gradual reduction in the replacement rate by half, (2) sudden cut in the replacement rate by half, (3) introduction of a consumption tax, and (4) introduction of a capital income tax. We compute the transition paths of each case, and find that the introduction of a consumption tax and a capital income tax improves the welfare of young and future households, based on ex-ante welfare. We also reveal that two redistribution effects of the basic public pension are keys when considering social security reforms: (a) the insurance effect on lifetime income, and (b) the intertemporal effect that affects the asset and consumption profile.

Keywords: Social Security Reform, Consumption Tax, Capital Income Tax

JEL Classification: H55, D31, D33, E24, E25.

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1 Introduction

Many developed countries in Europe, as well as Japan, have become aging societies. Faced with aging, governments in such countries have taken social security reforms into account seriously to sustain the system. When considering social security reforms, we should examine the source of finance for the reforms, because some reforms may result in intergenerational and intragenerational redistribution. Many countries have adopted a flat payroll tax rate for social security; however, such a flat tax may adversely affect labor incentives and wealth accumulation. In this case, alternatives such as a consumption tax or a capital income tax may provide a possible means of improving social welfare.

In general, social security systems have large redistributive effects on lifetime income. Table 1 shows the redistribution effects of the social security system. For example, Italy and Greece provide social security payments at the same replacement rate for all types of households, and Germany provides almost the same replacement rate by earnings. In contrast, in Canada, Japan, and the UK, low earning households receive a relatively high public pension, which implies a high gross replacement rate, and the social security payment for the rich amounts to less than 30% of their earnings. Moreover, the replacement rate level differs among countries. There exist three groups: the high replacement rate group including Greece, Italy, and Sweden, the middle group including Canada and France, and the low-level group including Germany, Japan, and the UK. The social security system in some countries actually redistributes resources not only intergenerationally but also intragenerationally; however, the progressivity of the social security system differs among OECD countries. The pension Gini coefficient and the progressivity of the public pension, which is calculated as one minus pension Gini over the Gini coefficient of workers' earnings, differs significantly among countries. If the progressivity index is close to zero, the public pension does not have a redistribution effect, because it maintains the earnings inequality of workers even after retirement. According to OECD (2007), there are large differences among OECD countries in the redistribution effect of the social security system.

Following OECD (2007), we separate the role of the social security system into two parts: (a) insurance (annuity) part and (b) redistribution part. Concerning the

insurance part, it is widely believed that the social security system should be actuarially fair. On the other hand, in the redistribution part, a minimum floor is required for the consumption or redistribution of resources through the social security system. For this reason, in many countries, the social security system comprises a two-tier structure. The first tier comprises three types of redistribution schemes, basic pension schemes, resource-tested plans, and the minimum pension. In the developed countries, how the first tier is constructed differs significantly. For example, in the US, the government imposes a resource test for receipt of a public pension (Table 1).[1] On the other hand, in Japan, all residents receive the same amount of basic pension. Some countries adopt a mixture of the three roles, e.g., the UK. The second tier comprises two typical forms of social security systems, defined benefit and defined contribution. Although a limit is set on the second tier, it is basically earnings-related. The overall average entitlement of the first tier in the OECD amounts to 25%, which is not small. Therefore, in this study, we focus on the redistributive effect of the social security system, especially on the first tier.

Many theoretical studies have been conducted on social security reforms using an overlapping generations model. Moreover, because research on social security reform requires numerical values such as tax rates, quantitative studies on the social security system have attracted attention, since the pioneering research by Auerbach and Kotlikoff (1987). In particular, current research focuses on social security reform in an economy with heterogeneous agents, due to its redistribution effect. For example, İmrohoroğlu, İmrohoroğlu, and Joines (1995) investigate the optimal replacement rate based on a stationary state comparison in such an environment. Huang, İmrohoroğlu, and Sargent (1997), De Nardi, İmrohoroğlu, and Sargent (1999), and Conesa and Krueger (1999) consider the transitional dynamics of aging and social security reform. Nishiyama and Smetters (2007) extend a traditional research topic, the privatization of the public pension system, into a stochastic overlapping generations model with heterogeneous agents. Recently, Krueger and Ludwig (2006) and Attanasio, Kitao, and Violante (2007) extend the model into an open macroeconomy with aging to include the effects of international

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[1] In a partial equilibrium model, Hubbard, Skinner and Zeldes (1995) investigate the redistributive effects of the social security system, especially the effect of the means test.

capital flows. Although Storesletten, Telmer, and Yaron (1999) and Huggett and Ventura (1999) consider the redistributive effect of social security based on the US system, they focus on the stationary state. In this paper, we focus on the role of the first tier, the redistribution part, of the social security system in the stationary state and its transitional dynamics. Moreover, we consider the sources of finance for the reforms, such as a consumption tax and a capital income tax.

To consider social security reforms, we employ an overlapping generations model with heterogeneous agents. The features of our model are as follows. Our model is based on Conesa and Krueger (1999) and Nishiyama and Smetters (2005, 2007), who extend the stationary equilibrium model constructed by Aiyagari (1994) and Huggett (1996). There are infinitely many households who face idiosyncratic income risks. The government manages the social security system as a pay-as-you-go system. We assume that the social security payment is the basic type, i.e., constant payment for the retired, and we also assume that there are private annuity markets. Extending the research by Conesa and Krueger (1999), we examine the sources of finance for social security reform in an aging economy. We calibrate the parameters for the Japanese economy and calculate the stationary equilibrium and the transitional dynamics of the aging economy. We choose the Japanese economy as a target for the following two reasons: First, as in Table 1, the first tier of the social security system in Japan has a strong redistribution effect, and the basic public pension supports retired households. Our results in this paper also apply to any other country in which the government has introduced a basic public pension. To our knowledge, no research focuses on the pure role of the first tier of the social security system. Second, Japan is one of the most rapidly aging countries in the world. A population projection indicates that the percentage of retired households will exceed 40% by 2055. Therefore, the Japanese economy is a good example for considering social security reform in an aging society.

We examine four social security reforms: (1) gradual reduction in the replacement rate by half over 50 years, (2) a sudden cut of the replacement rate by half, (3) introduction of a consumption tax, and (4) introduction of a capital income tax. Transitional dynamics and social welfare of the reforms is evaluated based on a stochastic OLG model. In the literature, many studies focus on privatization or transition to a funded

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system. In addition to the traditional reforms, we also focus on the consumption and capital income taxes for the following reason. Conesa, Kitao, and Krueger (2008) show that the optimal capital income tax is positive in a life-cycle model with heterogeneous agents and incomplete markets. Moreover, in a simple model, a consumption tax is believed to provide a better form of taxation because it has no distortion effect. We find that introducing a consumption tax and a capital income tax improves the position of current young and middle households. In contrast, gradual privatization of social security reform is not supported even by current generations, although it would create large welfare gains in a future stationary state. Moreover, when the redistribution effect of social security is large, introduction of a consumption tax is preferred for current young and middle generations.

This paper is structured as follows. In Section 2, we provide details of our model. In Section 3, we calibrate the parameters of the model for the Japanese economy. We compare the stationary states of the model in Section 4. In Section 5, we consider the transitional dynamics and the welfare implications for the economy. In Section 6, we discuss the circumstances in which the reforms further improve welfare. Finally, we conclude the paper in Section 7.

2   Overlapping Generations Model

2.1   Demographic Structure

We consider the overlapping generations model with a continuum of households.[2] In the model, time is discrete. The lifespan of households is a maximum of 100 years, but they face mortality risks. The number of households aged $j \in \{0, \dots, 100\}$ in period t is denoted by $\mu_{j,t}$. A fraction of households $(1 - \phi_{j,t})$ exits the economy owing to death, and $\mu_{j+1,t+1} = \phi_{j,t}\mu_{j,t}$ is the population of households aged $j + 1$ at period $t + 1$. We assume that households begin economic activity at $j = 20$. Because households are in their childhood at $j = 0, 1, \dots, 19$, they do not engage in consumption or employment, but they are included in the population dynamics for computing the future fertility rate.

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[2]Our model includes the population dynamics and total factor productivity growth. Thus, to solve the model, we need to distinguish between normal and detrended variables. For details, see the Appendix.

We assume $\phi_{100,t} = 0$. Let $\mu_t = (\mu_{0,t}, \ldots, \mu_{100,t})$ denotes the population distribution in period t. Therefore, the population dynamics in our economy are expressed in the following matrix form:

$$
\mu_{t+1} =
\begin{bmatrix}
1 + \psi_t & 0 & 0 & \cdots & 0 \\
\phi_{0,t} & 0 & 0 & \cdots & 0 \\
0 & \phi_{1,t} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \phi_{99,t} & 0
\end{bmatrix}
\mu_t,
$$

where ψ_t is the population growth rate of age 0 from t to $t+1$.[3] New households enter the economy in period $t+1$ as $\mu_{0,t+1} = (1+\psi_t)\mu_{0,t}$. The aggregate population including children at t is $N_t = \sum_{j=0}^{100} \mu_{j,t}$. We denote the population growth rate from period t to $t+1$ as n_t, i.e., $N_{t+1} = (1+n_t)N_t$. Although the population distribution is constant over time in the stationary state (i.e., $\mu_{j+1,t+1}/N_{t+1} = \mu_{j,t}/N_t$), the population distribution varies in the transition paths. In the following section, we consider both the stationary economy and the transitional dynamics.

2.2 Households

2.2.1 Objective Function

A household born in period t has a lifespan of at most 81 periods, supplies labor elastically until age 65, and faces idiosyncratic uncertainty with respect to its individual labor productivity. The objective function of the household in period t is expressed as follows:

$$
U_t = E \left\{ \sum_{j=20}^{100} \beta^{j-20} \left(\prod_{i=20}^{j-1} \phi_{i,t} \right) u(c_{j,t+j-20}, \bar{h} - h_{j,t+j-20}) \right\},
$$

where $\beta > 0$ is a discount factor and $\phi_{19,t} = 1$. All households have labor endowment $\bar{h}$ and supply labor $h_{j,t+j-20} \in [0, \bar{h}]$ at j.

Since households of age $j \in \{20, \ldots, 65\}$ are of employable age, they can supply labor elastically. Thereafter, i.e., $j \in \{66, \ldots, 100\}$, households retire and receive social security benefit from the government.

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[3]See Ríos-Rull (2001) for details on the transition of population distribution.

2.2.2 Earnings Profile and Idiosyncratic Income Risk

All households have deterministic labor productivity. Average earnings must reflect age-specific average labor productivity. Average labor productivity grows when households are young and peaks in middle age around 50; in other words, the efficiency of a household has a hump shape over its working life. We denote the deterministic productivity measured by hourly wages as $\{\kappa_j\}_{j=20}^{65}$.

In addition to the average productivity, all households face idiosyncratic skill risks when they are in employment. Following Storesletten, Telmer, and Yaron (2004), we assume that the idiosyncratic risk comprises three components: (1) transitory shocks, (2) persistent shocks, and (3) the fixed effect. The idiosyncratic labor productivity process e_j is specified as follows:

$$\ln e_j = \alpha + z_j + \varepsilon_j, \ \alpha \sim \mathcal{N}\left(0, \sigma_\alpha^2\right), \ \varepsilon_j \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right) \tag{1}$$

$$z_j = \rho z_{j-1} + \eta_j, \ \eta_j \sim \mathcal{N}(0, \sigma_{\eta,j}^2). \tag{2}$$

The fixed effect is denoted by the variance of α, and the transitory shock as that of ε_t, both of which follow the log-normal distribution. A persistent component of the idiosyncratic shock is represented by z_t, which is composed of the persistence parameter ρ and the persistent shock η_j. Let $s \equiv (\alpha, z, \varepsilon) \in S$ represent a state of the idiosyncratic shocks for an individual household. We assume that the average efficiency profile and the stochastic process are independent of time t. Thus, the pre-tax labor earning of each age group is determined by $y_{j,t} = w_t \kappa_j e_j h_{j,t}$, where w_t is the economy-wide wage level.

2.2.3 Social Security System

The government grants social security benefits through a *flat* payroll tax from labor earnings, and retired households receive the social security benefit. The flat payroll tax rate is denoted as τ_t^{ss}. Moreover, we define the consumption tax rate and the linear capital income tax rate as τ_t^{con} and τ_t^{cap}, respectively. After retirement, a household receives a *lump-sum* social security benefit $\varphi_t w_t H_t$, where φ_t is a replacement rate and $w_t H_t$ is the average earnings of workers as defined later.

Since we assume that the social security benefit is constant for all households, the social security system in our model has large redistribution effects.[4] We assume that the social security benefit in our model consists of the first tier of the social security system, as mentioned in Section 1. Although the social security system differs significantly across countries, it generally contains large redistribution mechanisms. The earnings-related or defined contribution part of the public pension, i.e., the insurance part, has relatively small impact on redistribution. On the other hand, the redistribution part has a strong redistributive mechanism, especially at the minimum or resource-tested level. To focus on the redistribution effect, we assume that the social security benefit is constant, which corresponds to the first-tier of the social security system. As the insurance part of the social security system, we consider private annuity markets, originally introduced by Yaari (1965) and recently investigated by Hansen and İmrohoroğlu (2008). Hansen and İmrohoroğlu (2008) define partial annuitization by assuming the price of the annuity by $\Lambda_{j,t} = 1 - \lambda(1 - \phi_{j,t})$, where λ characterizes the fraction of the annuitized asset. We consider two extreme cases: (1) there exists a perfect annuity market, $\lambda = 1$, and (2) there exists no such market, $\lambda = 0$. Since households face mortality risk, some households may die with positive assets. If we assume the existence of private annuity markets, then the assets are annuitized. In this case, the role of insurance for long living in the social security system is eliminated completely. On the contrary, if there are no such markets, we assume that the accidental bequests are collected by the government and redistributed to all households as a lump-sum transfer b_t.

A household has some asset holdings $a_{j,t} \in A$ at age j and in period t. The assets contain annuitized and non annuitized components. The budget constraints for employees and retirees are as follows:

$$(1 + \tau_t^{\mathrm{con}})c_{j,t} + \Lambda_{j,t}a_{j+1,t+1} \le (1 + (1 - \tau_t^{\mathrm{cap}})r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\mathrm{ss}})w_t \kappa_j e_j h_{j,t}, \quad : \text{Employee}$$

$$(1 + \tau_t^{\mathrm{con}})c_{j,t} + \Lambda_{j,t}a_{j+1,t+1} \le (1 + (1 - \tau_t^{\mathrm{cap}})r_t)(a_{j,t} + b_t) + w_t \varphi_t H_t, \qquad : \text{Retiree}$$

where r_t is the net interest rate at t. We assume that households face a liquidity con-

[4]İmrohoroğlu et al. (1995) and Conesa and Krueger (1999) consider an efficient social security system with a constant social security benefit. On the other hand, Storesletten et al. (2004) assume that the social security benefit depends on the resource test. See Storeslettten et al. (1999) and Huggett and Ventura (1999) for the redistributive effects of the social security system in the US.

straint, i.e., $a_{j,t} \geq 0$.

2.3 Behavior of Firms and the Factor Prices

The aggregate production technology follows a Cobb-Douglas constant returns to scale production function

$$Y_t = A_t K_t^\theta H_t^{1-\theta},$$

where A_t denotes the total factor productivity (TFP) in period t, K_t is the aggregate capital, and H_t is the aggregate labor supply measured by efficiency units. We assume that a sequence of the TFP is deterministic. Therefore, there are no aggregate uncertainties in the economy, and the aggregate productivity and population growth can be forecasted accurately. We denote the gross growth rate of the TFP as $1 + g_t = (A_{t+1}/A_t)^{1/(1-\theta)}$.

The asset holdings and labor supply of each household differ even in the same cohort and age group, due to idiosyncratic income risks. We denote a fraction of households aged j with asset a, and realize productivity s as $\Phi_t(a, s, j)$.[5] By construction, $\int d\Phi_t(a, s, j) = 1$. The aggregate capital and labor supply are determined by the sums of each generation's capital and labor, as follows.

$$K_t = \sum_{j=20}^{100} \mu_{j,t} \int a_{j,t} d\Phi_t(a, s, j), \tag{3}$$

$$H_t = \sum_{j=20}^{65} \mu_{j,t} \int \kappa_j e_j h_{j,t} d\Phi_t(a, s, j). \tag{4}$$

The interest rate r_t and wage w_t are determined as follows.

$$r_t = \theta A_t \left(\frac{K_t}{H_t}\right)^{\theta-1} - \delta, \ w_t = (1-\theta) A_t \left(\frac{K_t}{H_t}\right)^{\theta},$$

where δ is the depreciation rate.

2.4 The Government

We assume that the government collects tax to finance social security benefits and redistributes it to retired households in a lump-sum manner, and we do not consider other

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[5]For details of the distribution function, see the Appendix.

government expenditures. The social security system is governed by the *Pay-As-You-Go* system. Accordingly, the government collects payments from employees and retirees through taxes $(\tau_t^{\text{ss}}, \tau_t^{\text{con}}, \tau_t^{\text{cap}})$, and grants social security benefits. We assume that the replacement rate φ_t is fixed exogenously, and that the corresponding payroll tax rate is determined endogenously. We denote the aggregate social security payments by the payroll tax as T_t^{SS}, by the consumption tax as T_t^{CON}, by the capital income tax as T_t^{CAP}, and aggregate social security benefit as B_t.

The government must satisfy the following budget constraints:

$$B_t = T_t^{\text{SS}} + T_t^{\text{CON}} + T_t^{\text{CAP}}, \tag{5}$$

$$T_t^{\text{SS}} = \sum_{j=20}^{65} \mu_{j,t} \int \tau_t^{\text{ss}} w_t \kappa_j e_j h_{j,t} d\Phi_t(a,s,j) = \tau_t^{\text{ss}} w_t H_t,$$

$$T_t^{\text{CON}} = \sum_{j=20}^{100} \mu_{j,t} \int \tau_t^{\text{con}} c_{j,t} d\Phi_t(a,s,j) = \tau_t^{\text{con}} C_t,$$

$$T_t^{\text{CAP}} = \sum_{j=20}^{100} \mu_{j,t} \int \tau_t^{\text{cap}} r_t a_{j,t} d\Phi_t(a,s,j) = \tau_t^{\text{cap}} r_t K_t,$$

$$B_t = \sum_{j=66}^{100} \mu_{j,t} w_t \varphi_t H_t = w_t \varphi_t H_t N_t^{\text{ret}},$$

where N_t^{ret} is the propotion of retired households in the total population. Note that the average labor earning of all workers is $w_t H_t$.

The lump-sum transfer of accidental bequests is determined by the following equation:

$$b_t = \sum_{j=20}^{100} \mu_{j,t} (1 - \lambda)(1 - \phi_{j,t}) a_{j,t}.$$

Note that if $\lambda = 1$, all assets are perfectly annuitized, and there are no accidental bequests.

From the above, the Bellman equation of age j in period t is

$$V_{j,t}(a_j, s_j) = \max \left\{ u(c_{j,t}, \bar{h} - h_{j,t}) + \phi_{j,t} \beta E V_{j+1,t+1}(a', s') \right\}, \tag{6}$$

subject to

$$(1 + \tau_t^{\text{con}}) c_{j,t} + \Lambda_{j,t} a_{j+1,t+1} \le (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\text{ss}}) w_t \kappa_j e_j h_{j,t}, \tag{7}$$

$$(1 + \tau_t^{\text{con}}) c_{j,t} + \Lambda_{j,t} a_{j+1,t+1} \le (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + \varphi_t w_t H_t, \tag{8}$$

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2.5 Definition of a Competitive Equilibrium

Our concern is the stationary state and the transitional dynamics of the economy. Therefore, we need two definitions of equilibrium, one in the stationary state and the other in transition.

Definition 1 (Recursive Competitive Equilibrium) *Given the government's policy $\{\varphi_t\}$ and the population dynamics, the Recursive Competitive Equilibrium is a set of value functions $\{V_t\}$, policy functions $\{g_{j,t}^c, g_{j,t}^h, g_{j,t}^a\}$, aggregate capital $\{K_t\}$, aggregate labor $\{H_t\}$, factor prices $\{r_t, w_r\}$, and social security taxes $\{\tau_t^{ss}, \tau_t^{con}, \tau_t^{cap}\}$ that satisfy the following conditions:*

(i) A Household's Optimality: Given the factor prices $\{r_t, w_t\}$ and the social security taxes $\{\tau_t^{ss}, \tau_t^{con}, \tau_t^{cap}\}$, the value function $\{V_t\}$ solves equation (6), and $\{g_{j,t}^c, g_{j,t}^h, g_{j,t}^a\}$ are the associated policy functions. The value and policy functions are measurable.

(ii) A Firm's Optimality: The factor prices are competitively determined as follows,

$$r_t = \theta A_t \left(K_t/H_t\right)^{\theta-1} - \delta, \ w_t = (1-\theta) A_t \left(K_t/H_t\right)^{\theta}.$$

(iii) Market Clearing: The market clearing conditions of equations (3) and (4) are satisfied.

(iv) the Government's Budget: The governments' budget (5) clears.

(v) Transition Law: $\Phi_{t+1} = \mathcal{T}(\Phi_t)$.

Definition 2 (Stationary Recursive Competitive Equilibrium) *The Stationary Recursive Competitive Equilibrium is a recursive competitive equilibrium with a stationarity of distribution $\Phi_{j,t+1} = \Phi_{j,t}(\forall t)$ for each age group j.*

The final purpose of the paper is to investigate the welfare implications of the competitive equilibrium on the transition path, which requires complex computation. In this paper, we follow the method proposed by Conesa and Krueger (1999) and Nishiyama and Smetters (2005,2007), who compute two stationary equilibria and their transition path. Thus, to compute the transition path, we need to calibrate the initial and final stationary states. We set the initial stationary state of Japanese economy in the year 2008 and the final state in the year 2200.[6]

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[6]In the actual numerical procedure, we compute a detrended path.

3 Calibration

3.1 Preference and Production Parameters

First, we calibrate the fundamental parameters in the model. As the target of the initial stationary state, we choose the Japanese economy in the year 2008.

Households enter into our economy at age 20, supply labor until 65, and live till at most 100. We assume that the instantaneous utility function is of the Cobb-Douglas type[7]

$$u\left(c_{j,t}, \bar{h} - h_{j,t}\right) = \frac{\left[c_{j,t}^{\sigma}(\bar{h} - h_{j,t})^{1-\sigma}\right]^{1-\gamma}}{1-\gamma}.$$

The elasticity of the intertemporal substitution (EIS) parameter is set as $\gamma = 2$. This value is standard in the macroeconomics literature. Abe, Inakura, and Yamada (2007) estimate the preference parameters in Japan by structural estimation using *Japanese Panel Study of Consumers* data compiled by the Institute of Household Economy, and determined that the EIS parameter ranges from 2 to 7. A share parameter for consumption and leisure is set as $\sigma = 0.55$ to match the average work hours in the model with the actual Japanese data. In the model, we use an equilibrium interest rate of 4%, which is the average return of capital in Japan in 2000, as estimated by Hayashi and Prescott (2000), in the model as a target to determine the discount factor, $\beta = 0.989$. For the available time endowment $\bar{h}$, it is assumed that all households have 16 hours $\times$ 5 days $\times$ 4 weeks $\times$ 12 month per year, i.e., $\bar{h} = 3840$.

Finally, we choose the parameters for the production function. The capital share parameter θ is fixed at 0.362, from Hayashi and Prescott (2002). The depreciation rate is also taken from Hayashi and Prescott (2002), and the value is specified at $\delta = 0.083$. These values are the average of the 1990s in Japan.[8] In our model, the TFP growth rate

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[7]We use the nonseparable utility function that is used in the broad macroeconomics literature because we consider a growth economy. In contrast, some empirical researches reveal that microeconomic behavior is consistent with the separable utility function, although this is in contradiction with a growing economy. Heathcote, Storesletten, and Violante (2008) investigate the importance of insurance for income risks when the utility function is separable and nonseparable with respect to leisure. İmrohorolu and Kitao (2008) indicate that differences in the elasticity of labor supply have a surprisingly small effect on social security reform, although they result in a large reallocation of working hours over the life cycle.

[8]For details of data description, see Hayashi and Prescott (2002).

is provided exogenously and can be forecasted accurately. From Hayashi and Prescott (2002), Chen et al. (2007), and Braun et al. (2007), the TFP growth is set as $1 + g_t = 1.02$, which is the average between 1960 and 2000.

3.2 Idiosyncratic Income Risk

Estimating parameters for the idiosyncratic income risks that all households face is difficult because of the scarcity of micro data in Japan. Ohtake and Saito (1998) indicate that the logarithm of the variance of income in Japan increases across age groups. Moreover, they show that the shape of the age-variance profile is convex over age groups. To account for the convexity of the variance profile, Abe and Yamada (2006) specify the labor income process and estimate the parameters In this paper, we use the estimated data shown in Appendix Table 2 in Abe and Yamada (2006). To incorporate the non-linearity of the income variances, we use an age-dependent income variance shock. We choose the income shock parameters to match the cross-sectional variance of income.

Following Storesletten et al. (2004), we assume that the idiosyncratic labor productivity process follows equations (1) and (2). Abe and Yamada (2006) report on the possibility of $\rho \geq 1$ because of the convexity of the variance profile. However, incorporating $\rho \geq 1$ makes the numerical computation far more difficult. Thus, we choose the persistence parameter to be close to one; moreover, the standard deviation of the persistence shock increases across age groups (i.e., $\rho = 0.98$, $\sigma_{\eta_{20}} = 0.05$, and $\triangle \sigma_\eta = 0.0005$).[9] After the specification, we approximate the persistent shock process as a seven-state Markov chain by Tauchen's (1986) method. Assuming the initial value of the persistent shock, z_{20}, to be zero, an intercept of the income variance profile, i.e., the income variance of age 20, implies the sum of the variances of the transitory shock and the fixed effect. Because the income variance of age 20 is about 0.1 from Abe and Yamada (2006), the standard deviation of the transitory shock and the fixed effect are estimated to be $\sigma_\varepsilon = 0.08$, and $\sigma_\alpha = 0.25$, respectively. Both are approximated by two states as $\{e^{-\sigma}, e^\sigma\}$.[10]

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[9] The variance of the persistent shock represents the slopes of the income variance profile over the life cycle. For details, see Storesletten et al. (2004).

[10] Based on this calibration, the model-generated income variances profile matches the actual income

3.3 Average Hourly Wage Profile

The efficiency unit of average productivity for each age $\{\kappa_j\}$ determines the average hourly wage profile. We conduct the calculation following the method proposed by Hansen (1993), and in particular, that by Braun et al. (2007), which is based on *the Report on the Special Survey of the Labor Force Survey* by the Statistics Bureau, the Management and Coordination Agency, Government of Japan. Table 2 lists the average hourly wage for each age group. we use a smoothed profile.

3.4 Demographic Structure

We set demographic parameters to replicate the actual and projected population dynamics. The National Institute of Population and Social Security Research (NIPSSR) provides population projections from 2005 to 2055.[11] We set the survival probability $\{\phi_{j,t}\}_{t=2005}^{2055}$ from the medium variant of the value estimated by the NIPSSR. The fertility rate ψ_t is also taken from the medium variants of the projections. Because the population growth in our model is represented by the growth rate of newborns, we use the ratio of the projected population of newborns between period t and $t + 1$.

As we need to compute two stationary states and the transition paths, we set the initial stationary state in the year 2008. After the population changes from 2008 to 2055, following the projection by the NIPSSR, the population growth rate is assumed to converge to zero over 10 years between 2055 and 2064. Although the population growth rate of the newborns converges immediately, it takes approximately 100 years to reach a new stationary population distribution.

One problem that arises here is how to choose an initial population distribution in the initial stationary state. The actual population distribution in 2008 does not seem to be stationary because of the existence of the baby boomer generation, which is shown in Figure 1.[12] However, to compute the initial stationary state, a population distribution is

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variances over the life cycle in Japan. For details, see Yamada (2008).

[11]Details are available from the web: http://www.ipss.go.jp/p-info/e/psj2008/PSJ2008.html

[12]The population distribution in 2005 is obtained from *Population Census* by the *Ministry of Internal Affairs and Communications* in Japan, and the distribution in 2008 is calculated by the mortality and fertility rates estimates between 2005 and 2008.

required. Therefore, we assume that the households in the model *believe* that the actual population in 2008 is stationary.

The projection by the NIPSSR indicates *three variants*, the high, medium, and low population projections. We plot the fraction of the child population (under the age of 19), the working population (20-65), and the retired population (66-100) in Figure 2. In the medium variant projection, the fraction of retired households peaks around 2070, and thereafter, the rate converges to a new stationary state. In the low variant, the fraction of the retired households reaches over 40% and the working population decreases sharply with fewer births.

4   Stationary State Comparison

4.1   Four Policy Experiments

Before considering the complicated Japanese economy with aging, we consider a simple demographic structure to focus on the pure redistributive effect of social security reforms. The population distribution is constant over time, i.e., the population growth rate is zero.[13]

As policy experiments, we consider four social security reform plans. We calculate the equilibrium paths of the following scenarios:[14]

Benchmark As a benchmark, the replacement rate is targeted to be 25%, i.e., $\varphi_t = 0.25$. Although the replacement rate seems to be low compared with previous research such as Conesa and Krueger (1999), we focus only on the redistribution part, as mentioned in Section 1. In other words, we examine the economic implications of the first tier of the social security system. The average is around 25% (See Table 1).

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[13]We use the survival probability in 2008 over time, and the population distribution is calculated using $\mu_{j+1} = \phi_{j,2008}\mu_j$.

[14]Conesa and Krueger (1999) consider three cases: (1) a sudden cut in social security benefit, (2) a gradual decrease in the replacement rate over 50 years, (3) and a cut in social security after 20 years.

Case (1) Gradual Decline We consider a gradual cut in the social security benefit to half over 50 years; i.e., the final replacement rate is 12.5%.[15]

Case (2) Sudden Cut We again consider a cut in the social security benefit by half. However the replacement rate is cut suddenly in 2009.

Case (3) Consumption Tax We introduce a consumption tax for financing social security payments to retired households. The consumption tax rate τ_t^{con} is set as 5%. We choose the tax rates such that the remaining (determined endogenously) payroll tax rate is almost the same as in the capital income tax case stated below.

Case (4) Capital Income Tax We introduce a linear capital income tax to finance the social security benefits in part. The tax rate τ_t^{cap} is set as 25%. In this tax rate, the remaining payroll tax rate is approximately 1%, which is similar to the consumption tax case. Moreover, when we employ consumption tax and capital income tax rates of 5% and 25%, respectively, the welfare gain of introducing the capital income tax is similar to the case of the consumption tax.

4.2 Welfare Evaluation Measures

To compare social security reforms, we need a criterion that evaluates the social welfare of households. First, to evaluate the welfare of households, following Aiyagari and Mc-Grattan (1998), Conesa and Krueger (1999), and Conesa, Kitao, and Krueger (2008), we employ the following ex-ante expected value:

$$EV_t = \sum \pi(s) V_{20,t}(0, s_{20}). \tag{9}$$

This welfare criterion implies that we use a measure of the expected value of households who enter the economy in period t at age 20. In other words, it is a lifetime discounted value of each cohort before entering the economy. By assumption, the households have no wealth at age 20. Moreover, we apply the following consumption equivalent variation (CEQ) measure:

$$CEQ_t = \left(\frac{EV_t^{\mathrm{Reform}}}{EV_t^{\mathrm{Bench}}} \right)^{\frac{1}{\sigma(1-\gamma)}} - 1, \tag{10}$$

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[15] For half privatization of the social security system, see Nishiyama and Smetters (2007).

which compares the consumption equivalent variation of cohorts between the benchmark and a social security reform.

As a second welfare measure, we introduce hypothetical voting of the existing generations, as in Conesa and Krueger (1999). Suppose that a household weakly prefers to reform the social security system, i.e., $V_{j,2008}^{\text{Reform}}(a,s) \geq V_{j,2008}^{\text{Bench}}(a,s)$, then the household with state (a,s,j) votes in agreement with the reform. Then, the total agreement of voting by age j is determined as follows:

$$TA_j = \int I(a,s,j)d\Phi(a,s,j),$$

where $I(a,s,j)$ is an indicator function defined as follows:

$$\begin{cases} I(a,s,j) = 1, & \text{if } V_{j,2008}^{\text{Reform}}(a_j,s_j) \geq V_{j,2008}^{\text{Bench}}(a_j,s_j), \\ I(a,s,j) = 0, & \text{else.} \end{cases} \tag{11}$$

4.3 Stationary State Analysis

Table 3 summarizes the macroeconomic and microeconomic statistics in the benchmark case and social security reforms, when the population distribution is constant.[16] Compared with the benchmark case, half privatization implies capital deepening, and as a result, the interest rate declines by 0.5%. As households receive a low social security payment after retirement, they accumulate more wealth. Moreover, they also supply labor more extensively. When the government reduces the replacement rate by half, the labor supply measured by efficiency (Earning) increases by 7.7%. An interesting point is that working hours do not increase as much as earnings. This implies that highly productive households supply labor more intensively. As a result, the output and aggregate consumption also rise. The welfare implication of half privatization is consistent with previous research, such as İmrohorolu et al. (1995). That is, the optimal replacement rate is close to zero, and in our calculation, half privatization implies a welfare gain of 2.7%, as calculated using equation (10).

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[16]In the stationary state comparison, there are no differences between gradual declines and a sudden cut of the replacement rate.

The introduction of a consumption tax and a capital income tax as a source of finance results in similar effects on the payroll tax rate, i.e., the resulting payroll tax rates are 1.1% and 1.4%, respectively. In other words, in our model, a consumption tax of 5% and a capital income tax of 25% collect similar amounts of social security payments. However, these taxes affect wealth accumulation in opposite ways. If the government introduces a consumption tax, the capital-output ratio increases slightly, and it has a small effect on work hours. Note that in a model with a labor/leisure choice, the consumption tax causes tax distortion, although the Euler equation does not include the consumption tax rate, because it is included in the intratemporal first order equation. On the other hand, a capital income tax offers a disincentive for accumulating wealth.

However, the introduction of either a consumption tax or a capital income tax results in similar welfare gains of 1.1% or 1.2%, respectively. Because households in our model face idiosyncratic income risks, based on our welfare criterion, the basic public pension system has an insurance effect on lifetime income. In other words, the basic public pension equalizes the lifetime income of all households in ex-ante criterion. This equalization effect is larger in the consumption and capital income taxes than in the payroll tax, because these taxes are also collected from retirees, who are much more unequal than young households. Moreover, the basic public pension has different effects on the wealth rich and the wealth poor. The wealth rich accumulate more near retirement for consumption smoothing, and the wealth poor decumulate wealth for the same reason. The capital income tax collects more social security payments from the wealth rich. Therefore, the welfare effect of the capital income tax measured by the consumption equivalent variation is slightly higher than that of the consumption tax, even though the remaining payroll tax is higher in the case of the capital income tax.

Therefore, social welfare has improved based on the stationary state comparison in all social security reforms. However, there may exist households who lose welfare in the transitional dynamics.

5 Transitional Dynamics with/without Population Aging

5.1 Constant Population Distribution

In general, transitional dynamics describes very complicated paths because demographics do not change monotonically. To focus on the social security reforms, as a first step, we consider a simple demographic structure: the population distribution is constant over time.

Figure 3 shows the transitional dynamics of the interest rate, the payroll tax, aggregate capital, and aggregate labor. The aggregate variables are normalized to be one at period 0. In the equilibrium path, the interest rate declines when the government reduces the replacement rate by half gradually or suddenly, which is not surprising because households need to accumulate more wealth for their retirement. Moreover, the interest rate declines when the social security payment is financed by a consumption tax. In contrast, introducing a capital income tax increases the interest rate because the capital income tax creates a disincentive for accumulating assets. Although a social security reform is introduced in year 1, the adjustment of capital and labor continues for more than 10 years. Thus, even though the payroll tax rate is reduced suddenly rather than by gradual decline, the interest rate adjusts slowly.

5.2 Welfare Comparison of Each Cohort

In the previous section, we focused on the general equilibrium effect of the social security reforms. Next, we consider the welfare implications of the reforms, especially those for intergenerational inequality. In Figure 4, we plot the social welfare of each cohort based on equation (10). In the long run, the consumption equivalent variation measures of the social security reforms converge to new stationary state values after the reform. In particular, the consumption tax and the capital income tax converge to a similar level in the long run. However, the reforms have different effects on the existing and near-future generations.

If the lines in Figure 4 are below 0%, such generations exhibit distaste for reforms. Not surprisingly, the gradual privatization damages the welfare of the current generations

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significantly, due to the tax burden and small benefits. Thus, the CEQ variation is below the benchmark for all generations who enter the economy before period 0. For example, households around age 40, who entered the economy 20 years earlier, reduce the CEQ by 1.4%. In contrast, the other three financing schemes have different implications. A sudden cut in the social security benefit improves the welfare of the current young and middle generations, based on the cohort's welfare. Although the sudden cut damages the older generations by more than 2%, as measured by the CEQ, such a policy improves the welfare of young cohorts, as they bear no cost and the optimal replacement rate is very low. Introducing a consumption tax and a capital income tax also improves the welfare of the current young and middle generations. Both taxes enhance intragenerational inequality in an economy with a basic public pension, because of its redistribution effect. In particular, because the asset profile is strongly hump-shaped, the capital income tax is preferred by young households. Compared with the capital income tax, the consumption tax shares the burden across all generations equally, because of the flat consumption profile. Old households prefer to stay with the status-quo social security system for the three reforms.

5.3   Social Security Reforms in Japanese Economy

In the transition path, many factors reallocate resources, including aging, an increasing tax burden, and changing factor prices. In particular, the fraction of the retirees in the total population exceeds 40% in Japan, as shown in Figure 2. Thus, the aging level of the Japanese economy will at least temporarily be considerably larger than in the final stationary equilibrium.

As a benchmark, we use the medium variant of the population projection by the NIPSSR. Figure 5 plots the general equilibrium paths of the interest rate, payroll tax rate, aggregate capital, and labor. Figure 1 shows that the population distribution in Japan is not very smooth, due to baby boomers and their children. Therefore, the general equilibrium path fluctuates erratically compared with Figure 3. Contrary to Figure 3, the interest rate declines after all social security reforms. There is a significant predicted capital deepening and aggregate capital increases for 30 years. According to

the aging, the payroll tax rate increases sharply, and becomes more than 10% without reform. Note that this value is the tax rate of the first tier only. Therefore, the total tax burden of social security may be more than 20% of earnings, when we consider the cost of sustaining the current total social security system in Japan.[17]

In an economy with significant aging, there exists crucial intergenerational inequality. Based on various population projections, Figure 6 plots the welfare of each cohort, and the CEQ is normalized such that households who enter the economy in 1950 appear as zero. Although this criterion calculates the dynamic general equilibrium effect of aging only by comparing different generations of household, young generations of Japanese households suffer from aging and a tax burden, and at the bottom of the welfare are the current and near-future young. Although the welfare rebounds weakly, some social security reform should be seriously considered.

5.4 Welfare Comparison and Majority Voting

Based on the transitional dynamics of the Japanese economy, we calculate the consumption equivalent variation of each cohort with social security reform in Panel (a) of Figure 7. The shape is similar to the case in Figure 4. Two privatization policies have a large impact on the young and old compared with the case of a constant population. The gradual decline in the replacement rate by half is not preferred by the current generations, as ever. In particular, the current young and middle generations suffer a consumption loss of more than 2% from such a reform because they pay more tax and receive less. As also in Figure 4, although the introduction of the consumption tax and capital income tax improves future generations, middle and old households do not prefer such reforms.

We finally consider intragenerational inequality. Conesa and Krueger (1999) consider the hypothetical voting of each household.[18] We consider voting conducted in 2008. If

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[17]In the social security reform in Japan in 2004, the government decided to set the ceiling of the payroll tax rate as about 18%. The government decumulates the social security funds and the payment of the public pension may also decline, although the government promises to maintain the replacement rate above 50%.

[18]Boldrin and Rustichini (2000) and Casamatta, Cremer, and Psetieau (2000) consider the political decision process of social security reforms more explicitly.

a household agrees to some reform, i.e., $V_{j,2008}^{\text{Reform}}(a_j, s_j) \geq V_{j,2008}^{\text{Bench}}(a_j, s_j)$, he/she votes. The aggregated value of voting by age is plotted in Panel (a) of Figure 8. Consistent with Figure 7, middle and old households do not vote in agreement with any social security reform. However, the sudden cut, a consumption tax, and a capital income tax are the candidates that young and middle households support. In the case of very young households, such as the early 20s, all households agree to the reforms. Because there is heterogeneity in a cohort, opinion is divided even at the same age. For example, the asset rich do not prefer the capital income tax as much as the asset poor of the same age. The capital income tax is preferred more around age 40, because the asset profile increases sharply, and there is thus a cutoff point for voting around this age. Note that half privatization obtains no agreement in an economy with aging. This is also consistent with Figure 7.[19]

6 When does the Social Security Reform Gains more Welfare?

6.1 Strong Redistribution Effect

To clarify the redistribution effect of social security reform, we consider a strong redistribution policy. Although the overall entitlement of the first tier of social security is generally less than 30% in many OECD countries, as listed in Table 1, we set the replacement rate as 40% when considering a strong redistribution effect. Panel (b) of Figure 7 shows that this does not change the shape of cohorts' welfare profile, although a privatization policy is more effective in this case.

Panel (b) of Figure 8 shows that the consumption tax and capital income tax are more supported by the young through voting, due to their insurance and redistribution effects. Moreover, if the redistribution effect is strong, the discrepancy between consumption tax and capital income tax in voting becomes small. When the replacement

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[19]This result may seem inconsistent with Conesa and Krueger (1999) who find that although there exists a status-quo bias, young households preferred to reduce the replacement rate. They discuss whether the vote on reforms declines if the heterogeneity of the model becomes larger. Our model is more heterogeneous than their model because we consider three types of income risks.

rate is high, the corresponding payroll tax rate becomes high and the consumption profile becomes steeper. In other words, the young consume less. Thus, young households support a consumption tax more, due to the lower tax burden. Because the welfare gain through privatization is large, gradual and sudden cuts are supported by many households compared with the benchmark case. In general, social security reform should be discussed regarding not only the source of finance, but also the extent of redistribution. Therefore, the source of finance for social security should focus more attention on the redistribution effects.

6.2   No Private Annuity Market

Finally, we investigate the role of a private annuity market in the model. If private annuity markets do not exist, the social security system offers insurance against long-living risks. Because households evaluate consumption weakly in old age due to high discounting, the asset holdings of middle and old households decline. As a result, the welfare gain from a capital income tax and a consumption tax also declines slightly, as shown in Panel (c) of Figure 7. In particular, even in the long-run stationary state, the capital income tax does not attain a high value with the tax rate of 25% compared with the economy with a consumption tax. Therefore, a capital income tax should be introduced with the private annuity market.

7   Concluding Remarks

In this paper, we consider the welfare implications of social security reforms using an overlapping generations model with idiosyncratic income risks and private annuity markets. We examined four reforms: (1) gradual privatization by half, (2) sudden cut in the replacement rate by half, (3) introduction of a consumption tax, and (4) introduction of a capital income tax. We find that all four cases improve welfare by a stationary state comparison. In contrast, no one supports gradual privatization through majority voting, because all current generations would find their welfare reduced during the transitional dynamics. A consumption tax and a capital income tax improve the welfare of the current young and middle generations below 40, although they are a minority in the

economy. We also reveal that in designing social security systems, we need to consider two redistribution effects of the basic pension. The redistribution effects consist of the insurance effect on lifetime income, i.e., intragenerational inequality, and the intertemporal effect that affects the asset and consumption profile. When the consumption profile is steep, for example, the replacement rate is high, a consumption tax is supported by the current young generations.

This research focuses on the redistribution effect of the basic public pension, i.e., the first tier. In future research, we should extend the research to the second tier, which may include earnings-related components of the social security system and private defined contributions. Moreover, we should seriously consider a resource-tested basic public pension. In this paper, the voting mechanism is very simple and all social security reforms are rejected. We believe that political support should be seriously considered when considering some social security reforms. For example, a nonlinear capital income tax may improve the welfare of the majority in an economy with heterogeneous agents.

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	Multiple of Mean					Resource-tested	Basic	Minimum	Overall Entitlement	Pension Gini	Progressivity Index
	0.5	0.75	1	1.5	2						
Canada	75.4	54.4	43.9	29.6	22.2	17	14	–	31	3.7	86.6
France	63.8	51.2	51.2	46.9	44.7	32	–	23	32	20.5	24.6
Germany	39.9	39.9	39.9	39.9	30.0	19	–	–	19	20.0	26.7
Greece	95.7	95.7	95.7	95.7	95.7	11	30	34	34	26.5	2.6
Ireland	65.0	43.3	32.5	21.7	16.2	27	30	–	30	0.0	100.0
Italy	67.9	67.9	67.9	67.9	67.9	–	–	–	22	26.4	3.1
Japan	47.8	38.9	34.4	29.8	27.2	–	16	–	16	14.4	46.9
Sweden	79.1	66.6	62.1	64.7	66.3	34	–	–	34	23.7	12.9
UK	53.4	37.8	30.8	22.6	17.0	20	15	15	30	5.1	81.1
US	55.2	45.8	41.2	36.5	32.1	22	–	–	22	16.1	40.9
OECD Average	73.0	62.7	58.7	53.7	49.2	–	–	–	25.75	17.2	36.9

Source: OECD (2007).

Table 1: Pension System Across OECD Countries

Age	Hourly Wage	Age	Hourly Wage
20–24	1,349	45–49	3,075
25–29	1,777	50–54	3,145
30–34	2,187	55–59	2,797
35–39	2,548	60–64	1,923
40–44	2,842	65–	1,617

Table 2: Average Hourly Wage for Each Age Group (Yen)

	Benchmark	Half	Con. Tax	Cap. Tax
K/Y	2.92	3.06	2.98	2.78
ch(K/Y) (%)	–	4.92	2.06	−4.64
r (%)	4.10	3.52	3.85	4.70
ch(w) (%)	–	2.76	1.16	−2.66
Payroll Tax Rate (%)	6.53	3.27	1.10	1.40
ch(K) (%)	–	9.20	3.26	−6.72
ch(Earning) (%)	–	7.77	7.04	3.17
ch(Hours) (%)	–	1.56	−0.06	0.62
ch(Y) (%)	–	4.08	1.18	−2.19
ch(C) (%)	–	1.87	0.24	−0.29
EV (%)	–	2.71	1.10	1.23

Table 3: Stationary State Comparison Based on Constant Population Distribution
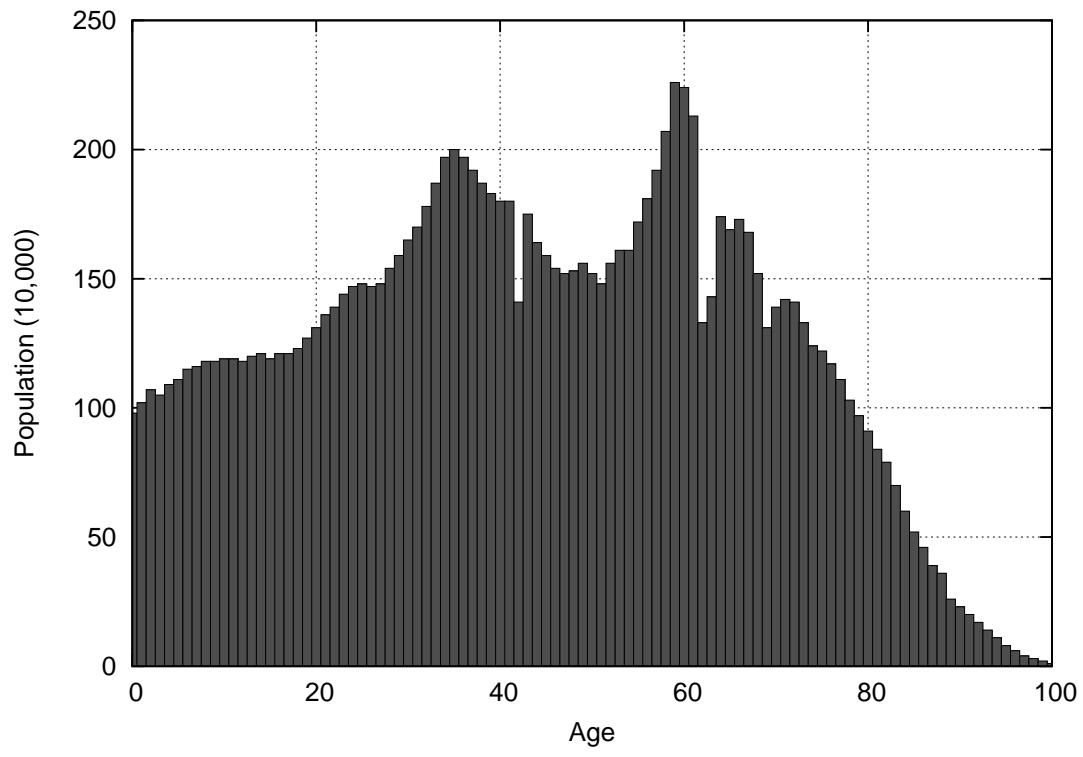
Figure 1: Population Distribution in Japan in 2008
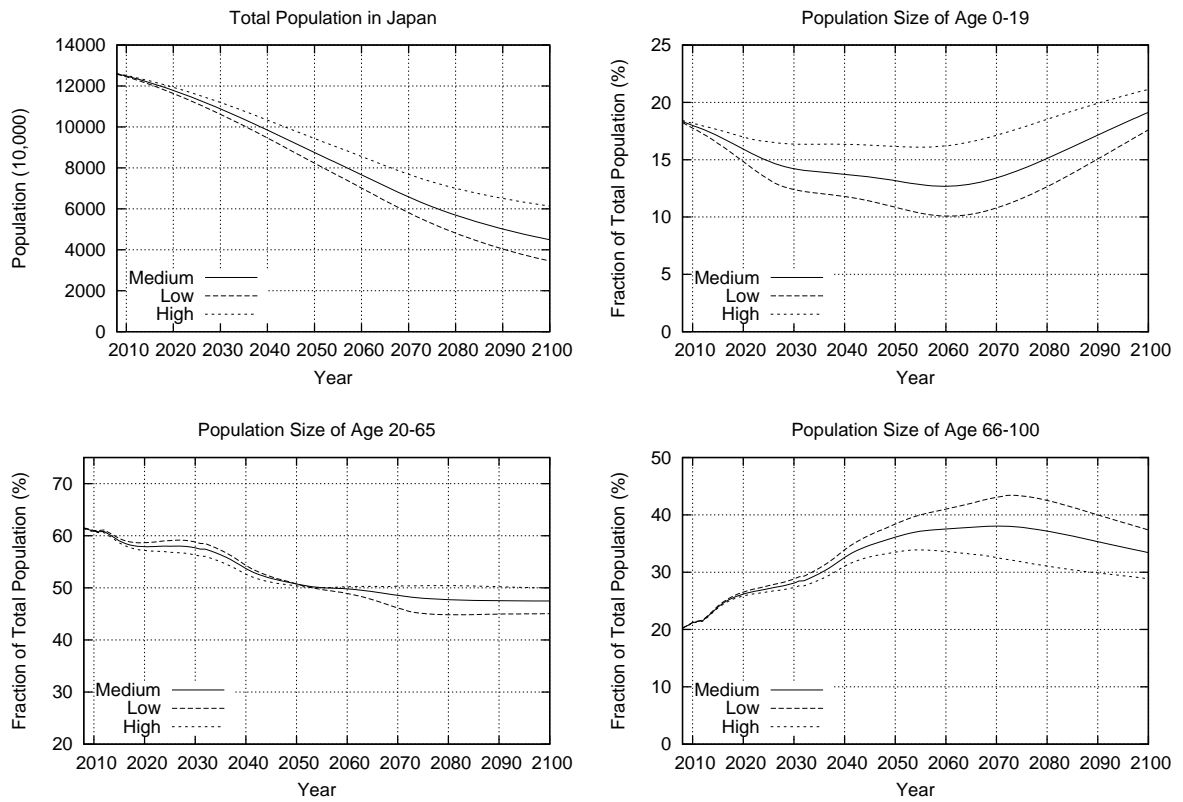
Figure 2: Projected Population Dynamics in Japan
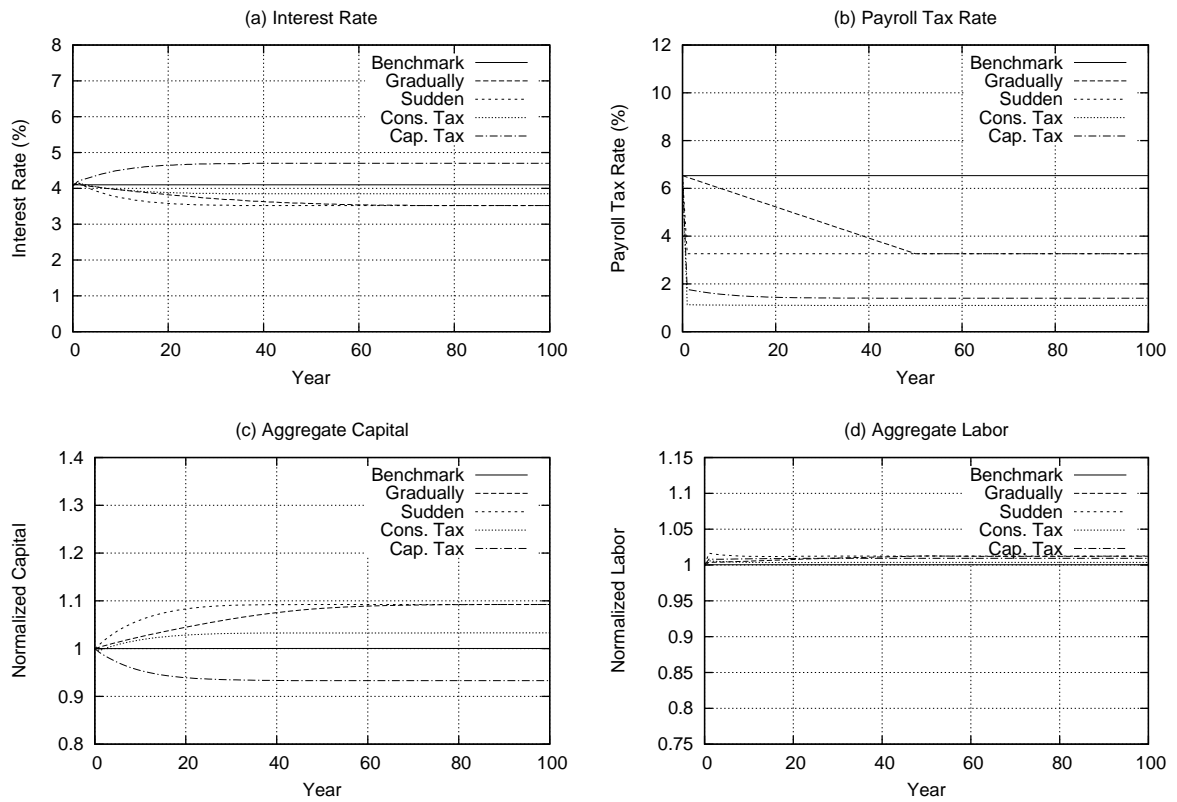
Figure 3: Transitional Dynamics with Constant Population Distribution
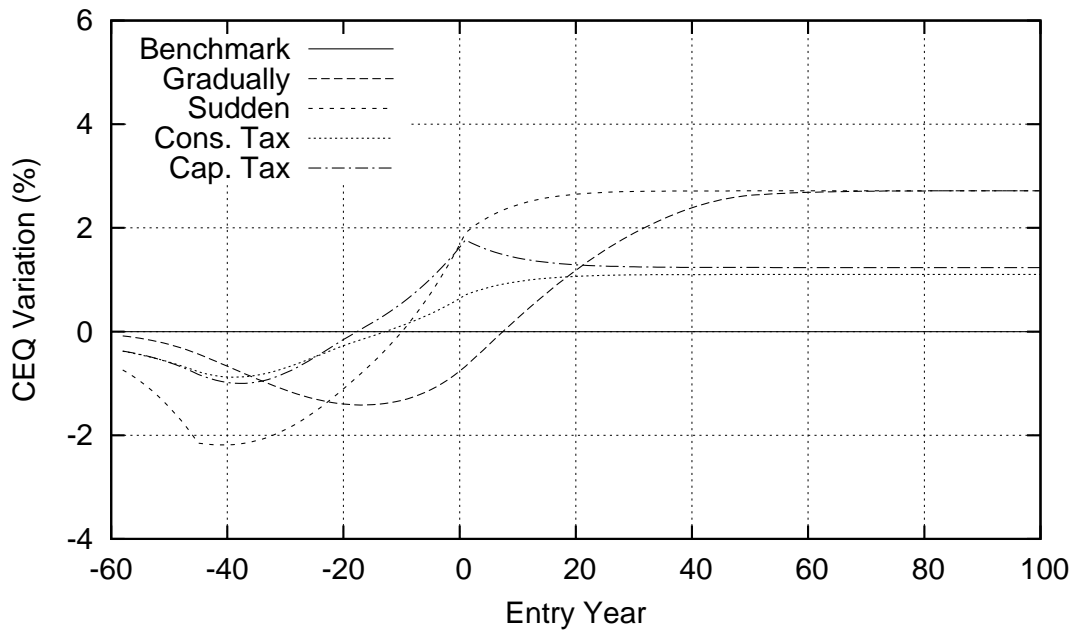
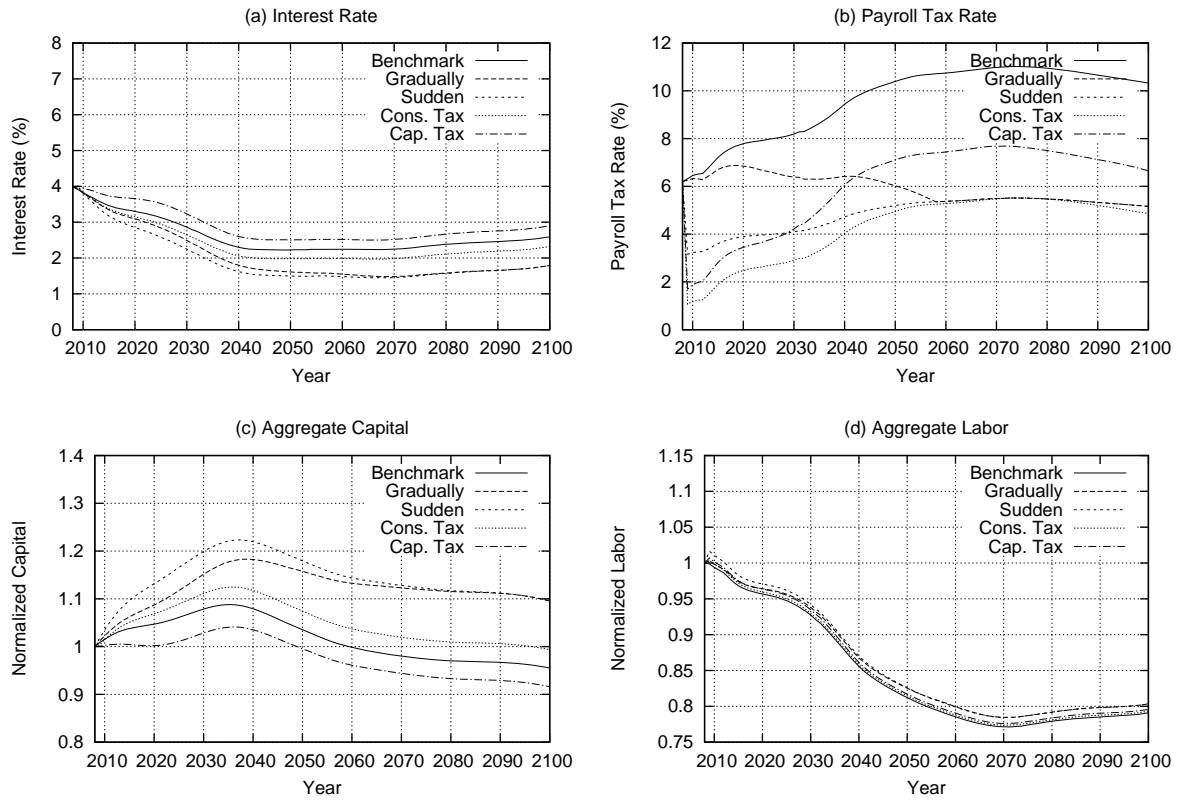Figure 4: Consumption Equivalent Variation Measure

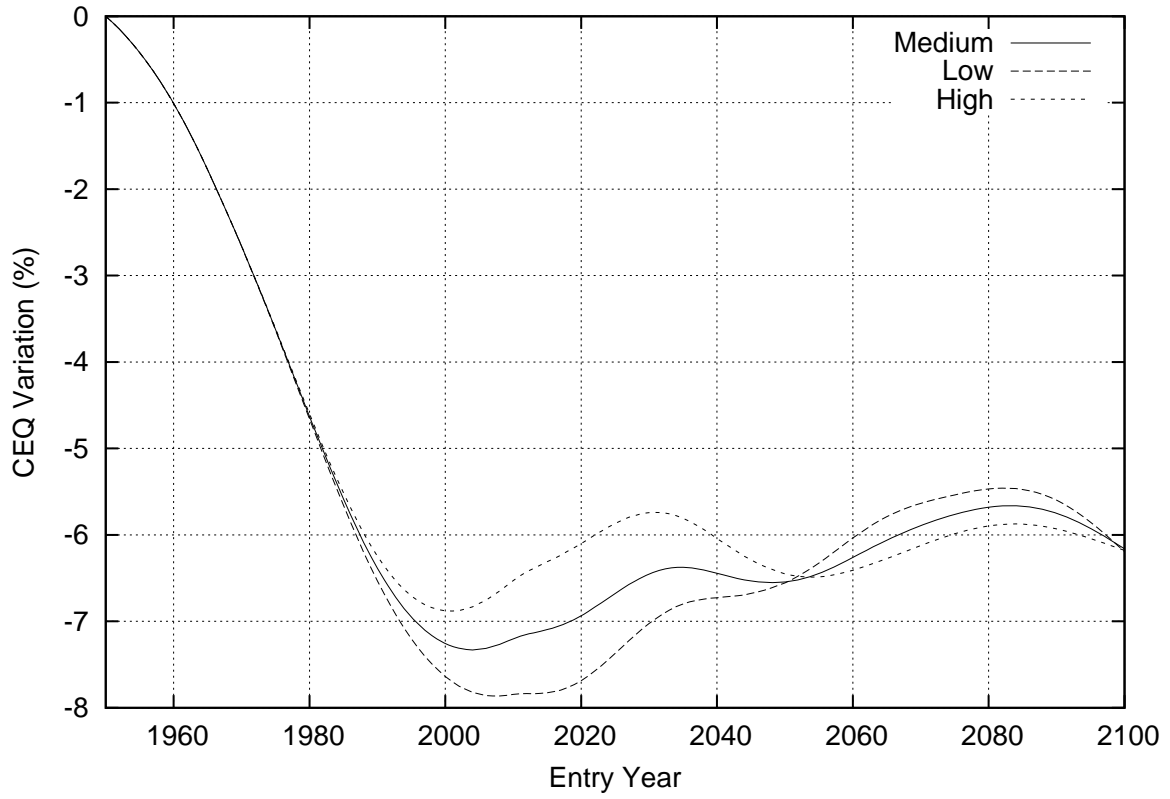Figure 5: Transitional Dynamics with Population Aging
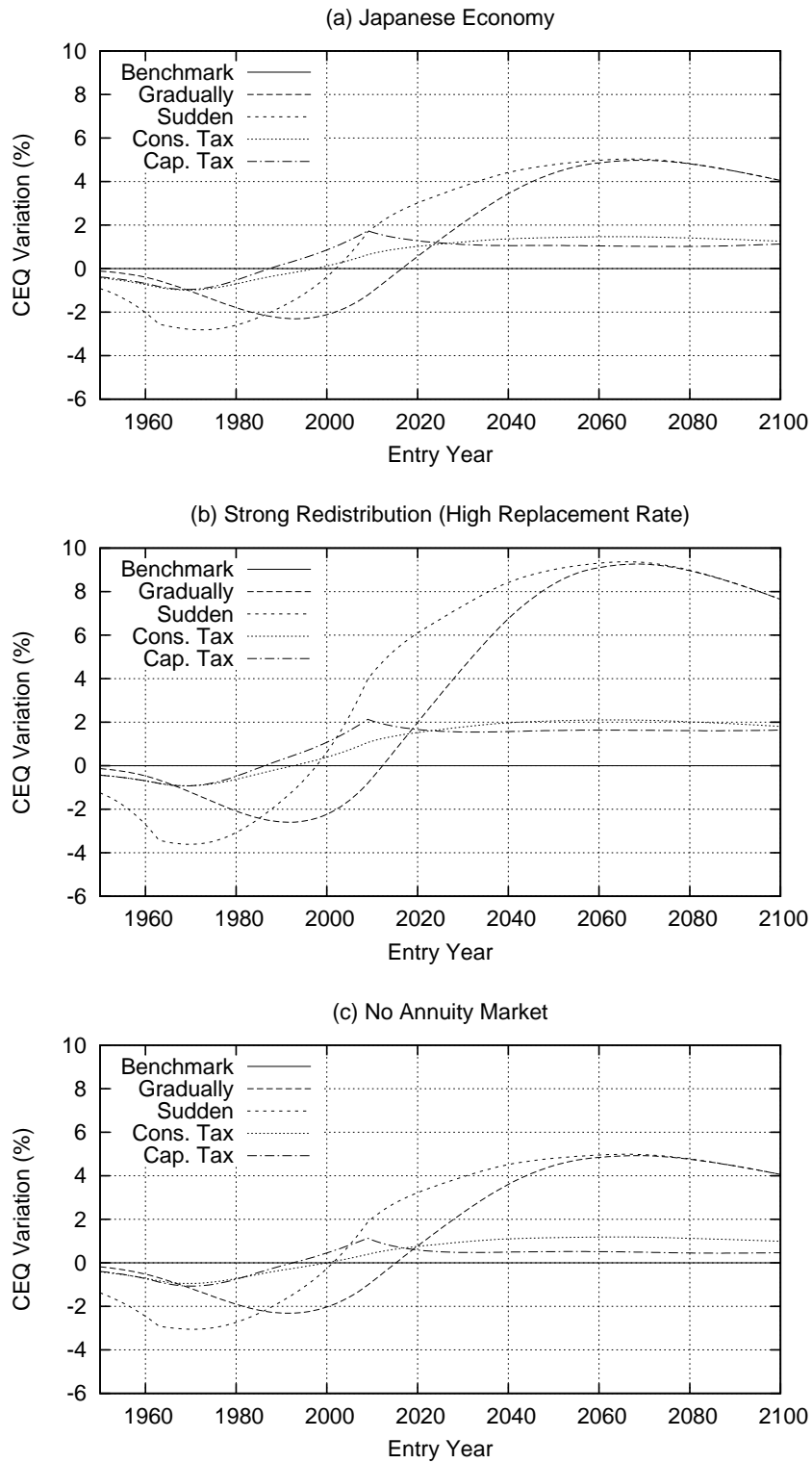
Figure 6: Intergenerational Inequality

Figure 7: Consumption Equivalent Variation Measure

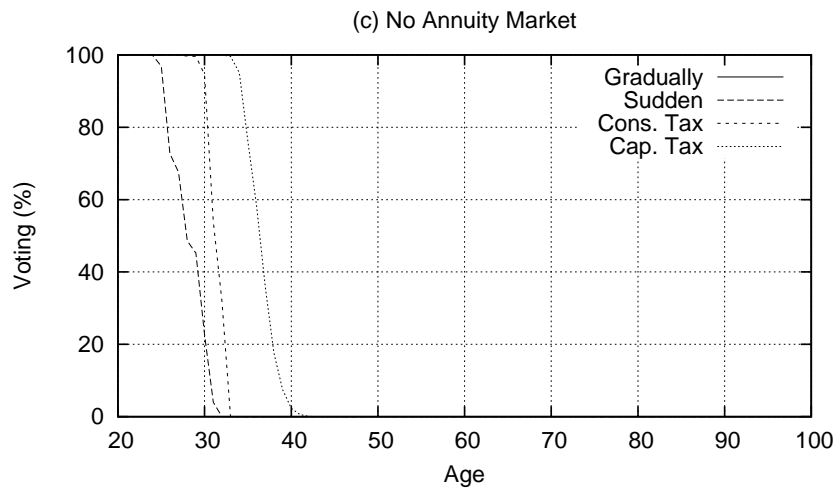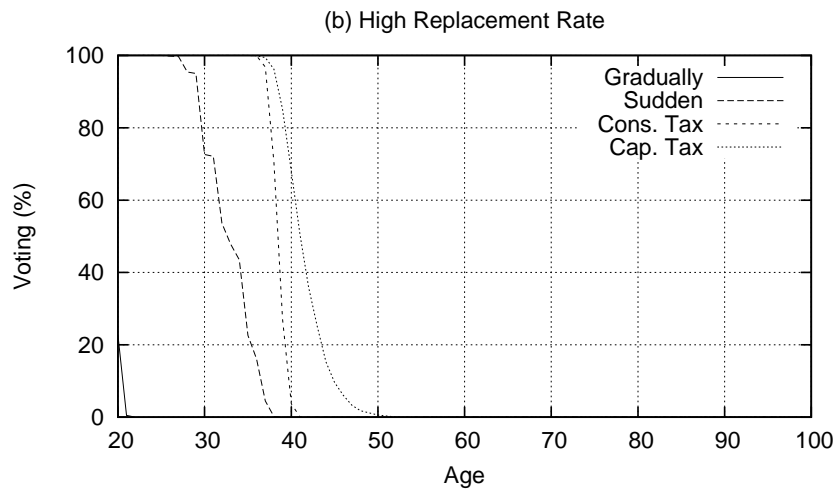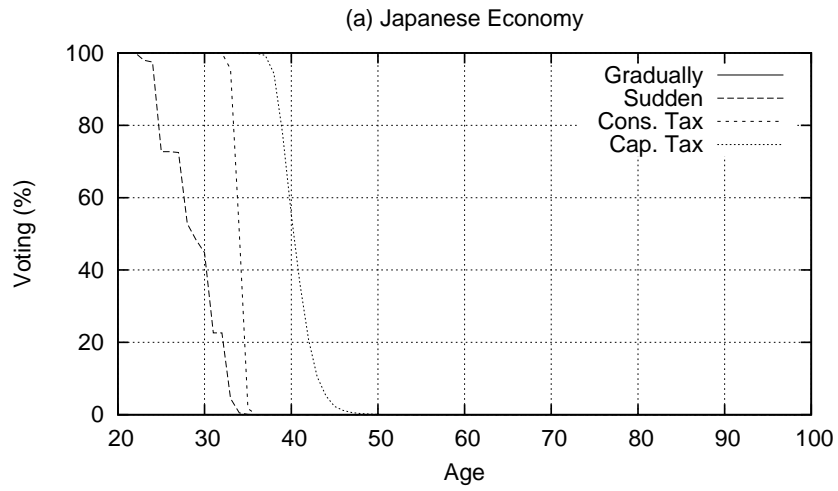38

Figure 8: Hypothetical Voting

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A Details of the Model

A.1 Detrended Macroeconomic Variables

As we consider an economy with TFP growth and population dynamics, we need to remove the trend to solve the equilibrium numerically. Define the TFP factor growth rate and population growth rate by

$$\frac{A_{t+1}^{1/(1-\theta)}}{A_t^{1/(1-\theta)}} \equiv 1 + g_t, \ \frac{N_{t+1}}{N_t} \equiv 1 + n_t.$$

Note that the adjustment parameter is not the TFP level A_t, but the TFP factor $A_t^{1/(1-\theta)}$. We divide all macroeconomic variables by $A_t^{1/(1-\theta)} N_t$ for detrending, excluding the aggregate labor supply. After normalization, the macroeconomic variables are re-defined as follows:

$$\tilde{Y}_t = Y_t/(A_t^{1/(1-\theta)} N_t), \ \tilde{K}_t = K_t/(A_t^{1/(1-\theta)} N_t), \ \tilde{H}_t = H_t/N_t.$$

The factor prices are

$$r_t = \theta \frac{\tilde{Y}_t}{\tilde{K}_t} - \delta, \ \tilde{w}_t = \frac{w_t}{A_t^{1/(1-\theta)}}.$$

Thus, the equilibrium wage level grows with the productivity.

A.2 Normalized Household Problem

In this paper, a household's optimization problem is defined as follows[1]:

$$V_{j,t}(a, s) = \max \left\{ u(c_{j,t}, \bar{h}_t - h_{j,t}) + \phi_{j,t} \beta E V_{j+1,t+1}(a', s') \right\}, \quad (1)$$

subject to

$$(1 + \tau_t^{\text{con}}) c_{j,t} + \Lambda_{j,t} a_{j+1,t+1} \leq (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + (1 - \tau_t^{\text{ss}}) w_t \kappa_j e_j h_{j,t},$$

$$(1 + \tau_t^{\text{con}}) c_{j,t} + \Lambda_{j,t} a_{j+1,t+1} \leq (1 + (1 - \tau_t^{\text{cap}}) r_t)(a_{j,t} + b_t) + w_t \varphi(\tau_t) H_t,$$

Because microeconomic variables are not affected by the population trend, we need to detrend them using the TFP factor growth rate alone. Thus, we define $c_{j,t}/A_t^{1/(1-\theta)} =$

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[1]For generality, following Hansen and İmrohoroğlu (2008), we denote the price of the annuity as $\Lambda_{j,t} = 1 - \lambda(1 - \phi_{j,t})$, and b_t is the lump-sum redistribution of unannuitized asset.

$\tilde{c}_{j,t}$, $a_{j,t}/A_t^{1/(1-\theta)} = \tilde{a}_{j,t}$, and $h_{j,t} = \tilde{h}_{j,t}$. Then, the normalized Bellman equation becomes as follows:

$$v_{j,t}(\tilde{a}_j, s) = \max \left\{ u(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) + \phi_{j,t}\tilde{\beta}_t E_j v_{j+1,t+1}(\tilde{a}', s') \right\} \qquad (2)$$

subject to

$$(1 + \tau_t^{\mathrm{con}})\tilde{c}_{j,t} + (1 + g_t)\Lambda_{j,t}\tilde{a}_{j+1,t+1} = (1 + (1 - \tau_t^{\mathrm{cap}})r_t)(\tilde{a}_{j,t} + \tilde{b}_t) + (1 - \tau_t^{\mathrm{ss}})\tilde{w}_t\kappa_j e_j \tilde{h}_{j,t}, \qquad (3)$$

$$(1 + \tau_t^{\mathrm{con}})\tilde{c}_{j,t} + (1 + g_t)\Lambda_{j,t}\tilde{a}_{j+1,t+1} = (1 + (1 - \tau_t^{\mathrm{cap}})r_t)(\tilde{a}_{j,t} + \tilde{b}_t) + \tilde{w}_t\varphi(\tau_t)\tilde{H}_t, \qquad (4)$$

where $\tilde{\beta}_t = \beta(1 + g_t)^{\sigma(1-\gamma)}$ and $\tilde{b}_t = b_t/A_t^{1/(1-\theta)}$. Moreover, by defining $\tilde{\varphi} \equiv \varphi_t/N_t$, we can formulate the normalized social security payment as a fraction of average earnings.

A.3 First Order Conditions

From the first-order conditions of the Bellman equation (2) and the envelope theorem, we obtain

$$u_c'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) - \xi(1 + \tau_t^{\mathrm{con}}) = 0,$$

$$-u_c'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t})\frac{(1 + g_t)}{(1 + \tau_t^{\mathrm{con}})}\Lambda_{j,t} + \phi_{j,t}\tilde{\beta}_t E_j \frac{\partial v_{j+1,t+1}(\tilde{a}', s')}{\partial \tilde{a}'} \leq 0,$$

$$\frac{\partial v_t(\tilde{a}_j, s)}{\partial \tilde{a}} = \frac{(1 + (1 - \tau_t^{\mathrm{cap}})r_t)}{(1 + \tau_t^{\mathrm{con}})}u_c'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t})$$

$$-u_h'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) + \xi(1 - \tau_t^{\mathrm{ss}})\tilde{w}_t\kappa_j e_j = 0,$$

where ξ is a Lagrange multiplier on a budget constraint.

From the Envelope Theorem, the intertemporal and intratemporal first-order conditions are as follows:

$$u_c'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t})\frac{(1 + g_t)}{(1 + \tau_t^{\mathrm{con}})}\Lambda_{j,t} = \frac{(1 + (1 - \tau_{t+1}^{\mathrm{cap}})r_{t+1})}{(1 + \tau_{t+1}^{\mathrm{con}})}\phi_{j,t}\tilde{\beta}_t E_j u_c'(\tilde{c}_{j+1,t+1}, \bar{h} - \tilde{h}_{j+1,t+1}),$$

$$\frac{u_h'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t})}{(1 - \tau_t^{\mathrm{ss}})\tilde{w}_t\kappa_j e_j} = \frac{u_c'(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t})}{(1 + \tau_t^{\mathrm{con}})},$$

If the utility function is of Cobb–Douglas type, the Euler equation is as follows:

$$\frac{[\tilde{c}_{j,t}^\sigma(\bar{h} - \tilde{h}_{j,t})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j,t}}\Lambda_{j,t} = \frac{(1 + \tau_t^{\mathrm{con}})}{(1 + \tau_{t+1}^{\mathrm{con}})}\frac{(1 + (1 - \tau_{t+1}^{\mathrm{cap}})r_{t+1})}{(1 + g_t)}\phi_{j,t}\tilde{\beta}_t E_j \left\{ \frac{[\tilde{c}_{j+1,t+1}^\sigma(\bar{h} - \tilde{h}_{j+1,t+1})^{1-\sigma}]^{1-\gamma}}{\tilde{c}_{j+1,t+1}} \right\}, \qquad (5)$$

Moreover, from the intratemporal first-order conditions, the labor supply function is

$$h_{j,t} = \max\left[\bar{h} - \left(\frac{1-\sigma}{\sigma}\right)\frac{(1+\tau_t^{\text{con}})\tilde{c}_{j,t}}{(1-\tau_t^{\text{ss}})\tilde{w}_t\kappa_j e_j}, 0\right],$$

where $h_{j,t} \in [0, \bar{h}]$.

A.4 Law of Motion

Define the probability space as $((\mathcal{A} \times \mathcal{S} \times \mathcal{J}), \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J})), \Phi_j)$ where $\mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))$ is a Borel σ-field and $\Phi_t(X)$ is a probability measure over $X \in \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))$. From the policy function and the transition probability of labor skill $\pi(s'|s) \equiv \Pr(\varepsilon') \times \Pr(z'|z)$, the transition function $Q_t(\cdot, \cdot)$ over household's states (a, s, j) and the distribution function $\Phi_{j,t}(a, s, j)$ is computable.[2] The probability measure is defined over household's state and also represents the fraction of households with state $X \in \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))$. Because we assume that households of age $j = 20$ have zero assets, Φ_{20} is equal to one on $a_{20,t} = 0$. The transition function $Q_j : (\mathcal{A} \times \mathcal{S} \times \mathcal{J}) \times \mathcal{B}((\mathcal{A} \times \mathcal{S} \times \mathcal{J}))) \rightarrow [0,1]$ is defined as

$$Q_j((\mathcal{A} \times \mathcal{S} \times \mathcal{J}), X) = \sum_{e' \in S}\begin{cases} \pi(s'|s) & \text{if } g_{j,t}^a(\tilde{a}, s) \in X \\ 0 & \text{else} \end{cases}, \text{ for all } j = 20, \ldots, 100.$$

Given initial distribution $\Phi_{20,t}$, the distribution function $\{\Phi_{j,t}\}_{j=21}^{100}$ for each j is mapped by the following equation.

$$\Phi_{j+1,t+1}(X) = \int Q_j((\mathcal{A} \times \mathcal{S} \times \mathcal{J}), X)\, d\Phi_{j,t}, \quad (\forall X \in \mathcal{B}(\mathcal{A} \times \mathcal{S} \times \mathcal{J})), j = 20, \ldots, 100,$$

$$\Phi_{t+1} = \mathcal{T}(\Phi_t).$$

Note that population change is adjusted by μ_t, and that the TFP growth is already included. Thus, this distribution is purely wealth distribution for each generation.

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[2]For details, see Stokey et al. (1989). For the computation of the distribution function, we follow Young's (2004) method. Also see Aiyagari and McGrattan (1998).

B Numerical Procedure

B.1 Endogenous Gridpoint Method

Among the many available procedures for computing the policy function, we apply the Endogenous Gridpoint Method (EGM) by Carroll (2006), because it is a safe and relatively fast method.[3]

Define the right-hand side of the Euler equation (5) as

$$\Gamma'_{j,t}(\tilde{a}_j, s_j) = \frac{(1 + \tau_t^{\text{con}})}{(1 + \tau_{t+1}^{\text{con}})} \frac{(1 + (1 - \tau_{t+1}^{\text{cap}})r_{t+1})}{(1 + g_t)} \phi_{j,t} \tilde{\beta}_t E_j u'_c(\tilde{c}_{j+1,t+1}, \bar{h} - h_{j+1,t+1}),$$

and take discretized grids on $\tilde{a}' \in [0, \bar{a}]$. We set the number of grids to be 80. From equation (5), the intertemporal first-order condition is rewritten as follows:

$$u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t}) = \frac{\Gamma'_{j,t}(\tilde{a}', s_j)}{\Lambda_{j,t}}. \tag{6}$$

Thus, if we can compute $\Gamma'_{j,t}$ for each discretized state $(\tilde{a}', s_j)$, after taking the inverse of the utility function, we obtain consumption $\tilde{c}_{j,t}$ for each state.

Suppose that next period's consumption and labor supply functions are already known as

$$\tilde{c}_{j+1,t+1} = \tilde{g}^c_{j+1,t+1}(\tilde{x}_{j+1,t+1}, s'),$$

$$h_{j+1,t+1} = \tilde{g}^h_{j+1,t+1}(\tilde{x}_{j+1,t+1}, s'), \text{ if } j \le 65,$$

where $\tilde{x}_{j+1,t+1} \equiv (1 + (1 - \tau_{t+1}^{\text{cap}})r_{t+1})(\tilde{a}' + \tilde{b}_{t+1}) + (1 - \tau_{t+1}^{\text{ss}})\tilde{w}_{t+1}\kappa_{j+1}e_{j+1}\bar{h}_{t+1}$ is cash on hand.[4] Then, we can compute the $\Gamma'_{j,t}(\tilde{a}', s_j)$ for each grid $\{\tilde{a}'_i\}_{i=1}^{80}$ for each age by backward induction. When we compute $\Gamma'_{j,t}$ for each discretized state $(\tilde{a}', s_j)$, if the marginal utility function is invertible, we obtain the equilibrium consumption $\tilde{c}_j$ for each state.

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[3]For details on the endogeneous gridpoint method with endogenous labor supply, see appendix in Krueger and Ludwig (2006) and Barillas and Fernández-Villaverde (2006).

[4]Note that as the labor supply is endogenous, the cash on hand in the next period is still not determined. Following Krueger and Ludwig (2006), we temporarily determine the cash on hand as asset holdings plus earning with maximum supply of labor.

Note that the marginal utility function is defined as follows:

$$u'_c(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) = \sigma \frac{\left[\tilde{c}^{\sigma}_{j,t}(\bar{h} - \tilde{h}_{j,t})^{1-\sigma}\right]^{1-\gamma}}{\tilde{c}_{j,t}}.$$

From the first order condition (6), by taking the inverse of the utility function $u'_c(\tilde{c}_{j,t}, \bar{h}_t - \tilde{h}_{j,t})$ with respect to $\tilde{c}_j$, we obtain $\tilde{c}_j$ for each choice variable $\tilde{a}'$. Using the Euler equation for leisure and removing $\tilde{h}_{j,t}$, we have

$$u'_c(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) = \tilde{c}^{-\gamma}_{j,t}\sigma\left(\frac{1-\sigma}{\sigma}\frac{(1+\tau^{con}_t)}{(1-\tau^{ss}_t)\tilde{w}_t\kappa_j e_j}\right)^{(1-\sigma)(1-\gamma)} \quad : h_{j,t} > 0$$

$$u'_c(\tilde{c}_{j,t}, \bar{h} - \tilde{h}_{j,t}) = \sigma\tilde{c}^{\sigma(1-\gamma)-1}_{j,t}\bar{h}^{(1-\sigma)(1-\gamma)} \quad : h_{j,t} = 0$$

This equation is apparently invertible. Thus, we have

$$\tilde{c}_{j,t} = u^{-1}\cdot\left(\frac{\Gamma'_{j,t}(\tilde{a}', s_j)}{\Lambda_{j,t}}\right).$$

From consumption $\tilde{c}_{j,t}$, we can directly induce $\tilde{h}^i_j$. From the set of $\{\tilde{c}^i_{j,t}, \tilde{h}^i_{j,t}, \tilde{a}^i_{j+1,t+1}\}$, we define new cash on hand $\tilde{x}^i_j \equiv (1 + g_t)\Lambda_{j,t}\tilde{a}^i_{j+1,t+1} + \hat{c}^i_{j,t}$, where $\hat{c}^i_j \equiv (1 + \tau^{con}_t)\tilde{c}^i_j + (1 - \tau^{ss}_t)\tilde{w}_t\kappa_j e_j(\bar{h}_t - \tilde{h}_{j,t})$.

B.2 Computation of Steady State

Computation of the stationary state is the same as in Aiyagari (1994) and Huggett (1996). There are three markets in the model, goods, labor, and capital. However the factor prices (r, w) are determined from the capital–labor ratio $\tilde{K}/\tilde{H}$. By the Walras law, we concentrate on $\tilde{K}/\tilde{H}$ and government budget clearing of $(\tau^{ss}, \tau^{con}, \tau^{cap})$.[5]

1. Given an initial guess of (K^0, H^0), compute a pair of $(r^0, \tilde{w}^0)$. We also need initial guess of $\tilde{C}^0$ for consumption tax.

2. Given $(r^0, \tilde{w}^0, \tilde{K}^0, \tilde{H}^0, \tilde{C}^0)$ and exogenous (τ^{con}, τ^{cap}), compute the payroll tax rate τ^{ss}_0 from the government budget condition.

3. Given $(r^0, \tilde{w}^0, \tau^{ss}; \tau^{con}, \tau^{cap})$, compute the policy function using the EGM and obtain the distribution function Φ^0 for each age.

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[5]We take 80 grids on asset a for computing policy function, and to compute the distribution we take 5000 grids.

4. Integrating the distribution function Φ^0, we obtain the aggregate capital and labor $\left(\tilde{K}^1, \tilde{H}^1\right)$.

5. If new $\left(\tilde{K}^1, \tilde{H}^1\right)$ and old $(\tilde{K}^0, \tilde{H}^0)$ are sufficiently close to each other, then stop; we have equilibrium prices for given $\tau^{\mathrm{ss},0}$.

6. From a new equilibrium condition $(r^1, \tilde{w}^1, \tilde{K}^1, \tilde{H}^1, C^1)$, re-compute a new payroll tax $\tau^{\mathrm{ss},1}$. Repeat steps $3-5$. If the iteration error of τ^{ss} is sufficiently small, stop. We have an equilibrium.

Note that all computations above are already detrended by $A_t^{1/(1-\theta)} H_t$.

B.3   Transition Dynamics

After the computation of the stationary state in 2008 and 2200, we compute the transitional path between the stationary states. The basic idea here is the same as in Conesa and Krueger (1999) and Nishiyama and Smetters (2005).

1. Set an exogenous path of tax rates pair $(\tau_t^{\mathrm{con}}, \tau_t^{\mathrm{cap}})$. Guess an equilibrium sequence of $\{r_t, \tilde{w}_t, \tau_t^{\mathrm{ss}}, \tilde{H}_t, \tilde{b}_t\}_{t=2008}^{2200}$, which is needed to solve a household's problem.[6] We assume that the benefit from social security and the sequence of TFP $\{\varphi(\tau_t), A_t\}_{t=2008}^{2200}$ are perfectly foreseen and exogenously given.

2. Because we have the policy function of the final stationary state in 2200, we compute a sequence of policy functions using the EGM by backward induction.

3. Given the policy functions, compute the distribution function from 2008 onwards and compute aggregate variables, $\{\tilde{K}_t, \tilde{H}_t, r_t, \tilde{w}_t\}_{t=2008}^{2200}$.

4. Check whether each market clearing condition and government budget balances are satisfied. If these are not in equilibrium, up-date the price sequences and repeat steps 2–3.[7]

5. If all markets clear in all periods, stop computation.

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[6]For simplicity, we start a linear case.

[7]There are many efficient methods for updating the price sequence. For example, Krueger and Ludwig (2006) and Ludwig (2008) use a modified version of the Gauss–Zeidel method for computing the transition path.

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