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On Initial Conferment of Individual Rights

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Abstract

An extended social choice framework is proposed for the analysis of initial conferment of individual rights. This framework captures the intuitive conception of decision-making procedure as a carrier of intrinsic value along with the instrumental usefulness thereof in realizing valuable culmination outcomes. The model of social decision-making consists of two stages. In the first stage, the society decides on the game-form rights to be promulgated. In the second stage, the promulgated game form rights, coupled with the revealed profile of individual preference orderings over the set of culmination outcomes, determine a fully-fledged game, the play of which determines a culmination outcome at the Nash equilibrium. A set of sufficient conditions for the existence of a social choice procedure, which can choose a game form in the first stage that is not only liberal, but also uniformly applicable to every revealed profile of individual preference orderings over the set of culmination outcomes, is identified.

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1 Introduction

1.1 Historical Background

Ever since Sen (1970, Chapter 6 & Chapter 6*; 1970a; 1976; 1983) acutely crystallized the logical conflict between the welfaristic outcome morality in the weak form of the Pareto principle and the non-welfaristic claim of libertarian rights into the *impossibility of a Paretian liberal*, a huge literature has evolved along several distinct avenues.[1] In the first place, some of the early literature either repudiated the importance of Sen's impossibility theorem, or tried to find an escape route from the logical impasse identified by Sen.[2,3] In the second place, capitalizing on the seminal observation by Nozick (1974, pp.164-166), alternative articulations of libertarian rights, which are game-theoretic in nature, were proposed by Gärdenfors (1981), Sugden (1985), Gaertner, Pattanaik and Suzumura (1992), Deb (1990/2004; 1994), Hammond (1995; 1996) and Peleg (1998). Recollect that Sen's original articulation of libertarian rights was in terms of the preference-contingent constraints on social choice rules by

[1]Some of these literature are succinctly surveyed and evaluated by Suzumura (1996; 2005).

[2]Representative work along these lines include Bernholz (1974), Gibbard (1974), Nozick (1974, pp.164-166), Blau (1975), Osborne (1975), Seidl (1975), Farrell (1976), and Buchanan (1976/1996). Sen (1976, 1992) commented on, and in some cases rejected, these early proposals. See also Sen (2002, Part VI) for his more recent evaluation on the issues of freedom and social choice.

[3]In the recent literature, Samet and Schmeidler (2003) characterized the liberal rule within the class of what they call consent rules. Since the consent rules are specific types of voting rules, it is quite natural, as Samet and Schmeidler (2003) pointed out, that the liberal rule in their model has a similar property with those discussed in Gibbard (1974).

means of individual decisiveness.[4] In contrast, these game-theoretic articulations captured the essence of libertarian rights by means of individual freedom of choosing admissible strategies in the game-theoretic situations where individual liberties are at stake. Unlike the first class of work, these game-theoretic articulations were meant to provide more legitimate methods of capturing the essence of what libertarian rights should mean.[5] In the third place, the crucial problem of initial conferment of libertarian rights was often mentioned in the literature without providing a fully-fledged analytical framework.[6] Suffice it to cite just one salient example. In his rebuttal to the game-form articulation proposed by Gaertner, Pattanaik and Suzumura (1992), Sen (1992, p.155) con-

[4]Suppose that there are two social states, say $\mathbf{x}$ and $\mathbf{y}$, which differ only in somebody's personal matters and nothing else. If the person in question prefers $\mathbf{x}$ to $\mathbf{y}$, then Sen would confer on him the decisive power of rejecting the social choice of $\mathbf{y}$ from any social opportunity set in which $\mathbf{x}$ is available.

[5]Note that these alternative articulations of libertarian rights do not claim to resolve the impossibility of a Paretian liberal. As a matter of fact, Pattanaik (1996), and Deb, Pattanaik and Razzolini (1997) showed that there are several natural variants of the impossibility of a Paretian liberal even when libertarian rights are articulated in terms of game forms.

[6]Pattanaik and Suzumura (1994; 1996) and Suzumura (1996; 2005) identified three distinct issues in the analysis of libertarian rights. The first issue is the *formal structure* of rights. The second issue is the *realization* of conferred rights. The third issue is the *initial conferment* of rights. In Sen's theory of libertarian rights, the formal structure of rights was articulated in terms of the preference-contingent constraints on social choice rules, whereas the issue of the realization of conferred rights could be boiled down to the existence of a social choice rule which respects the preference-contingent constraints on social choice rules. However, Sen has never addressed himself to the issue of initial conferment of rights. This is presumably because his interest was focussed squarely on the conflict between the non-welfaristic claim of libertarian rights and the welfaristic claim of the Pareto principle, so that it was unnecessary for him to develop a fully-fledged theory of the initial conferment of libertarian rights.

cluded with the following observation: "Gaertner *et al.* (1992) do, in fact, pose the question, 'How does the society decide which strategies should or should not be admissible for a specific player in a given context?' This, as they rightly note, is 'an important question'. ... [I]t is precisely on the answer to this further question that the relationship between the game-form formulations and social-choice formulations depend ... . We must not be too impressed by the 'form' of the 'game forms'. We have to examine its contents and its rationale. The correspondence with social-choice formulations becomes transparent precisely there." The purpose of this paper is to contribute to this less cultivated issue within the theory of libertarian rights.

1.2 Basic Problem

To illustrate the nature of the problem of initial conferment of game-form rights, consider the following example.

Example 1: There are two passengers 1 and 2 in a train's compartment, where 1 is a smoker and 2 is a non-smoker. The train company is contemplating whether to respect the smoker′s desire to smoke freely, or to respect the non-smoker′s desire not to be imposed secondary smoking by the smoker. The company′s problem is to choose from the set of various game forms, which includes the following two game forms.

The first game form $\gamma = (M_1^\gamma \times M_2^\gamma, g^\gamma)$ is defined by $M_1^\gamma = \{s, ns\}$, where $s =$ "to smoke" and $ns =$ "not to smoke", $M_2^\gamma = \{(l|s, r|ns), r\}$, where $(l|s, r|ns) =$ "to leave the compartment if the smoker smokes, to remain in the compartment if the smoker does not smoke" and $r =$ "to remain in the compartment no matter what", and g^γ is defined by

1 \ 2	$(l\,\vert\, s,\ r\,\vert\, ns)$	r
s	(s, l)	(s, r)
ns	(ns, r)	(ns, r)

where (s, l) is the *culmination outcome* such that the smoker smokes and the non-smoker leaves the compartment, and (ns, r) and (s, r) may be interpreted similarly.

The second game form $\gamma^* = (M_1^{\gamma^*} \times M_2^{\gamma^*}, g^{\gamma^*})$ is defined by $M_1^{\gamma^*} = \{(s|p, ns|np), ns\}$, where $(s|p, ns|np)$ = "to smoke if the non-smoker permits it, not to smoke if the non-smoker does not permit it" and ns = "not to smoke no matter what", $M_2^{\gamma^*} = \{p \cdot r, p \cdot l, np\}$, where $p \cdot r$ = "to permit the smoker to smoke and remain in the compartment", $p \cdot l$ = "to permit the smoker to smoke and leave the compartment if and only if the smoker indeed smokes" and np = "not to permit the smoker to smoke", and g^{γ^*} is defined by

1 \ 2	$p \cdot r$	$p \cdot l$	np
$(s\,\vert\, p,\ ns\,\vert\, np)$	(s, r)	(s, l)	(ns, r)
ns	(ns, r)	(ns, r)	(ns, r)

Note that the set of culmination outcomes is given by $A = \{(s, l), (ns, r), (s, r)\}$. Note also that the company confers on the smoker (*resp.* the non-smoker) the right for free smoking (*resp.* the right for clean air) if it chooses the game form γ (*resp.* γ^*). ■

The gist of this example is that the social choice of a game form is tantamount to the initial conferment of individual rights. This social choice issue should be solved by designing and implementing a democratic social decision procedure for initial conferment of individual rights.

This analysis can be based on the conceptual framework developed by Pattanaik and Suzumura (1994; 1996), which proposed to capture the intuitive conception of decision-making procedure as a carrier of intrinsic value beyond the instrumental usefulness thereof in realizing valuable culmination outcomes. The model of social decision-making consists of two stages. In the first stage, the society decides on the game-form rights to be promulgated. In the second stage, the promulgated game-form rights, coupled with the profile of individual preference orderings over the set of culmination outcomes, determine a fully-fledged game, and the play of this game determines a culmination outcome at the Nash equilibrium.

We may illustrate this two-stage framework by means of **Example 1**. Suppose that the two passengers have their own preference orderings over the set of culmination outcomes A, together forming the following profile $\mathbf{R} = (R_1, R_2)$:

$$R_1 : (s,l) \succ_1 (s,r) \succ_1 (ns,r) \,; \; R_2 : (ns,r) \succ_2 (s,l) \succ_2 (s,r) \,,$$

where $a \succ_i b$ denotes that $i \in \{1,2\}$ prefers a to b. Given this profile $\mathbf{R}$, (s,l) is the unique pure strategy Nash equilibirum outcome of the game $(\gamma, \mathbf{R})$, whereas (ns,r) is the unique pure strategy Nash equilibirum outcome of the game $(\gamma^*, \mathbf{R})$.

In the first stage of social decision-making procedure, which is to choose a game form from the set of admissible game forms including γ and γ^*, each and every individual is assumed to have an *ordering function* Q_i, which assigns an extended ordering $Q_i(\mathbf{R})$ over the pairs of game forms and realized culmination outcomes to the profile $\mathbf{R}$. For example, $((ns,r), \gamma^*)\, Q_i(\mathbf{R})\, ((s,l), \gamma)$ implies that the social situation where (ns,r) is realized as a Nash equilibrium outcome of the game $(\gamma^*, \mathbf{R})$ is at least as desirable for i as the social situation where (s,l) is realized as a Nash equilibrium outcome of the game

$(\gamma, \mathbf{R})$. Let Ψ be the social aggregator to be called the *extended constitution function*, which maps each admissible profile of individual ordering functions into a social ordering function, where an individual or a social ordering function specifies an individual or a social preference ordering over the set of pairs of culmination outcomes and game forms for each profile of individual preference orderings over the set of culmination outcomes. It is this social ordering function that determines the game-form rights to be socially chosen and promulgated as the rule of the game to be played in the second stage. For the sake of further argument, let γ^* be the game form which is chosen by means of the social ordering $Q(\mathbf{R}) = \Psi(Q_1(\mathbf{R}), Q_2(\mathbf{R}))$.

Let us turn now to the second stage of the two-stage social decision-making procedure. Since γ^* is assumed to be chosen by means of $Q\left(\mathbf{R}\right)$ when the profile $\mathbf{R}$ is revealed, the two individuals play the game $(\gamma^*, \mathbf{R})$ in the second stage, and the unique pure strategy Nash equilibrium outcome (ns, r) will emerge as a consequence. It may deserve emphasis that this two-stage social choice procedure has a sharply contrasting feature vis-à-vis the classical Arrow social choice framework. In the Arrow framework, it is the culmination outcome that is socially chosen, whereas the two-stage social choice framework *à la* Pattanaik and Suzumura visualizes a procedure where it is the game-form right that is socially chosen, and the culmination outcome is determined through the decentralized play of the game.

Given this scenario of the two-stage social decision-making procedure, the crucial task in the analysis of social choice of game form rights is to show the existence of a reasonable extended constitution function Ψ. In this paper, we will introduce some axioms on Ψ to identify the conditions which qualifies an extended constitution function to be reasonable. Also, we will propose some conditions which identify the class of liberal game forms. Since the concept of game forms itself has very little, if any, to do with liberal rights-structures, we should discuss what conditions are needed to characterize the liberal rights-structures. To sum up, our purpose in this paper is to investigate the possibility of reasonable extended constitution functions, in terms of which a liberal game form can be rationalized.

1.3 Other Related Literature

A motivation similar to ours is pursued in Koray (2000). Both Koray (2000) and the present paper address themselves to the social choice of social decision rules. One of the crucial differences is that the social decision rules envisaged by Koray are the conventional social choice functions, whereas we focus on the social decision rules as game forms. Another difference is that Koray (2000) was concerned only about the consequential values of social decision rules, whereas we are interested in both the consequential values and non-consequential values of social decision rules as game forms. It may also be worth noting that the main result of Koray (2000) is an impossibility theorem, whereas our main results are possibility theorems. This contrast is mainly due to the existence of social concerns about the non-consequential values of game forms in our framework.

Apart from this introduction, the paper consists of four sections and an appendix. Section 2 explains our basic model of extended social alternatives and game form rights. It also defines the extended constitution function. Section 3 introduces the basic Arrovian axioms which identify democratic extended constitution functions, and explains what we mean by game forms being liberal. Section 4 asserts the existence of an extended constitution function which enables the society to decide on the initial conferment of game-form rights. Section 5 concludes, and Appendix gathers all the involved proofs.

2 Basic Model

2.1 Description of Social States

The society consists of n individuals, where $2 \leq n < +\infty$. N denotes the set of all individuals, viz. $N = \{1, \cdots, i, \cdots, n\}$, which is fixed throughout this paper. Let A be the set of *feasible social states.* In what follows, it is assumed that $3 \leq \#A < +\infty$.

For each individual $i \in N$, $R_i \subseteq A \times A$ denotes i's (weak) preference ordering defined over A. For any $\mathbf{x}, \mathbf{y} \in A, (\mathbf{x}, \mathbf{y}) \in R_i$ means that $\mathbf{x}$ is at least as good as $\mathbf{y}$ from i's viewpoint. $P(R_i)$ and $I(R_i)$ denote, respectively, the strict preference relation and the indifference relation corresponding to R_i. Thus, $(\mathbf{x}, \mathbf{y}) \in P(R_i)$ if and only if $[(\mathbf{x}, \mathbf{y}) \in R_i \ \& \ (\mathbf{y}, \mathbf{x}) \notin R_i]$, and $(\mathbf{x}, \mathbf{y}) \in I(R_i)$ if and only if $[(\mathbf{x}, \mathbf{y}) \in R_i \ \& \ (\mathbf{y}, \mathbf{x}) \in R_i]$. $\mathcal{R}$ denotes the universal set of preference orderings defined over A. An n-tuple $\mathbf{R} = (R_1, R_2, \cdots, R_n)$ of individual preference orderings, one ordering for each individual $i \in N$, is called a *profile* of individual preference orderings over A. $\mathcal{R}^n$ denotes the universal set of logically conceivable profiles.

To articulate individual rights within our framework, we introduce rights-systems as game forms. A *game form* is a pair $\gamma = (M, g)$, where $M \equiv \prod_{i \in N} M_i$ and M_i denotes a set of *permissible strategies* for individual $i \in N$, and $g : M \to A$ is an *outcome function* which specifies, for each strategy profile $\mathbf{m} \in M$, a feasible outcome $g(\mathbf{m}) \in A$. Moreover, we assume that g is *surjective,* viz., $g(M) = A$. The universal set of game forms is denoted by Γ.

Given a profile $\mathbf{R} \in \mathcal{R}^n$ and a game form $\gamma = (M, g) \in \Gamma$, a pair $(\gamma, \mathbf{R})$ defines a *non-cooperative game*. Throughout this paper, we adopt the Nash equilibrium concept. Given a game $(\gamma, \mathbf{R})$, a strategy profile $\mathbf{m}^* \in M$ is called a *Nash equilibrium in pure strategies*, Nash equilibrium for short, if $\left(g(\mathbf{m}^*), g\left(m_i, \mathbf{m}^*_{-i}\right)\right) \in R_i$ holds for all $i \in N$ and all $m_i \in M_i$.[7] The set of all Nash equilibria of the game $(\gamma, \mathbf{R})$ is denoted by $\epsilon_{NE}(\gamma, \mathbf{R})$. A conceivable social outcome $\mathbf{x}^* \in A$ is called a *Nash equilibrium outcome* of the game $(\gamma, \mathbf{R})$ if there exists a Nash equilibrium $\mathbf{m}^* \in \epsilon_{NE}(\gamma, \mathbf{R})$ satisfying $\mathbf{x}^* = g(\mathbf{m}^*)$. The set of all Nash equilibrium outcomes of the game $(\gamma, \mathbf{R})$ is denoted by $\tau_{NE}(\gamma, \mathbf{R})$.

2.2 Social Decision Procedure for Rule Selection

Let us visualize the two-stage social decision procedure in the general setting. To begin with, every individual expresses his value judgements on the social desirability of alternative methods of conferring game-form rights. Then, all individuals engage in debates about each other′s value judgements, providing justifications for their own values, and offering criticisms of values held by others. Sooner or later, there comes a stage where debate must stop and action must be taken by the society. In the *primordial stage of rule selection*, the social decision is made on the rights-system to be promulgated by aggregating the individuals′ value judgements regarding the initial rights-conferment through some democratic social decision procedure. After the rights-system as a game form $\gamma \in \Gamma$ is promulgated, and the profile of individual preference or-

[7]For every $i \in N$, and every $\mathbf{m} \in M$, $\mathbf{m}_{-i} \equiv (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$ and $M_{-i} \equiv \Pi_{j \neq i} M_j$. For every $i \in N$, every $m_i^0 \in M_i$, and every $\mathbf{m}_{-i} \in M_{-i}$, $(m_i^0, \mathbf{m}_{-i}) \equiv (m_1, \ldots, m_{i-1}, m_i^0, m_{i+1}, \ldots, m_n)$.

derings $\mathbf{R} \in \mathcal{R}^n$ on the set of culmination outcomes is revealed, a fully-fledged game $(\gamma, \mathbf{R})$ is played in the *realization stage of the conferred game-form rights*, which determines a Nash equilibrium social outcome $\mathbf{x}^* \in \tau_{NE}(\gamma, \mathbf{R})$ if $\tau_{NE}(\gamma, \mathbf{R}) \neq \varnothing$.

To make this scenario precise, we invoke the extended social choice framework introduced by Pattanaik and Suzumura (1994; 1996).[8] Let Γ^* denote the *admissible* class of game forms. For every $\mathbf{x} \in A$ and every $\gamma \in \Gamma^*$, a pair $(\mathbf{x}, \gamma) \in A \times \Gamma^*$ is called an *extended (social) alternative.* Given a profile $\mathbf{R} \in \mathcal{R}^n$, an extended alternative $(\mathbf{x}, \gamma)$ is said to be *realizable under* $\mathbf{R}$ if and only if $\mathbf{x} \in \tau_{NE}(\gamma, \mathbf{R})$. The intended interpretation is that the social outcome $\mathbf{x}$ is realized through the exercise of the rights-system γ when the profile $\mathbf{R}$ prevails. In what follows, $\Lambda(\mathbf{R})$ denotes the set of all realizable extended alternatives under $\mathbf{R}$, viz.,

$$\Lambda(\mathbf{R}) = \{(\mathbf{x}, \gamma) \mid \mathbf{x} \in \tau_{NE}(\gamma, \mathbf{R}) \ \& \ \gamma \in \Gamma^*\}.$$

The social decision procedure is formulated as follows. First, each individual i's value judgements on the desirability of rights-systems is assumed to be represented by an *ordering function* $Q_i : \mathcal{R}^n \twoheadrightarrow (A \times \Gamma^*)^2$ such that, for

[8]This extended social choice framework à la Pattanaik and Suzumura capitalizes on the insightful observation by Arrow (1963, pp.89-90) to the following effect: "Up to now, no attempt has been made to find guidance by considering the components of the vector which defines the social state. One especially interesting analysis of this sort considers that, among the variables which taken together define the social state, one is the very process by which the society makes its choice. This is especially important if the mechanism of choice itself has a value to the individuals in the society. For example, an individual may have a positive preference for achieving a given distribution through the free market mechanism over achieving the same distribution through rationing by the government." See, also, Suzumura (1996; 1999; 2000; 2005).

each $\mathbf{R} \in \mathcal{R}^n$, $Q_i(\mathbf{R}) \subseteq \Lambda(\mathbf{R}) \times \Lambda(\mathbf{R})$ is a complete and transitive relation (ordering) defined over $\Lambda(\mathbf{R})$. $P(Q_i(\mathbf{R}))$ and $I(Q_i(\mathbf{R}))$ stand for the asymmetric part and the symmetric part of $Q_i(\mathbf{R})$, respectively. By definition, $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_i(\mathbf{R})$, or $(\mathbf{x}, \gamma)Q_i(\mathbf{R})(\mathbf{x}', \gamma')$ for the sake of brevity, means that, according to i's judgements, having a social outcome $\mathbf{x}$ through the play of the game $(\gamma, \mathbf{R})$ is at least as good for the society as having a social outcome $\mathbf{x}'$ through the play of the game $(\gamma', \mathbf{R})$. Let $\mathcal{Q}$ be the set of all logically possible ordering functions.

In the second place, the democratic procedure for aggregating individual value judgements is defined as follows.

Definition 1: *An* extended constitution function (**ECF**) *is a function* Ψ *which maps each and every profile of individual ordering functions* $\mathbf{Q} = (Q_i)_{i \in N}$ *in an appropriate domain* $\Delta_\Psi \subseteq \mathcal{Q}^n$ *into a social ordering function* Q*,* viz., $\Psi(\mathbf{Q}) = Q \in \mathcal{Q}$ for every $\mathbf{Q} \in \Delta_\Psi$.

The concept of extended constitution function is due originally to Pattanaik and Suzumura (1996), which is a natural extension of the Arrovian *social welfare function* or *constitution function* [Arrow (1963)]. Note that, in the present framework as well as in the framework of Pattanaik and Suzumura (1996), there are two types of individual preference orderings. One is an individual$'$s preference ordering R_i over A, which represents i's *subjective tastes* over the set of culmination outcomes, and the other is i's ordering function Q_i, which represents i's *value judgements* over the set of extended alternatives.[9]

[9]Note that the individual ordering function does not have to be *ethical* in nature. It may generate an extended preference ordering which is *selfish* in nature, where Q_i expresses i's *selfish judgements* if and only if, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_i(\mathbf{R})$ (resp. $P(Q_i(\mathbf{R}))$) if and only if $(\mathbf{x}, \mathbf{x}') \in R_i$ (resp. $P(R_i)$) holds.

The latter preferences constitute the informational basis of the **ECF** to select a rights-system in the primordial stage of rule selection, whereas the former preferences constitute the informational basis for realizing a feasible social outcome in the realization stage of conferred game-form rights.

When an **ECF** Ψ is specified, we can define the associated *rational social choice function* as follows. For each profile of individual ordering functions $\mathbf{Q} \in \Delta_\Psi$, Ψ determines a social ordering function $Q = \Psi(\mathbf{Q})$ which, in turn, determines the set of best extended social alternatives for each $\mathbf{R} \in \mathcal{R}^n$ by

(1) $B_Q(\mathbf{R}) \equiv \{(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R}) \mid \forall (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})\text{: } ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q(\mathbf{R})\}$,

where $Q = \Psi(\mathbf{Q})$. The set of game forms chosen through Ψ is then given by

(2) $C_\Psi(Q; \mathbf{R}) \equiv \{\gamma \in \Gamma^* \mid \exists\ \mathbf{x} \in A : (\mathbf{x}, \gamma) \in B_Q(\mathbf{R})\}$,

where $Q = \Psi(\mathbf{Q})$. In what follows, C_Ψ is called the *rational social choice function chosen through* Ψ.

3 Basic Axioms

3.1 Democratic Conditions for Extended Constitution Functions

As one of the desirable properties to be satisfied by the rational social choice function chosen through Ψ, we introduce the following condition.

Uniformity of Rational Choice (URC): *For every* $\mathbf{Q} \in \Delta_\Psi$,

$$\cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R}) \neq \varnothing,$$

where $Q = \Psi(\mathbf{Q})$.

If the Condition **URC** is satisfied and a game form $\gamma^* \in \cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R})$ is chosen, γ^* applies *uniformly* to each and every future realization of $\mathbf{R} \in \mathcal{R}^n$. Since the game form is nothing other than the formal method of specifying the distribution of rights in the society prior to the realization of the profile of individual preference orderings over culmination outcomes, it seems desirable, if at all possible, to design the extended constitution function Ψ satisfying the condition **URC**. Note that if we implement a $\gamma^* \in \cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R})$, then γ^* prevails as the basic rights-system no matter how frivolously the profile $\mathbf{R}$ undergoes a change.[10]

Our next requirement on Ψ is that it is *minimally democratic* in the sense that the unanimous individual value judgements must be faithfully reflected in the social value judgements in the following Paretian senses.

Strong Pareto Principle (SP): *For every* $\mathbf{Q} \in \Delta_\Psi$, *every* $\mathbf{R} \in \mathcal{R}^n$, *and every* $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$,

$$((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in (\cap_{i \in N} Q_i(\mathbf{R})) \cap (\cup_{i \in N} P(Q_i(\mathbf{R}))) \Rightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R})),$$

where $Q = \Psi(\mathbf{Q})$.

Pareto Indifference Principle (PI): *For every* $\mathbf{Q} \in \Delta_\Psi$, *every* $\mathbf{R} \in \mathcal{R}^n$, *and every* $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$,

[10] It is true that the condition **URC** is strong, as it requires that the promulgated rules of the game remains insensitive to the unforeseen changes in the individual preference orderings on the set of culmination outcomes. As a reflection of this fact, the conditions which guarantee the satisfaction of the condition **URC** cannot but be stringent and go beyond the consequentialist border of informational constraints.

$$((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in \cap_{i\in N} I(Q_i(\mathbf{R})) \Rightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in I(Q(\mathbf{R})),$$

where $Q = \Psi(\mathbf{Q})$.

The next requirement is a version of the independence of irrelevant alternatives [Arrow (1963)] in the framework of extended alternatives.

Independence (I): *For every* $\mathbf{R} \in \mathcal{R}^n$*, every* $\mathbf{Q}$*,* $\mathbf{Q}' \in \Delta_\Psi$*, and every* $(\mathbf{x},\gamma)$, $(\mathbf{x}',\gamma') \in \Lambda(\mathbf{R})$, *if*

$$((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in Q_i(\mathbf{R}) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in Q_i'(\mathbf{R})$$

holds for all $i \in N$, *then*

$$((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in Q(\mathbf{R}) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in Q'(\mathbf{R})$$

holds as well, where $Q = \Psi(\mathbf{Q})$ *and* $Q' = \Psi(\mathbf{Q}')$.

For every $\mathbf{R} \in \mathcal{R}^n$ and given an **ECF** Ψ, an individual $d \in N$ is called an $\mathbf{R}$-*dictator under* Ψ if, for every $\mathbf{Q} \in \Delta_\Psi$ and every $(\mathbf{x},\gamma),(\mathbf{x}',\gamma') \in \Lambda(\mathbf{R})$, $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P(Q_d(\mathbf{R}))$ implies $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P(Q(\mathbf{R}))$, where $Q = \Psi(\mathbf{Q})$. We are now ready to introduce the last democratic requirement on Ψ as follows.

Non-Dictatorship (**ND**): *For every* $\mathbf{R} \in \mathcal{R}^n$, *there is no* $\mathbf{R}$-*dictator under* Ψ.

Note that the above four requirements on the extended constitution function are natural extensions of the Arrovian axioms on the standard Arrovian constitution function [Arrow (1963)], except for the parametric role played by the profile $\mathbf{R}$ in the definition of a dictator under Ψ. In this arena, each and every profile $\mathbf{R}$ is a necessary datum for identifying a *social choice environment* and the domain $\Lambda(\mathbf{R})$ of individual and social ordering functions.

3.2 Nash Solvability, Minimal Liberalism, and Efficiency

In this subsection, we discuss some properties of a rights-system as a game form. They embody a property of stability in social decision-making, a property of minimal liberalism, and a property of outcome morality, respectively. The first property is due to van Hees (1999), which is well-known in game theory as the *Nash solvability* of a game form.

Definition 2: *A game form* $\gamma \in \Gamma^*$ *is* Nash-solvable *if* $\tau_{NE}(\gamma, \mathbf{R})$ *is non-empty for each and every profile* $\mathbf{R} \in \mathcal{R}^n$.

Let Γ_{NS} denote the subclass of Γ which consists solely of the Nash-solvable game forms.

The Nash-solvability plays an important role in the game form formulation of libertarian rights. Indeed, Peleg (1998) formulated the Gibbard paradox in the game form formulation by means of the fact that the game form is not Nash-solvable. Furthermore, Peleg, Peters and Storchen (2002) identified a necessary and sufficient condition for the Nash solvability so as to provide a resolution of the Gibbard paradox.

The second property is related to the intrinsic value of libertarian rights. As an auxiliary step, let us introduce the α*-effectivity function* of a game form, which gives us information on the *(veto) power structure* which a game form assigns to individuals. Given a game form $\gamma = (M, g)$, the associated α-effectivity function E^γ can be defined by $E^\gamma(\varnothing) = \varnothing$ and, for each and every non-empty $S \subseteq N$,

$$E^\gamma(S) \equiv \left\{B \subseteq A \mid \exists \mathbf{m}_S = (m_i)_{i\in S} \in M_S, \forall \mathbf{m}_{N\backslash S} \in M_{N\backslash S} : g(\mathbf{m}_S, \mathbf{m}_{N\backslash S}) \in B\right\},$$

where $M_S \equiv \Pi_{i\in S} M_i$ for every $S \subseteq N$. The universal class of α-effectivity functions associated with Γ^* is denoted by $\mathcal{E}(\Gamma^*)$. Since N and A are finite sets, $\mathcal{E}(\Gamma^*)$ is also finite.

By using the α-effectivity function of a game form, let us define two types of game forms:

Definition 3: *A game form* $\gamma = (M, g) \in \Gamma^*$ *is* dictatorial *if there exists a unique individual* $i \in N$, *to be called the* dictator of γ, *such that* $E^\gamma(i) = 2^A \setminus \{\varnothing\}$ *and* $E^\gamma(j) = \{A\}$ *for every* $j \neq i$. *A dictatorial game form in which* $i \in N$ *is the dictator is called the* i-dictatorial game form.

For each $i \in N$, $\Gamma(i)$ denotes the set of all i-ditatorial game forms.

Definition 4 [Peleg (1998)]**:** *A game form* $\gamma = (M, g) \in \Gamma^*$ *satisfies* minimal liberalism *if there exist at least two individuals* $i, j \in N$ *such that there are* $B^i \in E^\gamma(i)$ *and* $B^j \in E^\gamma(j)$ *with* $B^i \neq A \neq B^j$.

Note that the requisite of minimal liberalism is actually the minimal condition for individual rights to embody the value of individual liberty.[11] As a matter of fact, this requisite of minimal liberalism may not be attractive in more-than-two-person society, as it is compatible with the possibility of duopolistic distributions of effective powers in the presence of numerous individuals with no power whatsoever. To avoid such a duopolistic situation in more-than-two-person society, let us introduce a slightly stronger version of minimal liberalism.

[11]There have been some other proposed conditions for libertarian rights in the game form formulation of rights. *Two Independent Vetoes*^{k} and *Dichotomous Veto Powers*^{k} by Deb, Pattanaik, and Razzolini (1997), and *Maximal Freedom* by van Hees (1999) are two salient examples thereof.

Definition 5: *A game form* $\gamma = (M, g) \in \Gamma^*$ *is said to be* liberal *if each and every individual* $i \in N$ *has an effective power in the sense that there exists* $B^i \in E^\gamma(i)$ *such that* $B^i \neq A$.

Let Γ_L denote the subclass of Γ which consists solely of liberal game forms.

The third property is on the consequentialist value of rights-systems.

Definition 6: *A game form* $\gamma \in \Gamma^*$ *is* efficient *if, for each and every profile* $\mathbf{R} \in \mathcal{R}^n$, *there exists a Pareto efficient Nash equilibrium outcome in* A *whenever* $\tau_{NE}(\gamma, \mathbf{R})$ *is non-empty.*

Let us denote the set of efficient game forms by Γ_{PE}.

This condition is particularly relevant in the context of *liberal paradox* in the game form formulation of individual rights. Recollect that Deb, Pattanaik, and Razzolini (1997) proposed two notions of liberal paradox: *strong liberal paradox* and *weak liberal paradox*. The former says that, for some preference profile, every Nash equilibrium outcome is Pareto inefficient, whereas the latter says that, for some preference profile, there is a Nash equilibrium outcome which is Pareto inefficient. According to this classification, the existence of an efficient game form defined above resolves the strong paradox, but not the weak paradox. Although the resolution of the weak paradox is preferable to that of the strong one, it is a desideratum which is impossible to aspire for, since any game form satisfying minimal liberalism should have a Pareto inefficient outcome for some preference profile, as Peleg (1998) has shown.

We can show that there exists a game form which satisfies all of the above three requisites.

Proposition 1: *There exists a game form* $\gamma^* \in \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$.

Note that Peleg, Peters and Storchen (2002) showed that the Nash solvability is equivalent to the claim that, for every preference profile, there exists a *weakly Pareto efficient* Nash equilibrium outcome. Since we are requiring not weak Pareto efficiency, but strong Pareto efficiency, we cannot simply invoke their equivalence theorem in the context of verifying the validity of **Proposition 1**.

4 Social Choice of Rights Systems through ECF

Under what domain restrictions on the acceptable class of profiles of individual ordering functions can we construct an **ECF** which is not only consistent with the four Arrovian axioms of **SP**, **PI**, **I**, and **ND**, but also is capable of choosing a liberal game form? What about the stringent, but highly desirable property of uniformly rational choice of game-form rights? If **URC** is not satisfied, the associated rational choice function may switch from one game form to the other when the profile **R** undergoes a frivolous change, which one may find rather disturbing.

In section 4.1, we define a subclass of individual ordering functions which may be called the *self-interested* class, and examine the existence of an **ECF** which is workable for every profile of individual ordering functions within this specified class. Although the answer to our question is still negative on this restricted domain, we can show in section 4.2 that the answer turns out to be positive if a suitable further restriction is introduced on the self-interested domain. In section 4.3, we find another restricted domain on which the existence of a democratic **ECF** is guaranteed. Unlike the first two restricted domains, this third type of restricted domain contains a class of ethical ordering functions.

4.1 Self-Interested Domain Restriction

Let a subset $\mathcal{S}$ of $\mathcal{Q}$ be such that, for every $i \in N$, $Q_i \in \mathcal{S}$ implies, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, that:

(a) if $\gamma = \gamma'$ holds, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_i(\mathbf{R})$ if and only if $(\mathbf{x}, \mathbf{x}') \in R_i$; and

(b) if $\mathbf{x} = \mathbf{x}'$ holds, then $E^{\gamma}(i) \supseteq$ (resp. $\supsetneq$) $E^{\gamma'}(i)$ implies $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_i(\mathbf{R})$ (resp.$P(Q_i(\mathbf{R}))$).

The meaning of the first restriction (a) should be clear: whenever the two extended alternatives $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ share the same game form $\gamma = \gamma'$, then the evaluation by Q_i is in accordance with his personal preferences R_i on the pair of culmination outcomes $\{\mathbf{x}, \mathbf{x}'\}$. It means that this individual transcribes his selfish preferences over the set of culmination outcomes at least partly into his value judgements over the set of extended alternatives. The second restriction (b) says that whenever the two extended alternatives $(\mathbf{x}, \gamma)$ and $(\mathbf{x}', \gamma')$ share the same culmination outcome $\mathbf{x} = \mathbf{x}'$, Q_i prefers the extended alternative $(\mathbf{x}, \gamma)$ to another extended alternative $(\mathbf{x}', \gamma')$ at every $\mathbf{R} \in \mathcal{R}^n$ with $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$ as long as γ endows i with more effective power than γ' does. This implies, *inter alia*, that the individual i prefers i-dictatorial game form to any other game form.

Let us illustrate how the above two conditions defining $\mathcal{S}$ restrict the domain of ordering functions by another use of **Example 1**.

Example 2: Consider the problem of smoker and non-smoker in **Example 1**. Let us consider $\{\gamma, \gamma^*, \gamma^1, \gamma^2\} \subseteq \Gamma^*$, where γ and γ^* are defined as in **Example 1**, and γ^1 (*resp.* γ^2) is the 1- (*resp.* 2-) dictatorial game form.

Then, the α-effectivity functions of these game forms are given by:

$$
\begin{aligned}
E^{\gamma}(1) &= \Omega(\{(s,l),(s,r)\}) \cup \Omega(\{(ns,r)\});\\
E^{\gamma}(2) &= \Omega(\{(s,l),(ns,r)\}) \cup \Omega(\{(s,r),(ns,r)\});\\
E^{\gamma^*}(1) &= \Omega(\{(ns,r)\});\ E^{\gamma^*}(2) = \Omega(\{(ns,r)\});\\
E^{\gamma^1}(1) &= \Omega(\{(s,l)\}) \cup \Omega(\{(s,r)\}) \cup \Omega(\{(ns,r)\});\ E^{\gamma^1}(2) = \{A\};\ \text{and}\\
E^{\gamma^2}(1) &= \{A\};\ E^{\gamma^2}(2) = \Omega(\{(s,l)\}) \cup \Omega(\{(s,r)\}) \cup \Omega(\{(ns,r)\}),
\end{aligned}
$$

where $\Omega(B) \equiv \{B' \subseteq A \mid B' \supseteq B\}$ for any $B \subseteq A$.

Take the profile $\mathbf{R} = (R_1, R_2) \in \mathcal{R}^n$ which was defined in section 1.2. Then, $\{(s,l)\} = \tau_{NE}(\gamma; \mathbf{R})$, $\{(ns,r)\} = \tau_{NE}(\gamma^*; \mathbf{R})$, $\{(s,l)\} = \tau_{NE}(\gamma^1; \mathbf{R})$, and $\{(ns,r)\} = \tau_{NE}(\gamma^2; \mathbf{R})$.

Given this $\mathbf{R} \in \mathcal{R}^n$, any $\mathbf{Q} \in \mathcal{S}^n$ has the following property:

$$
\begin{aligned}
&\left(\left((s,l),\gamma^1\right),((s,l),\gamma)\right), \left(((ns,r),\gamma^*), \left((ns,r),\gamma^2\right)\right) \in P(Q_1(\mathbf{R}));\ \text{and}\\
&\left(\left((ns,r),\gamma^2\right),((ns,r),\gamma^*)\right), \left(((s,l),\gamma), \left((s,l),\gamma^1\right)\right) \in P(Q_2(\mathbf{R})).
\end{aligned}
$$

Thus, any preference of individual 1 (*resp.* individual 2) over $\{((s,l),\gamma^1),((s,l),\gamma)\}$ and $\{((ns,r),\gamma^*),((ns,r),\gamma^2)\}$ (*resp.* $\{((ns,r),\gamma^2),((ns,r),\gamma^*)\}$, and $\{((s,l),\gamma),((s,l),\gamma^1)\}$) is identical in the self-interested domain $\mathcal{S}^n$. ■

We are now ready to state the following:

Theorem 1: *Let $\Delta_\Psi = \mathcal{S}^n$ and $\Gamma^* = \Gamma$. Then, for every Ψ which satisfies* **SP**, **PI**, *and* **I**, *there exists $d \in N$ such that $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) = C_\Psi(Q_d; \mathbf{R})$ for every $\mathbf{Q} \in \mathcal{S}^n$. Moreover, if Ψ satisfies* **URC**, *then the class of d-*dictatorial game forms *is uniformly chosen*, viz., $\Gamma(d) \subseteq \cap_{\mathbf{R}\in\mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ *for every $\mathbf{Q} \in \mathcal{S}^n$, and $\cap_{\mathbf{Q}\in\mathcal{S}^n} \cap_{\mathbf{R}\in\mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) = \Gamma(d)$.*

The domain $\mathcal{S}^n$ is a strong restriction vis-à-vis the universal domain $\mathcal{Q}^n$, yet **Theorem 1** shows that even in such a restricted domain, every **ECF** Ψ satisfying the Pareto principles and independence condition should have the dictatorial property in the sense that its associated rational social choice function becomes dictatorial. Moreover, if Ψ satisfies **URC**, then it uniformly rationalizes only the dictatorial game forms. Nevertheless, the fact remains that this kind of restricted domain $\mathcal{S}^n$ is plausible in the realistic context of social choice. The regrettable message of **Theorem 1** is that there is no resolution of the emergence of dictatorship in the social choice of rights-systems as long as the society consists solely of self-interested individuals.

4.2 How to Protect Liberal Rights on the Self-Interested Domain

Although the previous subsection arrived at a pessimistic conclusion, there still remains an interesting issue to be explored. Can a society with self-interested individuals find a method which confers a liberal rights-system through a non-dictatorial social choice procedure? To answer this question in the affirmative, we must introduce a further restriction on the self-interested domain $\mathcal{S}^n$.

As an auxiliary step, let us define, for each and every $j \in N$, a subset $\Gamma_j^0 \subseteq \Gamma$ by

$$\Gamma_j^0 \equiv \{\gamma \in \Gamma^* \mid E^\gamma(j) = \{A\}\}.$$

By construction, Γ_j^0 consists of admissible game forms in which j is *powerless*. Note in particular that the set of all i-dictatorial game forms satisfies the following set-inclusion:

$$\Gamma(i) \subseteq \cap_{j \in N \setminus \{i\}} \Gamma_j^0.$$

For each $i \in N$, let $\Gamma^u(i) \subseteq \Gamma^*$ and $\Gamma^p(i) \subseteq \Gamma^*$ be defined, respectively, by

$$\Gamma^u(i) \equiv \cup_{j \in N \setminus \{i\}} \Gamma^0_j$$

and

$$\Gamma^p(i) \equiv \Gamma^* \setminus \Gamma^u(i) = \cap_{j \in N \setminus \{i\}} (\Gamma^* \setminus \Gamma^0_j).$$

By construction, $\Gamma^u(i)$ consists of admissible game forms in which somebody other than $i \in N$ is *unprivileged* in the sense of being powerless, whereas $\Gamma^p(i)$ consists of admissible game forms in which nobody other than $i \in N$ is unprivileged in the sense of being powerless.

With these auxiliary concepts at hand, we define a class of coalitions $\mathcal{N}_i(\mathbf{Q}) \subseteq 2^{N \setminus \{i\}}$, where $i \in N$ and $\mathbf{Q} \in \mathcal{S}^n$, as follows: for every $S \subseteq N \setminus \{i\}, S \in \mathcal{N}_i(\mathbf{Q})$ if and only if, for every $\gamma \in \Gamma^p(i)$, every $\gamma' \in \Gamma^u(i)$ with $E^{\gamma'}(S) = \{A\}$, every $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one $j \in S$ such that the following condition is satisfied:

$$((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_j(\mathbf{R}) \Leftrightarrow ((\mathbf{y}, \gamma), (\mathbf{y}', \gamma')) \in Q_j(\mathbf{R}'). \qquad \textbf{(n-c)}$$

In words, $S \in \mathcal{N}_i(\mathbf{Q})$ implies that if γ' deprives all members in S of their effective power, and γ does not deprive any member in S of his/her effective power, then some member $j \in S$ ranks at Q_j the relative desirability of γ at least as high as γ', regardless of the culmination outcomes which γ and γ' may happen to bring about at $\mathbf{R}$ and $\mathbf{R}'$, respectively. Thus, taking the condition (b) of $\mathcal{S}$ into consideration, the set $\mathcal{N}_i(\mathbf{Q})$, where $\mathbf{Q} \in \mathcal{S}^n$, is the class of coalitions, each element of which contains at least one member who consistently values at $\mathbf{Q}$, regardless of the culmination outcomes which may happen to emerge, the protection of rights of all members of S higher than the potential dictatorship by i.

Example 3: Consider the problem of smoker and non-smoker in **Example 1** again. Let $\mathbf{R} \in \mathcal{R}^n$ be the profile defined in section 1.2, and let $\mathbf{R}^0 \in \mathcal{R}^n$ be the profile such that every individual is universally indifferent over $A = \{(s,l), (s,r), (ns,r)\}$. In this case, any culmination outcome $\mathbf{x} \in A$ is a Nash equilibrium outcome of the game $(\gamma', \mathbf{R}^0)$ for any $\gamma' \in \Gamma^*$. Take any $\mathbf{Q} \in \mathcal{S}^n$. By virtue of the condition (b) of the self-interested domain, $(((s,l),\gamma^1),((s,l),\gamma^2) \in P(Q_1(\mathbf{R}^0))$, whereas the condition (a) of the self-interested domain brings about $(((s,l),\gamma^2),((ns,r),\gamma^2)) \in I(Q_1(\mathbf{R}^0))$. $Q_1(\mathbf{R}^0)$ being transitive, we then obtain $(((s,l),\gamma^1),((ns,r),\gamma^2)) \in P(Q_1(\mathbf{R}^0))$. Similar reasoning leads us to

$$\begin{aligned}&(((s,l),\gamma),((ns,r),\gamma^2)) \in P(Q_1(\mathbf{R}^0)); \text{ and}\\&(((ns,r),\gamma^2),((s,l),\gamma^1)),(((ns,r),\gamma^*),((s,l),\gamma^1)) \in P(Q_2(\mathbf{R}^0)).\end{aligned}$$

Thus, it follows from the **(n-c)** condition that we have

$$\begin{aligned}&\{1\} \in \mathcal{N}_2(\mathbf{Q}) \Rightarrow (((s,l),\gamma^1),((ns,r),\gamma^2)),(((s,l),\gamma),((ns,r),\gamma^2))\\&\quad \in P(Q_1(\mathbf{R}));\\&\{2\} \in \mathcal{N}_1(\mathbf{Q}) \Rightarrow (((ns,r),\gamma^2),((s,l),\gamma^1)),(((ns,r),\gamma^*),((s,l),\gamma^1))\\&\quad \in P(Q_2(\mathbf{R}))\end{aligned}$$

for any $\mathbf{Q} \in \mathcal{S}^n$. Moreover, for any $\mathbf{Q} \in \mathcal{S}^n$ and any $\mathbf{R}' \in \mathcal{R}^n$, we have:

$$\begin{aligned}&\{1\} \in \mathcal{N}_2(\mathbf{Q}) \Rightarrow ((\mathbf{x},\gamma^1),(\mathbf{x}',\gamma^2)),((\mathbf{x}'',\gamma),(\mathbf{x}',\gamma^2)),((\mathbf{x}''',\gamma^*),(\mathbf{x}',\gamma^2))\\&\quad \in P(Q_1(\mathbf{R}));\\&\{2\} \in \mathcal{N}_1(\mathbf{Q}) \Rightarrow ((\mathbf{x}',\gamma^2),(\mathbf{x},\gamma^1)),((\mathbf{x}'',\gamma),(\mathbf{x},\gamma^1)),((\mathbf{x}''',\gamma^*),(\mathbf{x},\gamma^1))\\&\quad \in P(Q_1(\mathbf{R}))\end{aligned}$$

whenever $(\mathbf{x},\gamma^1),(\mathbf{x}',\gamma^2),(\mathbf{x}'',\gamma),(\mathbf{x}''',\gamma^*) \in \Lambda(\mathbf{R})$ for any $\mathbf{x},\mathbf{x}',\mathbf{x}'',\mathbf{x}''' \in A$.

Thus, $\{1\} \in \mathcal{N}_2(\mathbf{Q})$ implies that individual 1 values at $\mathbf{Q}$ the protection of his own rights than the potential dictatorship by individual 2, in the sense that any extended alternative with the 2-dictatorial game form γ^2 is ranked worst by Q_1, no matter what culmination outcome is realized as a Nash equilibrium outcome under γ^2. The same statement applies to $\{2\} \in \mathcal{N}_1(\mathbf{Q})$. ■

Then we may assert the following:

Theorem 2: *Let $\Gamma^* = \Gamma$. For any $i \in N$, there exists an* **ECF** Ψ *with $\Delta_\Psi \subseteq \mathcal{S}^n$ satisfying* **SP**, **PI**, **I**, *and* **ND** *such that $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \neq \varnothing$ and $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ hold for every $\mathbf{Q} \in \Delta_\Psi$ and every $\mathbf{R} \in \mathcal{R}^n$ if $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}} \setminus \{\varnothing\}$ holds.*

Remark: Although the condition stated in **Theorem 2** is only a sufficient condition for the existence of an **ECF** with the asserted properties, it turns out to be a necessary condition as well if the domain Δ_Ψ of an **ECF** is rich enough. The exact statement of the required domain richness condition and the proof of the asserted necessity may be obtained from the authors upon request.

In the domain Δ_Ψ of **Theorem 2**, every individual other than i insists that the complete deprivation of his rights should be rejected, regardless of the social choice environment which prevails within $\mathcal{R}^n$ and regardless of the culmination outcomes . If every individual other than i always reveals such a strong view against the social decision-making in accordance with the potential dictatorship by i, then his right in terms of effective power can be protected through the democratic social decision procedure Ψ. Since the game form $\gamma^* \in C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ is such that $\gamma^* \in \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$, γ^* is not only non-dictatorial, but also confers liberty on every individual, i inclusive.

Observe, however, that this theorem falls short of guaranteeing the uniform rational choice of a liberal game form. To secure this uniformity property, we must consider another domain restriction. Given $\Gamma^p(i)$ and $\mathcal{S}^n$, define a class of coalitions $\mathcal{M}_i(\mathbf{Q}) \subseteq 2^N$ as follows: for any $S \subseteq N$, $S \in \mathcal{M}_i(\mathbf{Q})$ if and only if, for every $\gamma, \gamma' \in \Gamma^p(i)$, every $\mathbf{R}$, $\mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one individual $j \in S$ such that Q_j satisfies the condition **(n-c)**. Then:

Theorem 3: *Let* $\Gamma^* = \Gamma$. *For any* $i \in N$, *there exists an* **ECF** Ψ *with* $\Delta_\Psi \subseteq \mathcal{S}^n$ *satisfying* **SP**, **PI**, **I**, *and* **ND** *such that* $\varnothing \neq \cap_{\mathbf{R}\in\mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ *holds for every* $\mathbf{Q} \in \Delta_\Psi$ *if* $\cap_{\mathbf{Q}\in\Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}} \setminus \{\varnothing\}$ *and* $\{i\} \in \cap_{\mathbf{Q}\in\Delta_\Psi} \mathcal{M}_i(\mathbf{Q})$ hold.

According to **Theorem 3**, if every individual has only the non-consequential preferences on rights-systems, it is not only possible to resolve the Arrovian impossibility impasse, but it is also possible to protect every individual′s liberty in terms of effective power, and to choose uniformly rational, liberal, Nash-solvable, and Pareto efficient game form as a rights-system.

4.3 Ethical Individuals and Liberal Social Ordering Functions

In this subsection, we go beyond the self-interested class of individual ordering functions and introduce the possibility of ethical individual ordering functions. In so doing, we look for the domain restrictions on **ECF**s under which an ethical social ordering function can be derived without violating the Arrovian axioms.

To begin with, we introduce a condition for social ordering functions to be *liberal* as follows.

Definition 7: *An ordering function* $Q \in \mathcal{Q}$ *is said to be* liberal *if and only if, for every* $\mathbf{R} \in \mathcal{R}^n$ *and for whatever* $(\mathbf{x}, \gamma)$, $(\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $\gamma \in \Gamma_L$ *and* $\gamma' \in \Gamma \setminus \Gamma_L$ *necessarily imply* $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R}))$.

The idea behind this definition is simple. An ordering function which respects the *intrinsic value* of individual liberty should give every liberal game form the absolute priority over any other non-liberal game form no matter what culmination outcomes they may respectively bring about. In what follows, the class of liberal ordering functions will be denoted by $\mathcal{Q}^{\mathbf{L}}$.

Next, we define a condition which qualifies an ordering function to be not only liberal, but also *non-consequentialist liberal* as follows.

Definition 8: *An ordering function* $Q \in \mathcal{Q}$ *is* non-consequentialist liberal *if and only if* $Q \in \mathcal{Q}^{\mathbf{L}}$ *and, for every* $\gamma, \gamma' \in \Gamma_L$, *every* $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, *every* $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$ *and every* $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, Q *satisfies the* condition **(n-c)**.

It may deserve emphasis that a non-consequentialist liberal ordering function is one which embodies two distinct features of non-consequentialism. In the first place, it embodies a feature of non-consequentialism in the sense that a liberal game form, which confers some effective power on each and every individual, is judged strictly better than an illiberal game form, which does not do so, *no matter what consequences they may respectively bring about*. In the second place, it embodies another feature of non-consequentialism in the following sense: if both γ and γ' are liberal game forms, attaining a consequence $\mathbf{x} \in$

$\tau_{NE}(\gamma, \mathbf{R})$ through the play of the game $(\gamma, \mathbf{R})$ is judged at least as good as attaining a consequence $\mathbf{x}' \in \tau_{NE}(\gamma', \mathbf{R})$ through the play of the game $(\gamma', \mathbf{R})$ in terms of the ordering $Q(\mathbf{R})$ if and only if attaining a consequence $\mathbf{y} \in \tau_{NE}(\gamma, \mathbf{R}')$ through the play of the game $(\gamma, \mathbf{R}')$ is judged at least as good as attaining a consequence $\mathbf{y}' \in \tau_{NE}(\gamma', \mathbf{R}')$ through the play of the game $(\gamma', \mathbf{R}')$ in terms of the ordering $Q(\mathbf{R}')$. In what follows, the class of non-consequentialist liberal ordering functions is denoted by $\mathcal{Q}^{\mathbf{NCL}}$. It is clear that $\mathcal{Q}^{\mathbf{NCL}} \subseteq \mathcal{Q}^{\mathbf{L}}$.

We are now ready to discuss the necessary and sufficient conditions for the existence of **ECF**s which not only satisfy the Arrovian conditions, but also always generate non-consequentialist liberal ordering functions.

To begin with, let us define a subclass $\mathcal{F} \subsetneq \mathcal{Q}$ as follows: $Q \in \mathcal{F}$ holds if and only if the following two conditions hold for any $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$:

(a) if $\mathbf{x} = \mathbf{x}'$ holds, then $\left(\forall h \in N\text{: } E^{\gamma}(h) \supseteq E^{\gamma'}(h)\right)$ implies $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q(\mathbf{R})$;

(b) if $\mathbf{x} = \mathbf{x}'$ holds, then $\left(\forall h \in N\text{: } E^{\gamma}(h) \supseteq E^{\gamma'}(h) \;\&\; \exists j \in N\text{: } E^{\gamma}(j) \supsetneq E^{\gamma'}(j)\right)$ implies$((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R}))$.

This restricted class of ordering functions embodies ethical value judgements in the following sense. According to the defining condition (a) [resp. (b)] of $\mathcal{F}, Q$ judges that, if all individuals′ liberties in terms of effective power do not decrease [resp. do not decrease with at least one individual′s liberty being strictly increasing], it is not a worse [resp. a better] change. This is essentially the ethical value judgements, which is motivated by the notion of *Maximal Freedom* introduced by van Hees (1999). Using $\mathcal{F}$, let us define

$$(\mathcal{F}, \mathcal{Q}_{-i}) \equiv \underbrace{\mathcal{Q} \times \ldots \times \mathcal{Q}}_{(i-1)\text{-times}} \times \mathcal{F} \times \underbrace{\mathcal{Q} \times \ldots \times \mathcal{Q}}_{(n-i)\text{-times}},$$

which will serve as a crucial domain restriction in our subsequent possibility theorems.

As an auxiliary step in defining two crucial families of subsets of N, $\mathcal{K}(\mathbf{Q})$ and $\mathcal{L}(\mathbf{Q})$, for each and every $\mathbf{Q} \in \mathcal{Q}^n$, let a class of ordering functions $\mathcal{T} \subseteq \mathcal{Q}$ be defined by

$$\mathcal{T} \equiv \{Q \in \mathcal{Q} \mid \exists Q' \in \mathcal{S} : Q \text{ and } Q' \text{ coincide on } (A \times \Gamma_L) \times (A \times (\Gamma \backslash \Gamma_L))\}.$$

For every $\mathbf{Q} \in \mathcal{Q}^n$, let a class $\mathcal{K}(\mathbf{Q})$ of subsets of N be defined as follows: for every $S \subseteq N$, $S \in \mathcal{K}(\mathbf{Q})$ if and only if, for every $\gamma \in \Gamma_L$ and $\gamma' \in \Gamma \backslash \Gamma_L$ with $E^{\gamma'}(S) = \{A\}$, every $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$ and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one individual $j \in S$ such that $Q_j \in \mathcal{T}$ satisfies the condition **(n-c)**. Likewise, a class of coalitions $\mathcal{L}(\mathbf{Q}) \subseteq 2^N$ is defined as follows: for every $S \subseteq N$, $S \in \mathcal{L}(\mathbf{Q})$ if and only if, for every $\gamma, \gamma' \in \Gamma_L$, every $\mathbf{R}, \mathbf{R}' \in \mathcal{R}^n$, every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$ and every $(\mathbf{y}, \gamma), (\mathbf{y}', \gamma') \in \Lambda(\mathbf{R}')$, there exists at least one individual $j \in S$ such that $Q_j \in \mathcal{F}$ satisfies the condition **(n-c)**.

The meaning of $\mathcal{K}(\mathbf{Q})$ is almost the same as that of $\mathcal{N}_i(\mathbf{Q})$ discussed in the previous subsection: if a group S consists of individuals who are deprived of any effective power in γ', there exists a member of S who expresses a non-consequentialist evaluation between γ' and any liberal game form γ. In contrast, $\mathcal{L}(\mathbf{Q})$ is essentially gathering the ethical individuals together who have non-consequentialist preferences in favor of the set of liberal game forms.

We are now ready to assert the following:

Theorem 4: *Let* $\Gamma^* = \Gamma$. *For any* $i \in N$, *there exists an* **ECF** Ψ *satisfying* **SP**, **PI**, **I**, *and* **ND** *such that, for any* $\mathbf{Q} \in \Delta_\Psi$, $\Psi(\mathbf{Q}) \in \mathcal{Q}^{\mathbf{NCL}}$ *and* $\varnothing \neq C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ *for every* $\mathbf{R} \in \mathcal{R}^n$ *hold if and only if* $\Delta_\Psi \subseteq$

$(\mathcal{F}, \mathcal{Q}_{-i})$, $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{K}(\mathbf{Q}) = 2^N \setminus \{\varnothing\}$ *and* $\{i\} \in \cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{L}(\mathbf{Q})$ *hold.*

This theorem says that the existence of a democratic **ECF** Ψ is guaranteed, and that this Ψ generates a non-consequentialist social ordering function in $\mathcal{Q}^{\mathbf{NCL}}$ if and only if (1) any group of individuals objects to the deprivation of its members$'$ liberal rights in terms of effective power; and (2) there exists an individual $i \in N$ who not only always has ethical ordering functions in the restricted domain $(\mathcal{F}, \mathcal{Q}_{-i})$, but also always behaves as a non-consequentialist in the sense that $\{i\} \in \cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{L}(\mathbf{Q})$.

Capitalizing on **Theorem 4**, the next theorem is on the uniform rationalizability of efficient and liberal game forms through the democratic social decision procedure Ψ even within a broader domain than the self-interested domain.

Theorem 5: *Let* $\Gamma^* = \Gamma$. *For any* $i \in N$, *there exists an* **ECF** Ψ *on* $\Delta_\Psi \subseteq \mathcal{Q}^n$ *satisfying* **SP**, **PI**, **I**, *and* **ND** *such that* $\varnothing \neq \cap_{\mathbf{R}\in\mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_{NS} \cap \Gamma_L \cap \Gamma_{PE}$ *for any* $\mathbf{Q} \in \Delta_\Psi$ *if* $\Delta_\Psi \subseteq (\mathcal{F}, \mathcal{Q}_{-i})$, $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{K}(\mathbf{Q}) = 2^N \setminus \{\varnothing\}$ *and* $\{i\} \in \cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{L}(\mathbf{Q})$ *hold.*

This theorem says that the necessary and sufficient condition in **Theorem 4** also guarantees the solution for the issue of uniform rationalizability of efficient and liberal game forms.

It is worth emphasizing the two important roles of ethical individual $i \in \cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{L}(\mathcal{Q})$ in **Theorem 4** and **Theorem 5**. The first role is that, by committing himself to the restricted class of ethical ordering functions, viz $\mathcal{F}$, i can guarantee a nice property of the choice set $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ for any $\mathbf{Q} \in \Delta_\Psi$ and any $\mathbf{R} \in \mathcal{R}^n$ to the effect that $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ consists only of game forms representing the maximal freedom in the sense of van Hees (1999) as well as guaranteeing the Nash-solvability and efficiency of the rationalized game forms. The second role is that, by committing himself to behave as a non-consequentialist in the sense that $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$, i can ensure that Ψ not only always generates non-consequentialist liberal social ordering functions over its domain Δ_Ψ, but also the rational choice function C_Ψ chosen through Ψ meets the condition of uniform rationalizability.

In contrast to the role of $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$ and $\mathcal{F}$, the role of any coalition belonging to $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{K}(\mathbf{Q})$ is to guarantee liberalism of the rationalized game forms.

5 Concluding Remarks

According to the tripartite classification of the issues of libertarian rights due to Pattanaik and Suzumura (1996; 1996) and Suzumura (1996; 2005), there are three distinct issues to be identified and separately addressed to. The first issue of the formal structure of rights and the second issue of the realization of conferred rights have received detailed logical scrutiny in the social-choice theoretic approach due to Sen (1970, Chapter 6*; 1970a; 1976; 1983,1992) as well as in the game-form theoretic approach due to Sugden (1985), Deb (1990/2004; 1994), Gaertner et al. (1992), Deb et al. (1997), Peleg (1998), van Hees (1999), Peleg et al. (2002) and others, but the third issue of the initial conferment of libertarian rights has received only scanty, if any at all, analysis in the literature. This paper is devoted to this scarcely cultivated issue within the extended analytical framework of game-form rights.

Our results are focused on the conditions under which the extended social choice rule satisfying essentially Arrovian axioms exists, which can choose game forms embodying libertarian values and generating Pareto efficient culmination outcomes at the Nash equilibria. Recollect that the original Arrovian social choice framework is such that individuals express their respective subjective values over the set of culmination outcomes, and the social choice rule aggregates these individual values into the social value which, in its turn, is invoked in the social choice of culmination outcomes. In contrast, our extended framework has two idiosyncratic features. In the first place, the inputs into the extended social choice rule are individual ordering functions, and the outputs of the extended social choice rule are social ordering functions. Thus, unlike the traditional approach where individual and social preferences are consequentialist pure and simple, this extended framework can accommodate individual′s as well as society′s concern about the procedural aspects of social choice. In the second place, unlike the traditional Arrovian edifice, the object of social choice is not the culmination outcome *per se*, but the game form, the play of which in the realization stage results in the culmination outcome at the Nash equilibrium of the game. The main purpose of this paper will be served if this novel structure of social choice theory is found relevant in the concrete context of social choice of game-form rights.

In concluding this paper, a general observation on the meaning of our analysis may be in order. Recollect that it was North (1990, pp.3-4) who crystallized the notion of *institutions as rules of the game* in a society. Note also that Hurwicz (1996, p.115) reminded us of the fact that "it is the game form, rather than the game, that corresponds to the intuitive notion of the 'rules of the game'," as "the players' preferences are not part of the rules." Following the North-Hurwicz notion of institutions as the game forms, we may suggest that our two-stage structure of the analysis of game-form rights can be construed as a possible framework for the social-choice theoretic analysis of institutional choice and its decentralized realization. However, the fully-fledged development of this interesting scenario cannot but be relegated to the future opportunity.

Appendix: Proofs

Given $\mathbf{R} \in \mathcal{R}^n$ and $\mathbf{Q} \in \Delta_\Psi$, let $Q_N(\mathbf{R}) \equiv \cap_{h \in N} Q_h(\mathbf{R})$, $I(Q_N(\mathbf{R})) \equiv \cap_{h \in N} I(Q_h(\mathbf{R}))$, and $P(Q_N(\mathbf{R})) \equiv (\cap_{h \in N} Q_h(\mathbf{R})) \setminus I(Q_N(\mathbf{R}))$. Let a profile $\mathbf{R}^0 \in \mathcal{R}^n$ be such that every individual is universally indifferent over A. Given $\gamma = (M, g) \in \Gamma^*$, for any $h \in N$, and any $m_h \in M_h$, let $B^h_{m_h} \equiv g(m_h, M_{-h})$. Then, $E^\gamma(h) = \cup_{m_h \in M_h} \Omega(B^h_{m_h})$ for each and every $h \in N$, where and hereafter $\Omega(B)$ denotes the family of sets consisting of B and all its supersets in A.

Γ^* is assumed to be large enough in the following proofs, where the meaning of "large enough" is that it contains actual game forms we construct in the proofs. Let us say that a game form $\gamma \in \Gamma^*$ is *power-dominated* by another game form $\gamma' \in \Gamma^*$ if and only if $E^{\gamma'}(i) \supseteq E^\gamma(i)$ for all $i \in N$ and $E^{\gamma'}(j) \supsetneq E^\gamma(j)$ for some $j \in N$. A game form $\gamma^* \in \Gamma^*$ represents a *maximal power structure* if there is no other game form $\gamma \in \Gamma^*$ which power-dominates γ^*. Let us denote by $\mu(\Gamma^*)$ the set of game forms, each member of which represents a maximal power structure in Γ^*.

A.1 Proof of Theorem 1

Lemma 1: *Let a game form $\gamma \in \Gamma^*$ represent a* maximal power structure. *Then, there exists a* Nash-solvable *and* efficient *$\gamma^* \in \Gamma^*$ such that $E^{\gamma^*} = E^\gamma$.*

Proof. Let $\gamma \in \mu(\Gamma^*)$. Then, by the definition of $\mu(\Gamma^*)$, E^γ satisfies *maximal* *freedom* in the sense of van Hees (1999). According to van Hees (1999, Theorem 1), there exists $\gamma^* \in \Gamma^*$ which is Nash-solvable and efficient, and satisfies $E^{\gamma^*} = E^{\gamma}$. ■

Lemma 2: *Let a game form $\gamma \in \Gamma^*$ be such that $\gamma \notin \mu(\Gamma^*)$. Then, for any $\mathbf{R} \in \mathcal{R}^n$ and any $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, there exists $\gamma' \in \Gamma^*$ which power-dominates γ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$.*

Proof. Since $\gamma \notin \mu(\Gamma^*)$ by assumption, there exists $\gamma' \in \Gamma^*$ which power-dominates γ. It follows from $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$ that there exists a Nash equilibrium $(m_h^{\mathbf{x}})_{h \in N} \in \epsilon_{NE}(\gamma, \mathbf{R})$ such that $\mathbf{x} = g((m_h^{\mathbf{x}})_{h \in N}) \in \tau_{NE}(\gamma, \mathbf{R})$. Also, there exists $\left(B^h_{m_h^{\mathbf{x}}}\right)_{h \in N} \in \Pi_{h \in N} E^{\gamma}(h)$ such that $\cap_{h \in N} B^h_{m_h^{\mathbf{x}}} = \{\mathbf{x}\}$. By definition, $B^h_{m_h^{\mathbf{x}}} \in E^{\gamma'}(h)$ for every $h \in N$. That is, for $\gamma' = (M', g') \in \Gamma^*$ and for each and every $h \in N$, there exists $m'_h \in M'_h$ such that $g'\left(m'_h, M'_{-h}\right) \subseteq B^h_{m_h^{\mathbf{x}}}$. Then, $g'\left((m'_h)_{h \in N}\right) \in \cap_{h \in N} g'\left(m'_h, M'_{-h}\right) \subseteq \cap_{h \in N} B^h_{m_h^{\mathbf{x}}} = \{\mathbf{x}\}$. Take any $j \in N$, and note that $\cap_{h \in N \setminus \{j\}} B^h_{m_h^{\mathbf{x}}} = g\left(\mathbf{m}^{\mathbf{x}}_{N \setminus \{j\}}, M_j\right)$. Thus, $\cap_{h \in N \setminus \{j\}} g'\left(m'_h, M'_{-h}\right) = g'\left(\mathbf{m}'_{N \setminus \{j\}}, M'_j\right) \subseteq g\left(\mathbf{m}^{\mathbf{x}}_{N \setminus \{j\}}, M_j\right)$ for every $j \in N$. This implies that $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$, since $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$. ■

Lemma 3: *There exist at least three game forms $\gamma', \gamma'', \gamma''' \in \mu(\Gamma^*)$ such that, for any two $\gamma, \gamma^* \in \{\gamma', \gamma'', \gamma'''\}$, for every $h \in N$, neither $E^{\gamma}(h) \subseteq E^{\gamma^*}(h)$ nor $E^{\gamma}(h) \supseteq E^{\gamma^*}(h)$.*

Proof. Since $\#A \geq 3$, there exist at least three alternatives $\mathbf{x}', \mathbf{x}'', \mathbf{x}''' \in A$. Given any game form $\gamma = (M, g) \in \Gamma^*$, define $\gamma' = (M', g') \in \Gamma^*$ as follows: for each $h \in N$, $M'_h = M_h \cup \{\mathbf{x}'\}$, and the outcome function g' is such that, for each and every $\mathbf{m} \in M'$,

$$\begin{cases} g'(\mathbf{m}) = \mathbf{x}' & \text{if } m_h = \mathbf{x}' \in M'_h \text{ for some } h \in N; \\ g'(\mathbf{m}) = g(\mathbf{m}) & \text{otherwise.} \end{cases}$$

Then, $E^{\gamma'}(h) = \Omega(\{\mathbf{x}'\})$ for each $h \in N$. It is easy to check that $\gamma' \in \mu(\Gamma^*)$ holds true. In a similar way, we can construct $\gamma'', \gamma''' \in \mu(\Gamma^*)$ such that $E^{\gamma''}(h) = \Omega(\{\mathbf{x}''\})$ and $E^{\gamma'''}(h) = \Omega(\{\mathbf{x}'''\})$ for each $h \in N$. Note that, for any two $\gamma, \gamma^* \in \{\gamma', \gamma'', \gamma'''\}$, for every $h \in N$, $\{\mathbf{x}^*\} \notin E^{\gamma}(h)$, $\{\mathbf{x}^*\} \in E^{\gamma^*}(h)$, $\{\mathbf{x}\} \notin E^{\gamma^*}(h)$, and $\{\mathbf{x}\} \in E^{\gamma}(h)$. ■

Lemma 4: *Let a game form $\gamma \in \Gamma^*$ have the α-effectivity function E^{γ} such that $E^{\gamma}(i) = \{A\}$ for some $i \in N$. Then, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, there exists $\gamma' \in \Gamma^*$ such that $E^{\gamma'}(i) \neq \{A\}$ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. Moreover, if $\{\mathbf{x}\} = \tau_{NE}(\gamma, \mathbf{R})$, then $\{\mathbf{x}\} = \tau_{NE}(\gamma', \mathbf{R})$.*

Proof. By definition, $E^{\gamma}(h) = \cup_{m_h \in M_h} \Omega(B^h_{m_h})$ for each $h \in N \backslash \{i\}$, and $E^{\gamma}(i) = \{A\}$. Since $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, there exists $\left(m^{\mathbf{x}}_i, \mathbf{m}^{\mathbf{x}}_{-i}\right) \in \epsilon_{NE}(\gamma, \mathbf{R})$ such that $\left(\cap_{h \in N \backslash \{i\}} B^h_{m^{\mathbf{x}}_h}\right) \cap B^i_{m^{\mathbf{x}}_i} = \left(\cap_{h \in N \backslash \{i\}} B^h_{m^{\mathbf{x}}_h}\right) \cap A = \{\mathbf{x}\}$.

Let us construct a new game form $\gamma' = (M', g') \in \Gamma^*$ as follows. For each $h \in N \backslash \{i\}$, $M'_h \equiv M_h$ and $M'_i = \{A, \{\mathbf{x}\}\}$, and, for each $h \in N \backslash \{i\}$ and each $m_h \in M'_h$, the outcome function g' is defined by:

$$\begin{cases} \left\{g'\left((m_h)_{h \in N \backslash \{i\}}, m'_i\right)\right\} = g\left((m_h)_{h \in N \backslash \{i\}}, M_i\right) & \text{if } m'_i = A \in M'_i; \\ g'\left((m_h)_{h \in N \backslash \{i\}}, m'_i\right) = \mathbf{x} & \text{if } m'_i = \{\mathbf{x}\} \in M'_i. \end{cases}$$

Then, by construction, $\left(\{\mathbf{x}\}, \mathbf{m}^{\mathbf{x}}_{-i}\right)$ and $\left(A, \mathbf{m}^{\mathbf{x}}_{-i}\right)$ are in $\epsilon_{NE}(\gamma', \mathbf{R})$, and $g'(\{\mathbf{x}\}, \mathbf{m}^{\mathbf{x}}_{-i}) = g'\left(A, \mathbf{m}^{\mathbf{x}}_{-i}\right) = \mathbf{x}$. If $\{\mathbf{x}\} = \tau_{NE}(\gamma, \mathbf{R})$, then by construction, $\{\mathbf{x}\} = \tau_{NE}(\gamma', \mathbf{R})$ holds true. Moreover, $E^{\gamma'}(h) = \cup_{m_h \in M'_h} \Omega(B^h_{m_h} \cup \{\mathbf{x}\})$ for each $h \in N \backslash \{i\}$, and $E^{\gamma'}(i) = \Omega(\{\mathbf{x}\})$. ■

Lemma 5: *Let $\gamma \in \Gamma^*$ be an i-dictatorial game form. Then, for every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, there exists $\gamma' \in \mu(\Gamma^*)$ such that $E^{\gamma'}(j) \neq \{A\}$ for some $j \in N \backslash \{i\}$ and $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$.*

Proof. By a similar method used in the proof of **Lemma 4**, we can construct a desired game form γ'. ■

Lemma 6: *Let* $\gamma', \gamma'', \gamma''' \in \mu(\Gamma^*)$ *be the three game forms, the existence of which being assured in* **Lemma 3**. *Assume that* $(\mathbf{x}', \gamma') \in \Lambda(\mathbf{R}), (\mathbf{x}'', \gamma'') \in \Lambda(\mathbf{R})$, *and* $(\mathbf{x}''', \gamma''') \in \Lambda(\mathbf{R})$ *for some* $\mathbf{R} \in \mathcal{R}^n$. *Then, for every* **ECF** Ψ *satisfying* **SP, PI,** *and* **I**, *there exists a local dictator* $d \in N$ *over* $\{(\mathbf{x}', \gamma'), (\mathbf{x}'', \gamma''), (\mathbf{x}''', \gamma''')\}$. *Moreover, for every* $(\widetilde{\mathbf{x}}, \widetilde{\gamma}) \in \{(\mathbf{x}', \gamma'), (\mathbf{x}'', \gamma''), (\mathbf{x}''', \gamma''')\}$ *and every* $(\mathbf{x}^*, \gamma^*) \in \Lambda(\mathbf{R})$ *with* $\gamma^* \in \mu(\Gamma^*) \backslash [\cup_{h \in N} \Gamma(h)]$,
(i) *if* $((\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\mathbf{x}^*, \gamma^*)) \in P(Q_d(\mathbf{R}))$, *then* $((\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\mathbf{x}^*, \gamma^*)) \in P(Q(\mathbf{R}))$,
(ii) *if* $\# N = 2$ *and* $((\mathbf{x}^*, \gamma^*), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q_d(\mathbf{R}))$, *then* $((\mathbf{x}^*, \gamma^*), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q(\mathbf{R}))$,
where $Q = \Psi(\mathbf{Q})$.

Proof. Capitalizing on **Lemma 3**, we can find three game forms $\gamma', \gamma'', \gamma''' \in \mu(\Gamma^*) \cap \Gamma_L$ such that $(\mathbf{x}', \gamma'), (\mathbf{x}'', \gamma''), (\mathbf{x}''', \gamma''') \in \Lambda(\mathbf{R})$. Moreover, for every $\gamma \in \{\gamma', \gamma'', \gamma'''\}$, $E^{\gamma}(h) = \Omega(\{\mathbf{x}\})$ holds for each $h \in N$, where $\mathbf{x} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$. Then, we can see that the free triple property holds among $(\mathbf{x}', \gamma'), (\mathbf{x}'', \gamma'') (\mathbf{x}''', \gamma''')$ on the domain $\mathcal{S}^n$, so that there exists a (local) dictator, say d, for the social evaluation among these three extended alternatives by virtue of the Arrovian impossibility theorem.

Suppose that there exists $\gamma^* \in \mu(\Gamma^*) \backslash \{\gamma', \gamma'', \gamma'''\}$ satisfying $(\mathbf{x}^*, \gamma^*) \in \Lambda(\mathbf{R})$. If $\mathbf{x}^* \notin \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ and any other Nash equilibrium outcome of the game $(\gamma^*, \mathbf{R})$ does not belong to $\{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$, then it is easy to show that d can extend his dictatorship to include $(\mathbf{x}^*, \gamma^*)$, because of the free triple property. Suppose that $\mathbf{x}^* \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ or there exists another $\mathbf{x}^{*\prime} \in \tau_{NE}(\gamma^*, \mathbf{R})$ such that $\mathbf{x}^{*\prime} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$. Let $\mathbf{x}'' = \mathbf{x}^*$ or $\mathbf{x}'' = \mathbf{x}^{*\prime}$.

Proof of the Statement (i).

Suppose that $((\mathbf{x}^*,\gamma^*),(\mathbf{x}',\gamma')) \in Q(\mathbf{R})$ in spite of $((\mathbf{x}',\gamma'),(\mathbf{x}^*,\gamma^*)) \in P(Q_d(\mathbf{R}))$. Then, we can find a new game form $\gamma^{**} \in \Gamma^*$ such that, for each $h \in N$, $E^{\gamma^{**}}(h) \nsubseteq E^{\gamma^*}(h)$ and $E^{\gamma'''}(h) \nsubseteq E^{\gamma^{**}}(h)$. Choose $\pi : A \to A$ appropriately, and let $\gamma^{**} = (M^{**}, g^{**}) \in \Gamma^*$ be such that, for each $h \in N$, $M_h^{**} \equiv M_h^*$, and $g^{**}(\mathbf{m}) \equiv \pi(g^*(\mathbf{m}))$ for every $\mathbf{m} \in M^{**}$. Then, for any $h \in N$, $E^{\gamma^{**}}(h) \nsubseteq E^{\gamma^*}(h)$ holds. Note that, if $E^{\gamma^*}(h) \supsetneq E^{\gamma'''}(h)$ for some $h \in N$, it follows that $E^{\gamma^*}(h) \supsetneq \Omega(\{\mathbf{x}'''\})$. In this case, for any $h' \in N\backslash\{h\}$, $E^{\gamma^*}(h') \nsupseteq \Omega(\{\widetilde{\mathbf{x}}\})$ for any $\widetilde{\mathbf{x}} \in A$. Then, since $\gamma^* \notin \cup_{h\in N}\Gamma(h)$, there exists $\widehat{\mathbf{x}} \in A$ such that $E^{\gamma^*}(h) \nsupseteq \Omega(\{\widehat{\mathbf{x}}\})$. Thus, by choosing $\pi : A \to A$ so that $\pi(\widehat{\mathbf{x}}) = \mathbf{x}'''$, $\pi(\mathbf{x}''') = \widehat{\mathbf{x}}$, and $\pi(\widetilde{\mathbf{x}}) = \widetilde{\mathbf{x}}$ for all $\widetilde{\mathbf{x}} \in A\backslash\{\mathbf{x}''',\widehat{\mathbf{x}}\}$, we have $E^{\gamma'''}(h) \nsubseteq E^{\gamma^{**}}(h)$ for each $h \in N$. If $E^{\gamma^*}(h) \nsupseteq E^{\gamma'''}(h)$ for any $h \in N$, then choose $\pi : A \to A$ as the identity mapping. Thus, we have $E^{\gamma'''}(h) \nsubseteq E^{\gamma^{**}}(h)$ for each $h \in N$. Since $\gamma^* \in \mu(\Gamma^*)$, it follows that $\gamma^{**} \in \mu(\Gamma^*)$, so that there exists $\mathbf{x}^{**} \in \tau_{NE}(\gamma^{**},\mathbf{R})$ by **Lemma 1**.

Consider a profile $\widetilde{\mathbf{Q}} \in \mathcal{S}^n$ such that $((\mathbf{x}''',\gamma'''),(\mathbf{x}^{**},\gamma^{**})) \in \widetilde{Q}_N(\mathbf{R})$, $((\mathbf{x}^{**},\gamma^{**}),(\mathbf{x}^*,\gamma^*)) \in \widetilde{Q}_N(\mathbf{R})$, and $\widetilde{Q}_h(\mathbf{R}) \cap \{(\mathbf{x}^*,\gamma^*),(\mathbf{x}',\gamma')\}^2 = Q_h(\mathbf{R}) \cap \{(\mathbf{x}^*,\gamma^*),(\mathbf{x}',\gamma')\}^2$ for every $h \in N$. Moreover, $((\mathbf{x}',\gamma'),(\mathbf{x}''',\gamma''')) \in P\left(\widetilde{Q}_d(\mathbf{R})\right)$. Then, by **SP** and **PI**, and the transitivity of $\widetilde{Q}(\mathbf{R})$, $((\mathbf{x}''',\gamma'''),(\mathbf{x}^*,\gamma^*)) \in \widetilde{Q}(\mathbf{R})$. Also, by **I**, $((\mathbf{x}^*,\gamma^*),(\mathbf{x}',\gamma')) \in \widetilde{Q}(\mathbf{R})$. But this contradicts the consistency of $\widetilde{Q}(\mathbf{R})$, which follows from the transitivity thereof, since $((\mathbf{x}',\gamma'),(\mathbf{x}''',\gamma''')) \in P\left(\widetilde{Q}(\mathbf{R})\right)$. Thus, even if $\mathbf{x}^* \in \{\mathbf{x}',\mathbf{x}'',\mathbf{x}'''\}$ or there exists another $\mathbf{x}^{*\prime} \in \tau_{NE}(\gamma^*,\mathbf{R})$ such that $\mathbf{x}^{*\prime} \in \{\mathbf{x}',\mathbf{x}'',\mathbf{x}'''\}$, $((\mathbf{x}',\gamma'),(\mathbf{x}^*,\gamma^*)) \in P(Q_d(\mathbf{R}))$ cannot but imply $((\mathbf{x}',\gamma'),(\mathbf{x}^*,\gamma^*)) \in P(Q(\mathbf{R}))$.

Proof of the Statement (ii).

Suppose that $\#N = 2$, $\gamma^* \notin \cup_{h\in N}\Gamma(h)$, and $((\mathbf{x}',\gamma'),(\mathbf{x}^*,\gamma^*)) \in Q(\mathbf{R})$

hold in spite of $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q_d(\mathbf{R}))$. Note that, since $\#N = 2$ and $\gamma^* \notin \cup_{h\in N}\Gamma(h)$, $\gamma^* \in \mu(\Gamma^*) \cap \Gamma_L$ holds. Then, we can find a new game form $\gamma^{**} \in \mu(\Gamma^*) \cap \Gamma_L$ such that, for each $h \in N$, $E^{\gamma^{**}}(h) \not\supseteq E^{\gamma^*}(h)$ and $E^{\gamma'''}(h) \not\supseteq E^{\gamma^{**}}(h)$. To verify this fact, choose $\pi : A \to A$ appropriately, and define $\gamma^{**} = (M^{**}, g^{**}) \in \Gamma^*$ as follows: for each $h \in N$, $M_h^{**} \equiv M_h^*$, and $g^{**}(\mathbf{m}) \equiv \pi(g^*(\mathbf{m}))$ for each $\mathbf{m} \in M^{**}$. Then, for any $h \in N$, $E^{\gamma^{**}}(h) \not\supseteq E^{\gamma^*}(h)$ holds. Note that, if $E^{\gamma^*}(h) \subsetneq E^{\gamma'''}(h)$ for some $h \in N$, it follows that $E^{\gamma^*}(h) \subsetneq \Omega(\{\mathbf{x}'''\})$. In this case, for any $B \subseteq A\backslash\{\mathbf{x}'''\}$, $B \notin E^{\gamma^*}(h)$. Since $\gamma^* \in \mu(\Gamma^*)$, it is a *semi-tight* game form in the sense of van Hees (1999, Lemma 0.2), which implies $E^{\gamma^*}(N\backslash\{h\}) \supseteq \Omega(\{\mathbf{x}'''\})$. Thus, for another individual $h' \in N\backslash\{h\}$, $E^{\gamma^*}(h') \supseteq \Omega(\{\mathbf{x}'''\})$ holds by $\#N = 2$. Then, by choosing $\pi : A \to A$ as $\pi(\mathbf{x}''') = \widehat{\mathbf{x}}$, $\pi(\widehat{\mathbf{x}}) = \mathbf{x}'''$, and $\pi(\widetilde{\mathbf{x}}) = \widetilde{\mathbf{x}}$ for all $\widetilde{\mathbf{x}} \in A\backslash\{\mathbf{x}''', \widehat{\mathbf{x}}\}$, where $\Omega(\widehat{\mathbf{x}}) \nsubseteq E^{\gamma^*}(h')$, we have $E^{\gamma'''}(h) \not\supseteq E^{\gamma^{**}}(h)$ for each $h \in N$. Note that such $\widehat{\mathbf{x}}$ exists, since $\gamma^* \in \mu(\Gamma^*) \cap \Gamma_L$. If $E^{\gamma^*}(h) \nsubseteq E^{\gamma'''}(h)$ for any $h \in N$, then we may choose $\pi : A \to A$ as the identity mapping. Thus, we have $E^{\gamma'''}(h) \not\supseteq E^{\gamma^{**}}(h)$ for every $h \in N$.

Consider $\widetilde{\mathbf{Q}} \in \mathcal{S}^n$ such that $((\mathbf{x}^{**}, \gamma^{**}), (\mathbf{x}''', \gamma''')) \in \widetilde{Q}_N(\mathbf{R})$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}^{**}, \gamma^{**})) \in \widetilde{Q}_N(\mathbf{R})$, and $\widetilde{Q}_h(\mathbf{R}) \cap \{(\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')\}^2 = Q_h(\mathbf{R}) \cap \{(\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')\}^2$ for all $h \in N$. Moreover, $((\mathbf{x}''', \gamma'''), (\mathbf{x}', \gamma')) \in P\left(\widetilde{Q}_d(\mathbf{R})\right)$. Then, by **SP** and **PI**, and the transitivity of $\widetilde{Q}(\mathbf{R})$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}''', \gamma''')) \in \widetilde{Q}(\mathbf{R})$ holds. Also, by **I**, $((\mathbf{x}', \gamma'), (\mathbf{x}^*, \gamma^*)) \in \widetilde{Q}(\mathbf{R})$ holds. Then, $\widetilde{Q}(\mathbf{R})$ turns out to be inconsistent, since $((\mathbf{x}''', \gamma'''), (\mathbf{x}', \gamma')) \in P\left(\widetilde{Q}(\mathbf{R})\right)$, in contradiction with the transitivity thereof. Thus, even if $\mathbf{x}^* \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ or there exists another $\mathbf{x}^{*\prime} \in \tau_{NE}(\gamma^*, \mathbf{R})$ such that $\mathbf{x}^{*\prime} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q_d(\mathbf{R}))$ cannot but imply $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R}))$. ■

Lemma 7: *Let $\mathbf{R} \in \mathcal{R}^n$ be such that there exists an individual $j \in N$ who has* *the unique maximal element* $\mathbf{x}^j$ *in* A *in terms of* R_j. *Moreover, his maximal element* $\mathbf{x}^j \in A$ *at* R_j *is uniquely worst for any other individual* $h \neq j$ at R_h. *Then, for every game form* $\gamma = (M, g)$ with $\tau_{NE}(\gamma, \mathbf{R}) \neq \varnothing$, $\tau_{NE}(\gamma, \mathbf{R}) \subsetneq A$. *In particular, either* $\tau_{NE}(\gamma, \mathbf{R}) = \{\mathbf{x}^j\}$ *or* $\mathbf{x}^j \notin \tau_{NE}(\gamma, \mathbf{R})$.

Proof. Suppose $\tau_{NE}(\gamma, \mathbf{R}) = A$, so that $\mathbf{x}^j \in \tau_{NE}(\gamma, \mathbf{R})$. By definition, there exists $\mathbf{m}^{\mathbf{x}^j} \in \epsilon_{NE}(\gamma, \mathbf{R})$ such that $g(\mathbf{m}^{\mathbf{x}^j}) = \mathbf{x}^j$. Then, for every $h \neq j$, and every $\mathbf{x} \in g\left(M_h, \mathbf{m}^{\mathbf{x}^j}_{-h}\right)$, $(\mathbf{x}^j, \mathbf{x}) \in R_h$. This implies $g\left(M_h, \mathbf{m}^{\mathbf{x}^j}_{-h}\right) = \{\mathbf{x}^j\}$ for every $h \neq j$. Then, we must have $\cup_{h \neq j} g\left(M_h, \mathbf{m}^{\mathbf{x}^j}_{-h}\right) = g\left(\mathbf{m}^{\mathbf{x}^j}_j, M_{-j}\right) = \{\mathbf{x}^j\}$. Thus, $E^\gamma(j) \supseteq \Omega(\{\mathbf{x}^j\})$.

Since $\mathbf{x}^j$ is the unique maximal element for j at R_j, $E^\gamma(j) \supseteq \Omega(\{\mathbf{x}^j\})$ implies that any other $\mathbf{x} \in A \backslash \{\mathbf{x}^j\}$ cannnot be a Nash equilibrium outcome of the game $(\gamma, \mathbf{R})$, which is a contradiction. Thus, $\tau_{NE}(\gamma, \mathbf{R}) \subsetneq A$. In particular, if $\mathbf{x}^j \in \tau_{NE}(\gamma, \mathbf{R})$, then $\tau_{NE}(\gamma, \mathbf{R}) = \{\mathbf{x}^j\}$. ■

Proof of Theorem 1: Let Ψ be an **ECF** satisfying **SP**, **PI**, and **I**.

Case 1: $\#N \geq 3$.

Take any profile $\mathbf{R} \in \mathcal{R}^n$ such that each individual $h \in N$ has the unique maximal element $\mathbf{x}^h$ in A in terms of R_h. Moreover, for this $\mathbf{R} \in \mathcal{R}^n$, there exists $j \in N$ such that his maximal element $x^j \in A$ at R_j is uniquely worst for any other individual $h \neq j$ at R_h. Then, for every h-dictatorial game form $\gamma^h \in \Gamma(h)$, $\mathbf{x}^h$ is the unique Nash equlibrium outcome of the game $\left(\gamma^h, \mathbf{R}\right)$.

First, for any $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, if $\gamma \notin \mu(\Gamma^*)$, then $\gamma \notin C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ for every $\mathbf{Q} \in \mathcal{S}^n$. This is because there exists $\gamma' \in \mu(\Gamma^*)$ such that $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$, and γ' power-dominates γ by **Lemma 2**. Moreover, by the restriction (b) of $\mathcal{S}^n$, **SP** implies the above mentioned result. Thus, $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \mu(\Gamma^*)$ for every $\mathbf{Q} \in \mathcal{S}^n$.

Second, note that $\cup_{h\in N}\Gamma(h) \subsetneq \mu(\Gamma^*)$. Let us consider the social evaluation $Q(\mathbf{R}) = \Psi(\mathbf{Q}(\mathbf{R}))$ over the set of all dictatorial game forms, viz., $\cup_{h\in N}\Gamma(h)$. By the restriction (b) of $\mathcal{S}^n$, we can regard each $\Gamma(h)$ to be an essentially singleton set in the evaluation by $Q(\mathbf{R})$. Take $\left\{(\mathbf{x}^h, \gamma^h)\right\}_{h\in N} \subseteq \Lambda(\mathbf{R})$. Since $n \geq 3$ and $\#A \geq 3$, it follows from the condition of $\mathbf{R}$ that there are at least three distinct individuals j, k, l and three distinct culmination outcomes $\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l$ such that $(\mathbf{x}^j, \gamma^j), (\mathbf{x}^k, \gamma^k), (\mathbf{x}^l, \gamma^l) \in \Lambda(\mathbf{R})$. As mentioned above, $\mathbf{x}^j$ is the unique element of $\tau_{NE}(\gamma^j, \mathbf{R})$. Likewise, $\mathbf{x}^k$ (resp. $\mathbf{x}^l$) is the unique element of $\tau_{NE}(\gamma^k, \mathbf{R})$ (resp. $\tau_{NE}(\gamma^l, \mathbf{R})$). Then, we can see that the free triple property holds among $(\mathbf{x}^j, \gamma^j)$, $(\mathbf{x}^k, \gamma^k)$, $(\mathbf{x}^l, \gamma^l)$ even in the restricted domain $\mathcal{S}^n$. Thus, by the Arrovian impossibility theorem [Arrow (1963)], there exists a (local) dictator, say i, for the social evaluation among $(\mathbf{x}^j, \gamma^j), (\mathbf{x}^k, \gamma^k), (\mathbf{x}^l, \gamma^l)$.

Suppose there exists $t \in N \setminus \{j, k, l\}$ such that $(\mathbf{x}^t, \gamma^t) \in \Lambda(\mathbf{R})$. If $\mathbf{x}^t \notin \left\{\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l\right\}$, then it is easy to show that i can extend his dictatorship to include $(\mathbf{x}^t, \gamma^t)$. Suppose $\mathbf{x}^t \in \left\{\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l\right\}$, and $((\mathbf{x}^t, \gamma^t), (\mathbf{x}^j, \gamma^j)) \in Q(\mathbf{R})$ even if $((\mathbf{x}^j, \gamma^j), (\mathbf{x}^t, \gamma^t)) \in P(Q_i(\mathbf{R}))$. Then, there still exists $\mathbf{x}' \in \left\{\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l\right\}$ such that $\mathbf{x}' \neq \mathbf{x}^j$ and $\mathbf{x}' \neq \mathbf{x}^t$. Thus $\mathbf{x}' = \mathbf{x}^k$. Consider now a profile $\widetilde{\mathbf{Q}} \in \mathcal{S}^n$ such that $\left((\mathbf{x}^k, \gamma^k), (\mathbf{x}^t, \gamma^t)\right) \in \widetilde{Q}_N(\mathbf{R})$ and $\widetilde{Q}_h(\mathbf{R}) \cap \{(\mathbf{x}^j, \gamma^j), (\mathbf{x}^t, \gamma^t)\}^2 = Q_h(\mathbf{R}) \cap \{(\mathbf{x}^j, \gamma^j), (\mathbf{x}^t, \gamma^t)\}^2$ for all $h \in N$. Moreover, $((\mathbf{x}^j, \gamma^j), (\mathbf{x}^k, \gamma^k)) \in P(\widetilde{Q}_i(\mathbf{R}))$. Thus, by **SP** and **PI**, $\left((\mathbf{x}^k, \gamma^k), (\mathbf{x}^t, \gamma^t)\right) \in \widetilde{Q}(\mathbf{R})$. Also, by **I**, $((\mathbf{x}^t, \gamma^t), (\mathbf{x}^j, \gamma^j)) \in \widetilde{Q}(\mathbf{R})$. Then, $\widetilde{Q}(\mathbf{R})$ is not consistent since $((\mathbf{x}^j, \gamma^j), (\mathbf{x}^k, \gamma^k)) \in P(\widetilde{Q}(\mathbf{R}))$. This implies that i can extend his dictatorship to include $(\mathbf{x}^t, \gamma^t)$ even when $\mathbf{x}^t \in \left\{\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l\right\}$. In this way, we can see that i is the local dictator over $\left\{(\mathbf{x}^h, \gamma^h)\right\}_{h\in N}$ under $\mathbf{R}$.

Third, consider the social evaluation $Q(\mathbf{R}) = \Psi(\mathbf{Q}(\mathbf{R}))$ over the set $\mu(\Gamma^*) \setminus [\cup_{h\in N}\Gamma(h)]$. Capitalizing on **Lemma 3**, we can find three game forms

$\gamma', \gamma'', \gamma''' \in \mu(\Gamma^*) \cap \Gamma_L$ such that $(\mathbf{x}', \gamma'), (\mathbf{x}'', \gamma''), (\mathbf{x}''', \gamma''') \in \Lambda(\mathbf{R})$, and for every $\gamma \in \{\gamma', \gamma'', \gamma'''\}$, $E^{\gamma}(h) = \Omega(\{\mathbf{x}\})$ holds for each $h \in N$, where $\mathbf{x} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$. Since $\#N \geq 3$, we can assume $\{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\} = \{\mathbf{x}^j, \mathbf{x}^k, \mathbf{x}^l\}$. Thus, by the property of each $\gamma \in \{\gamma', \gamma'', \gamma'''\}$ and the assumption on $\mathbf{R}$, each $\mathbf{x} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ is the unique Nash equilibrium outcome of the game $(\gamma, \mathbf{R})$, where $\gamma \in \{\gamma', \gamma'', \gamma'''\}$. Then, by **Lemma 6**, there exists a (local) dictator, say d, for the social evaluation among $(\mathbf{x}', \gamma')$, $(\mathbf{x}'', \gamma'')$ and $(\mathbf{x}''', \gamma''')$.

Suppose that there exists $\gamma^* \in \mu(\Gamma^*)\backslash([\cup_{h \in N}\Gamma(h)] \cup \{\gamma', \gamma'', \gamma'''\})$ satisfying $(\mathbf{x}^*, \gamma^*) \in \Lambda(\mathbf{R})$. If $\mathbf{x}^* \notin \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ and any other Nash equilibrium outcome of the game $(\gamma^*, \mathbf{R})$ does not belong to $\{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$, then it is easy to show that d can extend his dictatorship to include $(\mathbf{x}^*, \gamma^*)$, because of the free triple property. Suppose, therefore, that $\mathbf{x}^* \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ or there exists another $\mathbf{x}^{*\prime} \in \tau_{NE}(\gamma^*, \mathbf{R})$ such that $\mathbf{x}^{*\prime} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$.

Suppose $((\mathbf{x}', \gamma'), (\mathbf{x}^*, \gamma^*)) \in P(Q_d(\mathbf{R}))$. Then, by **Lemma 6**, $((\mathbf{x}', \gamma'), (\mathbf{x}^*, \gamma^*)) \in P(Q(\mathbf{R}))$ holds. Suppose that $((\mathbf{x}', \gamma'), (\mathbf{x}^*, \gamma^*)) \in Q(\mathbf{R})$ in spite of $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q_d(\mathbf{R}))$. Then, we can find in the following a new game form $\gamma^{**} \in \mu(\Gamma^*)$ such that, for each $h \in N$, $E^{\gamma'''}(h) \not\supseteq E^{\gamma^{**}}(h)$. By **Lemma 7**, either $\mathbf{x}^j \notin \tau_{NE}(\gamma^*, \mathbf{R})$ or $\{\mathbf{x}^j\} = \tau_{NE}(\gamma^*, \mathbf{R})$. If $\mathbf{x}^j \notin \tau_{NE}(\gamma^*, \mathbf{R})$, then choose $\gamma^{**} \in \mu(\Gamma^*)$ so that $E^{\gamma^{**}}(h) = \Omega(\{\mathbf{x}^{**}\})$ holds for each $h \in N$, where $\mathbf{x}^{**} = \mathbf{x}^j$. If $\{\mathbf{x}^j\} = \tau_{NE}(\gamma^*, \mathbf{R})$, then choose $\gamma^{**} \in \mu(\Gamma^*)$ so that $E^{\gamma^{**}}(h) = \Omega(\{\mathbf{x}^{**}\})$ for every $h \in N$, where $\mathbf{x}^{**} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}\backslash\{\mathbf{x}^j, \mathbf{x}'\}$. Thus, we have $E^{\gamma'''}(h) \not\supseteq E^{\gamma^{**}}(h)$ for every $h \in N$. Furthermore, $(\mathbf{x}^{**}, \gamma^*) \notin \Lambda(\mathbf{R})$ and $(\mathbf{x}^*, \gamma^{**}) \notin \Lambda(\mathbf{R})$. The latter holds true, since $\{\mathbf{x}^{**}\} = \tau_{NE}(\gamma^{**}, \mathbf{R})$ by the definition of γ^{**} and $\mathbf{R}$.

Consider $\widetilde{\mathbf{Q}} \in \mathcal{S}^n$ such that $((\mathbf{x}^{**}, \gamma^{**}), (\mathbf{x}''', \gamma''')) \in \widetilde{Q}_N(\mathbf{R})$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}^{**}, \gamma^{**})) \in \widetilde{Q}_N(\mathbf{R})$, and $\widetilde{Q}_h(\mathbf{R}) \cap \{(\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')\}^2 = Q_h(\mathbf{R}) \cap \{(\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')\}^2$

for every $h \in N$. Moreover, $((\mathbf{x}''', \gamma'''), (\mathbf{x}', \gamma')) \in P\left(\widetilde{Q}_d(\mathbf{R})\right)$. Then, by **SP** and **PI**, and by the transitivity of $\widetilde{Q}(\mathbf{R})$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}''', \gamma''')) \in \widetilde{Q}(\mathbf{R})$. Also, by **I**, $((\mathbf{x}', \gamma'), (\mathbf{x}^*, \gamma^*)) \in \widetilde{Q}(\mathbf{R})$. Then, $\widetilde{Q}(\mathbf{R})$ turns out to be inconsistent by virtue of $((\mathbf{x}''', \gamma'''), (\mathbf{x}', \gamma')) \in P\left(\widetilde{Q}(\mathbf{R})\right)$, which contradicts the transitivity thereof. Thus, even if $\mathbf{x}^* \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$ or there exists another $\mathbf{x}^{*\prime} \in \tau_{NE}(\gamma^*, \mathbf{R})$ such that $\mathbf{x}^{*\prime} \in \{\mathbf{x}', \mathbf{x}'', \mathbf{x}'''\}$, $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q_d(\mathbf{R}))$ cannnot but imply $((\mathbf{x}^*, \gamma^*), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R}))$. Thus, d can extend his dictatorship to include $(\mathbf{x}^*, \gamma^*)$.

Thus far, d is a local dictator over $\mu(\Gamma^*)\backslash[\cup_{h\in N}\Gamma^(h)]$, whereas i is a local dictator over $\cup_{h\in N}\Gamma(h)$. We will show now that $d = i$. Suppose $d \neq i$. Capitalizing on **Lemma 4** and **Lemma 5** and noting that $(\mathbf{x}^j, \gamma^j), \left(\mathbf{x}^k, \gamma^k\right) \in \Lambda(\mathbf{R})$, we can find two game forms $\gamma_*^j, \gamma_*^k \in \mu(\Gamma^*) \setminus [\cup_{h\in N}\Gamma(h)]$ such that $\mathbf{x}^h$ is the unique Nash equilibrium outcome of the game $(\gamma_*^h, \mathbf{R})$ for each $h \in \{j, k\}$. Now, consider $(\mathbf{x}^j, \gamma^j), (\mathbf{x}^j, \gamma_*^j), (\mathbf{x}^k, \gamma_*^k), \left(\mathbf{x}^k, \gamma^k\right) \in \Lambda(\mathbf{R})$. Take $\mathbf{Q} \in \mathcal{S}^n$ such that $\left(\left(\mathbf{x}^k, \gamma^k\right), (\mathbf{x}^j, \gamma^j)\right) \in P(Q_i(\mathbf{R}))$, $\left(\left(\mathbf{x}^k, \gamma_*^k\right), (\mathbf{x}^j, \gamma_*^j)\right) \in P(Q_d(\mathbf{R}))$, and $\left((\mathbf{x}^j, \gamma_*^j), \left(\mathbf{x}^k, \gamma^k\right)\right), \left((\mathbf{x}^j, \gamma^j), \left(\mathbf{x}^k, \gamma_*^k\right)\right) \in Q_N(\mathbf{R})$. By virtue of the free triple property, we can find such a profile. Then, $\left(\left(\mathbf{x}^k, \gamma^k\right), (\mathbf{x}^j, \gamma^j)\right) \in P(Q(\mathbf{R}))$, $\left(\left(\mathbf{x}^k, \gamma_*^k\right), (\mathbf{x}^j, \gamma_*^j)\right) \in P(Q(\mathbf{R}))$, and $\left((\mathbf{x}^j, \gamma_*^j), \left(\mathbf{x}^k, \gamma^k\right)\right), \left((\mathbf{x}^j, \gamma^j), \left(\mathbf{x}^k, \gamma_*^k\right)\right) \in Q(\mathbf{R})$ by **SP** and **PI**. Thus, $Q(\mathbf{R})$ cannot be consistent, a contradiction. Thus, $d = i$, so that d can extend his dictatorship over $\mu(\Gamma^*)$. Hence $C_\Psi(Q; \mathbf{R}) = C_\Psi(Q_d; \mathbf{R})$ for every $\mathbf{Q} \in \mathcal{S}^n$, where $Q = \Psi(\mathbf{Q})$.

By virtue of **Lemma 4** and **Lemma 5**, for every $\mathbf{Q} \in \mathcal{S}^n$, $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \cap \Gamma(h) = \varnothing$ for every $h \neq d$. In fact, if $\left(\mathbf{x}^h, \gamma^h\right) \in \Lambda(\mathbf{R})$, where $\gamma^h \in \Gamma(h)$, then **Lemma 4** and **Lemma 5** guarantee the existence of $\gamma_*^h \in \mu(\Gamma^*) \backslash \Gamma(h)$ such that $\left(\mathbf{x}^h, \gamma_*^h\right) \in \Lambda(\mathbf{R})$ and $E^{\gamma_*^h}(d) \neq \{A\}$. Thus, $\left(\left(\mathbf{x}^h, \gamma_*^h\right), \left(\mathbf{x}^h, \gamma^h\right)\right) \in P(Q_d(\mathbf{R}))$ for every $\mathbf{Q} \in \mathcal{S}^n$, which implies $\left(\left(\mathbf{x}^h, \gamma_*^h\right), \left(\mathbf{x}^h, \gamma^h\right)\right) \in P(Q(\mathbf{R}))$

for every $\mathbf{Q} \in \mathcal{S}^n$. We can also verify that, for every $\gamma^* \in \mu(\Gamma^*)$ such that $E^{\gamma^*}(d) = \{A\}$, $\gamma^* \notin C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ holds for every $\mathbf{Q} \in \mathcal{S}^n$.

Finally, we must show that if Ψ satisfies **URC**, then $\Gamma(d) \subseteq \cap_{\widetilde{\mathbf{R}} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \widetilde{\mathbf{R}})$ for every $\mathbf{Q} \in \mathcal{S}^n$. Note that, for some $\mathbf{Q}' \in \mathcal{S}^n$, we have $C_\Psi(\Psi(\mathbf{Q}'); \mathbf{R}) = \Gamma(d)$. It follows from **URC** that $\cap_{\widetilde{\mathbf{R}} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}'); \tilde{\mathbf{R}}) = \Gamma(d)$. Let us take any other profile $\mathbf{R}' \in \mathcal{R}^n$ such that every individual h has the unique maximal element $\mathbf{x}'^h$ in A with respect to R'_h. Similarly, there exists a local $\mathbf{R}'$-dictator $d' \in N$ over $\mu(\Gamma^*)$ such that $C_\Psi(\Psi(\mathbf{Q}''); \mathbf{R}') = \Gamma(d')$ for some $\mathbf{Q}'' \in \mathcal{S}^n$. Then, by **URC**, $\cap_{\widetilde{\mathbf{R}} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}''); \tilde{\mathbf{R}}) = \Gamma(d')$. In particular, we have $C_\Psi(\Psi(\mathbf{Q}''); \mathbf{R}) \supseteq \Gamma(d')$. Since $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \cap \Gamma(h) = \varnothing$ for every $\mathbf{Q} \in \mathcal{S}^n$, and every $h \neq d$, it follows that $d' = d$.

Recollect that the profile $\mathbf{R}^0 \in \mathcal{R}^n$ is such that every alternative in A becomes a Nash equilibrium outcome in every game form with this preference profile. By **URC**, $C_\Psi(\Psi(\mathbf{Q}'); \mathbf{R}^0) \cap \Gamma(d) \neq \varnothing$, which implies $\Gamma(d) \subseteq C_\Psi(\Psi(\mathbf{Q}'); \mathbf{R}^0)$. Note that, for every $(\mathbf{x}, \gamma^d), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R}^0)$, where $\gamma^d \in \Gamma(d)$, $\mathbf{Q}(\mathbf{R}^0) \cap \{(\mathbf{x}, \gamma^d), (\mathbf{x}', \gamma')\}^2 = \mathbf{Q}'(\mathbf{R}^0) \cap \{(\mathbf{x}, \gamma^d), (\mathbf{x}', \gamma')\}^2$ holds for every $\mathbf{Q} \in \mathcal{S}^n$. Thus, by **I**, $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}^0) \cap \Gamma(d) = C_\Psi(\Psi(\mathbf{Q}'); \mathbf{R}^0) \cap \Gamma(d)$ for every $\mathbf{Q} \in \mathcal{S}^n$. Thus, by **URC**, $\Gamma(d) \subseteq \cap_{\tilde{\mathbf{R}} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \tilde{\mathbf{R}})$ for every $\mathbf{Q} \in \mathcal{S}^n$. Moreover, we obtain $\cap_{\mathbf{Q} \in \mathcal{S}^n} \cap_{\widetilde{\mathbf{R}} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}), \widetilde{\mathbf{R}}) = \Gamma(d)$ by the existence of $\mathbf{Q}'$.

Case 2: $\#N = 2$.

Take any profile $\mathbf{R} \in \mathcal{R}^n$ such that $(\mathbf{x}^1, \mathbf{x}^3), (\mathbf{x}^3, \mathbf{x}^2) \in P(R_1)$ and $(\mathbf{x}^2, \mathbf{x}^3)$, $(\mathbf{x}^3, \mathbf{x}^1) \in P(R_2)$. Then, for every $h \in N = \{1, 2\}$, and every h-dictatorial game form $\gamma^h \in \Gamma(h)$, $\mathbf{x}^h$ is the unique Nash equlibrium outcome of the game $(\gamma^h, \mathbf{R})$. Let $\gamma^3 \in \mu(\Gamma^*) \setminus [\cup_{h \in N} \Gamma(h)]$ be such that $E^{\gamma^3}(h) = \Omega(\{\mathbf{x}^3\})$ for each $h \in N$. Note that for γ^3, $\tau_{NE}(\gamma^3, \mathbf{R}) = \{\mathbf{x}^3\}$, since for any strategy profile which attains $\mathbf{x}^1$, the individual 2 has another strategy to change the social outcome from $\mathbf{x}^1$ to $\mathbf{x}^3$. The same is true for $\mathbf{x}^2$. Thus, the triple $(\mathbf{x}^1, \gamma^1), (\mathbf{x}^2, \gamma^2), (\mathbf{x}^3, \gamma^3) \in \Lambda(\mathbf{R})$ has a free triple property even within $\Delta_\Psi = \mathcal{S}^n$. This implies that, by the Arrovian impossibility theorem [Arrow (1963)], there exists a (local) dictator, say i, for the social evaluation among $(\mathbf{x}^1, \gamma^1), (\mathbf{x}^2, \gamma^2), (\mathbf{x}^3, \gamma^3)$.

Next, capitalizing on **Lemma 4** and **Lemma 5** and noting that $(\mathbf{x}^1, \gamma^1), (\mathbf{x}^2, \gamma^2) \in \Lambda(\mathbf{R})$, we can find two game forms $\gamma_*^1, \gamma_*^2 \in \mu(\Gamma^*) \setminus [\cup_{h \in N} \Gamma(h)]$ such that each $\mathbf{x}^h$ is the unique Nash equilibrium outcome of the game $(\gamma_*^h, \mathbf{R})$ for each $h \in N$. In particular, $E^{\gamma_*^1}(h) = \Omega(\{\mathbf{x}^1\})$ and $E^{\gamma_*^2}(h) = \Omega(\{\mathbf{x}^2\})$ hold for each $h \in N$. Then, since $(\mathbf{x}^1, \gamma_*^1), (\mathbf{x}^2, \gamma_*^2), (\mathbf{x}^3, \gamma^3)$ constitute a free triple, there exists a (local) dictator, say d, for the social evaluation among $(\mathbf{x}^1, \gamma_*^1), (\mathbf{x}^2, \gamma_*^2), (\mathbf{x}^3, \gamma^3)$. Since the two free triples overlap at $(\mathbf{x}^3, \gamma^3)$, we are assured that $d = i$.

Suppose that there exists $\gamma^* \in \mu(\Gamma^*) \setminus [(\cup_{h \in N} \Gamma(h)) \cup \{\gamma_*^1, \gamma_*^2, \gamma^3\}]$ satisfying $(\mathbf{x}^*, \gamma^*) \in \Lambda(\mathbf{R})$. Then, by **Lemma 6**, d can extend his dictatorship to include $(\mathbf{x}^*, \gamma^*)$. The remaining argument can be developed in the same way as in the case where $\#N \geq 3$. ■

A.2 Proofs of Theorem 2 and Theorem 3

Lemma 8: *Given $i \in N$ and $\mathbf{Q} \in \mathcal{S}^n$, let $\{j\} \in \mathcal{N}_i(\mathbf{Q})$. Then, for every $(\gamma, \gamma') \in \Gamma^p(i) \times \Gamma^u(i)$ with $E^{\gamma'}(j) = \{A\}$, every $\mathbf{R} \in \mathcal{R}^n$, and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q_j(\mathbf{R}))$ holds true.*

Proof. Let a profile $\mathbf{R}^0 \in \mathcal{R}^n$ be such that every individual is universally indifferent over A. Then, for every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in A \times \Gamma^*$, $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R}^0)$. Moreover, if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$ with $E^{\gamma'}(j) = \{A\}$, then

$((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q_j(\mathbf{R}^0)\right)$. This follows from $((\mathbf{x},\gamma),(\mathbf{x},\gamma'))\,,((\mathbf{x}',\gamma),(\mathbf{x}',\gamma'))$ $\in P\left(Q_j(\mathbf{R}^0)\right)$ and $((\mathbf{x},\gamma'),(\mathbf{x}',\gamma')) \in I\left(Q_j(\mathbf{R}^0)\right)$ by the property of $\mathcal{S}^n$ and the transitivity of $Q_j(\mathbf{R}^0)$. Thus, by the condition **(n-c)**, for every $\mathbf{R} \in \mathcal{R}^n$, if $(\mathbf{x},\gamma),(\mathbf{x}',\gamma') \in \Lambda(\mathbf{R})$, then $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P(Q_j(\mathbf{R}))$. ■

Proof of Theorem 2: If $\Delta_\Psi = \mathcal{S}^n$, there exists an individual $i \in N$ who is decisive over the whole set of extended alternatives. Let $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}}\setminus\{\varnothing\}$. For every $\mathbf{R} \in \mathcal{R}^n$, every $\mathbf{Q} \in \Delta_\Psi$, and every $(\mathbf{x},\gamma),(\mathbf{x}',\gamma') \in \Lambda(\mathbf{R})$, define Ψ as follows:

(i) if $\gamma,\gamma' \in \Gamma^p(i)$, then $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q_i(\mathbf{R})\right) \cup P\left(Q_N(\mathbf{R})\right)$, and $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in I\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in I\left(Q_i(\mathbf{R})\right)\setminus P\left(Q_N(\mathbf{R})\right)$;

(ii) if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$, $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q(\mathbf{R})\right)$; and

(iii) if $\gamma,\gamma' \in \Gamma^u(i)$, then $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q_j(\mathbf{R})\right) \cup P\left(Q_N(\mathbf{R})\right)$, and $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in I\left(Q(\mathbf{R})\right) \Leftrightarrow ((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in I\left(Q_j(\mathbf{R})\right)\setminus P\left(Q_N(\mathbf{R})\right)$ for some $\{j\} \in \cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q})$, where $Q = \Psi(\mathbf{Q})$.

Note that (i), (ii) and (iii) are mutually exclusive and jointly exhaustive. The above $Q(\mathbf{R})$ is complete, and has a two-tier structure. It is also an ordering. We must examine whether or not the part (ii) is consistent with the Arrovian four conditions and the domain restrictions. For every $(\mathbf{x},\gamma),(\mathbf{x}',\gamma') \in \Lambda(\mathbf{R})$, if $\gamma \in \Gamma^p(i)$ and $\gamma' \in \Gamma^u(i)$, then by the condition $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N\setminus\{i\}}\setminus\{\varnothing\}$ and **Lemma 8**, there exists at least one individual $j \in N\setminus\{i\}$ such that $E^{\gamma'}(j) = \{A\}$ and $((\mathbf{x},\gamma),(\mathbf{x}',\gamma')) \in P\left(Q_j(\mathbf{R})\right)$. Thus, the part (ii) is consistent with **SP** and **PI**.

By construction, it is easy to verify that Ψ satisfies **SP**, **PI**, **I**, and **ND**. Moreover, $\cup_{\mathbf{Q}\in\Delta_\Psi}\cup_{\mathbf{R}\in\mathcal{R}^n} C_\Psi(Q;\mathbf{R}) \subseteq \Gamma^p(i)$. Note that $\Gamma^p(i)$ contains a game form γ with $E^{\gamma}(i) = \{A\}$. However, such a game form cannot be rationally chosen, since, for every $\mathbf{R} \in \mathcal{R}^n$, if $(\mathbf{x}, \gamma) \in \Lambda(\mathbf{R})$, then there exists another game form $\gamma' \in \Gamma^p(i)$ with $E^{\gamma'}(i) \neq \{A\}$ such that $(\mathbf{x}, \gamma') \in \Lambda(\mathbf{R})$. This is guaranteed by **Lemma 4**. Thus, by the restriction (b) of $\mathcal{S}^n$, $((\mathbf{x}, \gamma'), (\mathbf{x}, \gamma)) \in P(Q_i(\mathbf{R}))$, which implies $((\mathbf{x}, \gamma'), (\mathbf{x}, \gamma)) \in P(Q(\mathbf{R}))$. In summary, we have:

$$\gamma \in \cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \Rightarrow \ \forall j \in N, \exists B^j \in E^{\gamma}(j) \text{ s.t. } B^j \neq A.$$

Thus, $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_L$. Note that every $\gamma \in \cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ also represents a maximal power-structure. In fact, **Lemma 2**, the restriction (b) of $\mathcal{S}^n$ and **SP** together guarantee that, for every $\gamma \notin \mu(\Gamma^*)$, $\gamma \notin \cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ holds. Thus, $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \mu(\Gamma^*)$. By **Lemma 1**, there exists a liberal game form in $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ for any $\mathbf{Q} \in \Delta_\Psi$ and any $\mathbf{R} \in \mathcal{R}^n$, which is Nash-solvable and efficient. ■

Proof of Theorem 3: If $\Delta_\Psi = \mathcal{S}^n$, there exists an individual $i \in N$ who is decisive over the whole set of extended alternatives.

Let $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{N}_i(\mathbf{Q}) \supseteq 2^{N \setminus \{i\}} \setminus \{\varnothing\}$ and $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{M}_i(\mathbf{Q})$. For every $\mathbf{R} \in \mathcal{R}^n$, every $\mathbf{Q} \in \Delta_\Psi$, and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, define Ψ as in the proof of **Theorem 2**. Then, our only task is to examine the uniform rationalizability by means of $Q = \Psi(\mathbf{Q})$. By construction, if $\gamma, \gamma' \notin \Gamma^u(i)$, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q(\mathbf{R}) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in Q_i(\mathbf{R})$. Also, $C_\Psi(Q; \mathbf{R}) \subseteq \mu(\Gamma^*) \setminus [\cup_{h \in N} \Gamma^u(h)]$. Take $\mathbf{R}^0 \in \mathcal{R}^n$ in which every individual is universally indifferent over A. Then, $C_\Psi(Q; \mathbf{R}^\mathbf{0})$ is identified. By the domain restriction (b) of $\mathcal{S}^n$, $C_\Psi(Q; \mathbf{R}^\mathbf{0}) = C_\Psi(Q_i; \mathbf{R}^\mathbf{0}) \subseteq \mu(\Gamma^*)$. Thus, it consists of Nash-solvable and efficient game forms. Since $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{M}_i(\mathbf{Q})$ for all $\mathbf{R} \in \mathcal{R}^n$, $C_\Psi(Q_i; \mathbf{R})$ remains invariant, which implies that $\cap_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(Q; \mathbf{R}) \neq \varnothing$. By the construction of Ψ, any dictatorial game form cannot be rationally chosen. ■

A.3 Proofs of Theorem 4 and Theorem 5

Proof of Theorem 4:

Proof of Sufficiency: For every $\mathbf{R} \in \mathcal{R}^n$, every $\mathbf{Q} \in \Delta_\Psi$, and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, define Ψ as follows:

(I) if $\gamma, \gamma' \in \Gamma_L$, then $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R})) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q_i(\mathbf{R})) \cup P(Q_N(\mathbf{R}))$, and $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I(Q(\mathbf{R})) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I(Q_i(\mathbf{R})) \backslash P(Q_N(\mathbf{R}))$;

(II) if $\gamma \in \Gamma_L$ and $\gamma' \notin \Gamma_L$, $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R}))$;

(III) otherwise, $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q(\mathbf{R})) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q_j(\mathbf{R})) \cup P(Q_N(\mathbf{R}))$, and $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I(Q(\mathbf{R})) \Leftrightarrow ((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in I(Q_j(\mathbf{R})) \backslash P(Q_N(\mathbf{R}))$ for some $j \neq i$,

where $Q = \Psi(\mathbf{Q})$.

This $Q(\mathbf{R})$ is an ordering. Also, since $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$, $Q \in \mathcal{Q}^{\mathbf{NCL}}$ holds. We must examine whether or not the part (II) is consistent with the Arrovian four conditions and the domain restrictions. For every $\mathbf{R} \in \mathcal{R}^n$ and every $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, if $\gamma \in \Gamma_L$ and $\gamma' \notin \Gamma_L$, then by the condition $\cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{K}(\mathbf{Q}) = 2^N \backslash \{\varnothing\}$, there exists at least one individual $j \in N$ such that $Q_j \in \mathcal{S}$; for some $S \subseteq N$ with $j \in S$, $E^{\gamma'}(S) = \{A\}$; and $((\mathbf{x}, \gamma), (\mathbf{x}', \gamma')) \in P(Q_j(\mathbf{R}))$. The last equation can be obtained by the (**n-c**) condition of Q_j and **Lemma 8**. Thus, the part (II) is consistent with **SP** and **PI**.

Let $\Gamma_L^* \equiv \Gamma_L \cap \mu(\Gamma^*)$. By construction, it is easy to see that Ψ satisfies **SP**, **PI**, **I**, and **ND**. Moreover, from the property (I) of Ψ, $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$, and by virtue of the restrictions (a) and (b) of $\mathcal{F}$, we have $\cup_{\mathbf{Q} \in \Delta_\Psi} \cup_{\mathbf{R} \in \mathcal{R}^n} C_\Psi(\Psi(\mathbf{Q}); \mathbf{R}) \subseteq \Gamma_L^*$, which implies that there exists $\gamma \in C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ for any $\mathbf{Q} \in \Delta_\Psi$ and any $\mathbf{R} \in \mathcal{R}^n$, which is a Nash-solvable, efficient, and liberal game form by **Lemma 1.** The non-emptiness of $C_\Psi(\Psi(\mathbf{Q}); \mathbf{R})$ is guaranteed by the restriction (a) of $\mathcal{F}$ and the condition **(n-c)** on $Q = \Psi(\mathbf{Q})$. This is because, by these conditions, Q evaluates the wellness of alternative liberal game forms on the basis of the structures of their corresponding α-effectivity functions, the class of which being finite by virtue of the finiteness of A.

Proof of Necessity: First, we show the necessity of $\cap_{\mathbf{Q}\in\Delta_\Psi}\mathcal{K}(\mathbf{Q}) = 2^N \setminus \{\varnothing\}$. Suppose that there exists $\mathbf{Q} \in \Delta_\Psi$ such that, for some $S \subseteq N$, for some $(\gamma, \gamma') \in \Gamma_L \times (\Gamma^* \backslash \Gamma_L)$ with $E^{\gamma'}(S) = \{A\}$, for some $\mathbf{R} \in \mathcal{R}^n$, and for some $(\mathbf{x}, \gamma), (\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $((\mathbf{x}', \gamma'), (\mathbf{x}, \gamma)) \in Q_j(\mathbf{R})$ holds for every $j \in S$ with $Q_j \in \mathcal{T}$. Then it follows that $\mathbf{x} \neq \mathbf{x}'$ by the fact that $Q_j \in \mathcal{T}$ satisfies the restriction (b) of $\mathcal{S}$.

Given the above two game forms γ and γ', we can consider the following two cases. The first case is that there exists at least one individual $j^* \in S$ such that, for some $B \in E^\gamma(j^*)$, either $B \subsetneq A \backslash \{\mathbf{x}\}$ or $\mathbf{x} \in B \subsetneq A$ holds; the second case is that, for every $j \in S$, if $B \in E^\gamma(j^*)$ implies $B \subsetneq A$, then $B = A \backslash \{\mathbf{x}\}$. For each of these two cases, we can construct $(\gamma^*, \gamma^{*\prime}) \in \Gamma_L \times (\Gamma^* \backslash \Gamma_L)$ with $E^{\gamma^*}(j^*) \not\supseteq E^\gamma(j^*)$ and $E^{\gamma^{*\prime}}(j^*) = \{A\}$ for some $j^* \in S$.

Consider the first case. Then, $\gamma^* = (M^*, g^*) \in \Gamma_L$ is defined as follows: let $M^*_{j^*} = M_{j^*}$ for j^*; $M^*_h = M_h \cup \{\mathbf{x}\}$ for any $h \neq j^*$; and for any $\mathbf{m} \in M^*$,

$$g^*(\mathbf{m}) = \begin{cases} g(\mathbf{m}) & \text{if } \mathbf{m} \in M; \\ \mathbf{x} & \text{otherwise.} \end{cases}$$

Also, $\gamma^{*\prime} = (M^{*\prime}, g^{*\prime}) \in \Gamma^* \backslash \Gamma_L$ is defined as follows: let $M^{*\prime}_{j^*} = M'_{j^*}$ for j^*; $M^{*\prime}_h = M'_h \cup \{\mathbf{x}'\}$ for any $h \neq j^*$; and for any $\mathbf{m} \in M^{*\prime}$,

$$g^{*\prime}(\mathbf{m}) = \begin{cases} g'(\mathbf{m}) & \text{if } \mathbf{m} \in M'; \\ \mathbf{x} & \text{otherwise.} \end{cases}$$

Then, $(\mathbf{x}, \gamma^*), (\mathbf{x}', \gamma^{*\prime}) \in \Lambda(\mathbf{R})$. Moreover, $E^{\gamma^*}(j^*) \subseteq E^\gamma(j^*)$ and $E^{\gamma^{*\prime}}(j^*) = \{A\}$.

Next, consider the second case. Let $\mathbf{m}^{\mathbf{x}} \in \epsilon_{NE}(\gamma, \mathbf{R})$ be such that $g(\mathbf{m}^{\mathbf{x}}) = \mathbf{x}$. Then, for every individual $j \in S$, $g\left(m_j^{\mathbf{x}}, M_{-j}\right) = A$. Take some $j^* \in S$ freely, and let $\widetilde{B} \equiv A \backslash \{\mathbf{x}''\}$ for some $\mathbf{x}'' \in A \backslash \{\mathbf{x}, \mathbf{x}'\}$. Define $\gamma^* = (M^*, g^*) \in \Gamma_L$ as follows. Let $M^*_{j^*} = M_{j^*} \cup \left\{\widetilde{B}\right\}$ for j^*; $M^*_h = M_h \cup \{\mathbf{x}\}$ for any $h \neq j^*$; and for any $\mathbf{m} \in M^*$,

$$g^*(\mathbf{m}) = \begin{cases} g(\mathbf{m}) & \text{if } \mathbf{m} \in M \\ \mathbf{x}^{\left(\widetilde{B}, \mathbf{m}_{-j^*}\right)} & \text{if } m_{j^*} = \widetilde{B} \text{ and } \mathbf{m}_{-j^*} \in M_{-j^*} \\ \mathbf{x} & \text{otherwise,} \end{cases}$$

where $\mathbf{x}^{\left(\widetilde{B}, \mathbf{m}_{-j^*}\right)}$ is such that $\left\{\mathbf{x}^{\left(\widetilde{B}, \mathbf{m}_{-j^*}\right)}\right\} = \widetilde{B} \cap g\left(m_j^{\mathbf{x}}, \mathbf{m}_{-j^*}\right)$ if $\widetilde{B} \cap g\left(m_j^{\mathbf{x}}, \mathbf{m}_{-j^*}\right) \neq \varnothing$, while $\mathbf{x}^{\left(\widetilde{B}, \mathbf{m}_{-j^*}\right)} = \mathbf{x}$ if $\widetilde{B} \cap g\left(m_j^{\mathbf{x}}, \mathbf{m}_{-j^*}\right) = \varnothing$. Then, $\widetilde{\mathbf{m}} = \left(\widetilde{B}, \widetilde{\mathbf{m}}_{-j^*}\right) \in M^*$ with $\widetilde{m}_h = \mathbf{x}$ for any $h \neq j^*$ constitutes a Nash equilibrium in the game $(\gamma^*, \mathbf{R})$ with $g^*(\widetilde{\mathbf{m}}) = \mathbf{x}$. Moreover, $E^{\gamma^*}(j^*) = \Omega(A \backslash \{\mathbf{x}''\})$, which implies $E^{\gamma^*}(j^*) \not\supseteq E^{\gamma}(j^*)$ and $E^{\gamma^*}(j^*) \not\subseteq E^{\gamma}(j^*)$.

Thus, in both of the above two cases, since $E^{\gamma^{*\prime}}(j^*) = \{A\}$, the restriction (b) of $\mathcal{T}$ implies that $((\mathbf{x}', \gamma'), (\mathbf{x}', \gamma^{*\prime})) \in I(Q_{j^*}(\mathbf{R}))$ for $j^* \in S$. Moreover, $\gamma, \gamma^* \in \Gamma_L$ implies that there exists $Q'_{j^*} \in \mathcal{T}$ such that $((\mathbf{x}, \gamma), (\mathbf{x}, \gamma^*)) \in Q'_{j^*}(\mathbf{R})$. Since $((\mathbf{x}', \gamma'), (\mathbf{x}', \gamma^{*\prime})) \in I\left(Q'_{j^*}(\mathbf{R})\right)$, we conclude that $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'_{j^*}(\mathbf{R})$ for $(\gamma^*, \gamma^{*\prime}) \in \Gamma_L \times (\Gamma^* \backslash \Gamma_L)$. Note that $E^{\gamma^*}(h) = \Omega(\{\mathbf{x}\})$ and $E^{\gamma^{*\prime}}(h) = \Omega(\{\mathbf{x}'\})$ for every $h \neq j^*$. Thus, $E^{\gamma^*}(h) \not\supseteq E^{\gamma^{*\prime}}(h)$ and $E^{\gamma^*}(h) \not\subseteq E^{\gamma^{*\prime}}(h)$ for every $h \neq j^*$. This implies that, for every $h \neq j^*$, there exists $Q'_h \in \mathcal{T}$ such that $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'_h(\mathbf{R})$. Thus, we can show that there exists a profile $\mathbf{Q}' \in \Delta_\Psi$ such that, for any Ψ satisfying the Arrovian four conditions, $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'(\mathbf{R})$, where $Q' = \Psi(\mathbf{Q}')$. This implies $Q' \notin \mathcal{Q}^L$.

In this case, we can show that if $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'(\mathbf{R})$ (resp. $P(Q'(\mathbf{R}))$), then $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'_i(\mathbf{R})$ (resp. $P(Q'_i(\mathbf{R}))$). Suppose $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'(\mathbf{R})$, but $((\mathbf{x}, \gamma^*), (\mathbf{x}', \gamma^{*\prime})) \in P(Q'_i(\mathbf{R}))$. Let us take a game form $\gamma'' \in \Gamma^*_L$

such that, for every $h \in N$, $E^{\gamma''}(h) = \Omega(\{\mathbf{x}''\})$. By **Lemma 3**, we can find such a game form. Since $\gamma^*, \gamma'' \in \Gamma_L$, we can assume that $((\mathbf{x}'', \gamma''), (\mathbf{x}, \gamma^*)) \in I(Q'_h(\mathbf{R}))$ for every $h \in N$. Thus, $((\mathbf{x}'', \gamma''), (\mathbf{x}, \gamma^*)) \in I(Q'(\mathbf{R}))$ by **PI**. Let us also take a game form $\widehat{\gamma}^{*\prime} \in \Gamma_L^*$ such that, for every $h \in N$, $E^{\widehat{\gamma}^{*\prime}}(h) = \Omega(\{\mathbf{x}'\})$. By **Lemma 3**, we can find such a game form. Then, $(\mathbf{x}', \widehat{\gamma}^{*\prime}) \in \Lambda(\mathbf{R})$, and $\left((\mathbf{x}', \widehat{\gamma}^{*\prime}), (\mathbf{x}', \gamma^{*\prime})\right) \in P(Q'(\mathbf{R}))$ by **SP**. Now, let us assume that $\left((\mathbf{x}'', \gamma''), (\mathbf{x}', \widehat{\gamma}^{*\prime})\right) \in P(Q'_i(\mathbf{R}))$. Note that, by **Lemma 6**, this individual i can be a dictator over $\left\{(\mathbf{x}'', \gamma''), (\mathbf{x}', \widehat{\gamma}^{*\prime})\right\}$, so that $\left((\mathbf{x}'', \gamma''), (\mathbf{x}', \widehat{\gamma}^{*\prime})\right) \in P(Q'(\mathbf{R}))$. Then, $Q'(\mathbf{R})$ turns out to be inconsistent. Thus, $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'(\mathbf{R})$ implies $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'_i(\mathbf{R})$. The case of strict part can be treated in a similar way.

Next, we can show that, for every $(\widetilde{\mathbf{x}}, \widetilde{\gamma}) \in \Lambda(\mathbf{R})$ with $\widetilde{\gamma} \in \Gamma_L^*$, $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'_i(\mathbf{R}))$ implies $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'(\mathbf{R}))$. Suppose $((\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\mathbf{x}', \gamma^{*\prime})) \in Q'(\mathbf{R})$ in spite of $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'_i(\mathbf{R}))$. Let $((\mathbf{x}'', \gamma''), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'_i(\mathbf{R}))$ and $((\mathbf{x}'', \gamma''), (\mathbf{x}, \gamma^*)) \in I(Q'_h(\mathbf{R}))$ for every $h \in N$. Then, by **Lemma 6**, $((\mathbf{x}'', \gamma''), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'(\mathbf{R}))$, and $((\mathbf{x}'', \gamma''), (\mathbf{x}, \gamma^*)) \in I(Q'(\mathbf{R}))$ by **PI**. Thus, whenever $((\mathbf{x}', \gamma^{*\prime}), (\mathbf{x}, \gamma^*)) \in Q'(\mathbf{R})$, $Q'(\mathbf{R})$ turns out to be inconsistent. Thus, for any $(\widetilde{\mathbf{x}}, \widetilde{\gamma}) \in \Lambda(\mathbf{R})$ with $\widetilde{\gamma} \in \Gamma_L^*$, $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'_i(\mathbf{R}))$ implies $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'(\mathbf{R}))$.

Let $(\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \Lambda(\mathbf{R})$ with $\widetilde{\gamma}' \in \mu(\Gamma^*) \setminus \Gamma_L$ be an extended alternative such that $((\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\mathbf{x}', \gamma^{*\prime})) \in Q(\mathbf{R})$. Let the set of such alternatives be denoted by $\mathcal{U}((\mathbf{x}', \gamma^{*\prime}), Q(\mathbf{R}); \Gamma_L^{*c})$. Then, for every $(\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \Lambda(\mathbf{R})$ with $(\widetilde{\gamma}, \widetilde{\gamma}') \in \Gamma_L^* \times (\mu(\Gamma^*) \setminus \Gamma_L)$, if $(\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \mathcal{U}((\mathbf{x}', \gamma^{*\prime}), Q'(\mathbf{R}); \Gamma_L^{*c})$ and $((\mathbf{x}', \gamma^{*\prime}), (\widetilde{\mathbf{x}}, \widetilde{\gamma})) \in P(Q'_i(\mathbf{R}))$, then $P(Q'_i(\mathbf{R})) \cap \{(\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\}^2 = P(Q'(\mathbf{R})) \cap \{(\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\}^2$. Thus, by **I**, for every $(\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \Lambda(\mathbf{R})$ with $(\widetilde{\gamma}, \widetilde{\gamma}') \in \Gamma_L^* \times (\mu(\Gamma^*) \setminus \Gamma_L)$, if $(\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \mathcal{U}((\mathbf{x}', \gamma^{*\prime}), Q'(\mathbf{R}); \Gamma_L^{*c})$, then $\left((\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\right) \in P(Q'_i(\mathbf{R}))$ implies

$\left((\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\right) \in P\left(Q'(\mathbf{R})\right)$. Moreover, by **I**, for every $\mathbf{Q} \in \Delta_\Psi$ and every $(\widetilde{\mathbf{x}}, \widetilde{\gamma}), (\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \Lambda(\mathbf{R})$ with $(\widetilde{\gamma}, \widetilde{\gamma}') \in \Gamma_L^* \times (\mu\left(\Gamma^*\right) \backslash \Gamma_L)$, $\left((\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\right) \in P\left(Q_i(\mathbf{R})\right)$ implies $\left((\widetilde{\mathbf{x}}', \widetilde{\gamma}'), (\widetilde{\mathbf{x}}, \widetilde{\gamma})\right) \in P\left(Q(\mathbf{R})\right)$, whenever $(\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \mathcal{U}((\mathbf{x}', \gamma^{*\prime}), Q''(\mathbf{R}); \Gamma_L^{*c})$ for some $\mathbf{Q}'' \in \Delta_\Psi$.

Let us construct a game form $\widetilde{\gamma}' \in \mu\left(\Gamma^*\right) \backslash \Gamma_L$ as $(\widetilde{\mathbf{x}}', \widetilde{\gamma}') \in \mathcal{U}((\mathbf{x}', \gamma^{*\prime}), Q''(\mathbf{R}); \Gamma_L^{*c})$ for some $\mathbf{Q}'' \in \Delta_\Psi$. Consider the case of $\# N \geq 3$ and $\#\left(N \backslash S\right) \geq 2$. Choose $i, k \in N \backslash S$, and define a game form $\widetilde{\gamma}' = \left(\widetilde{M}', \widetilde{g}'\right) \in \mu\left(\Gamma^*\right) \backslash \Gamma_L$ as follows: let $\widetilde{M}_i' = \{\{\mathbf{x}'\}, A \backslash \{\mathbf{x}'\}\}$; $\widetilde{M}_k' = \{\{\widetilde{\mathbf{x}}, \mathbf{x}'\} \mid \exists\, \widetilde{\mathbf{x}} \in A \backslash \{\mathbf{x}'\}\}$; $\widetilde{M}_h' = \{A\}$ for any other $h \neq i, k$; and for any $\mathbf{m} \in \widetilde{M}'$, $\widetilde{g}'\left(\mathbf{m}\right) = \widehat{\mathbf{x}}$ where $\{\widehat{\mathbf{x}}\} \equiv \cap_{h \in N} m_h$. Then, since $(\mathbf{x}', \gamma') \in \Lambda(\mathbf{R})$, $(\mathbf{x}', \widetilde{\gamma}') \in \Lambda(\mathbf{R})$ holds. Consider the case of $\# N = 2$ or $\#\left(N \backslash S\right) = 1$. Then, choose $i \in N \backslash S$, and take the i-dictatorial game form γ^i as $\widetilde{\gamma}' \in \mu\left(\Gamma^*\right) \backslash \Gamma_L$. In this case, since $\mathbf{x}'$ is a maximal element for i at R_i, $(\mathbf{x}', \gamma^i) \in \Lambda(\mathbf{R})$ holds. Note that, since $\widetilde{\gamma}' \in \mu\left(\Gamma^*\right) \backslash \Gamma_L$ and $\gamma^{*\prime} \in \Gamma^* \backslash \Gamma_L$, it is possible to have $(\mathbf{x}', \widetilde{\gamma}') \in \mathcal{U}\left((\mathbf{x}', \gamma^{*\prime}), Q''(\mathbf{R}); \Gamma_L^{*c}\right)$ for some $\mathbf{Q}'' \in \Delta_\Psi$. Thus, we can have $\mathbf{Q}^* \in \Delta_\Psi$ such that $C_\Psi(\Psi(\mathbf{Q}^*); \mathbf{R}) \nsubseteq \Gamma_L^*$.

Our next task is to show the necessity of $\Delta_\Psi \subseteq (\mathcal{F}, \mathcal{Q}_{-i})$ and $\{i\} \in \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$. Let us define $\mathcal{Q}_{A \times \Gamma_L^*} \equiv \left\{Q \cap \left(A \times \Gamma_L^*\right)^2 \mid Q \in \mathcal{Q}\right\}$ and $\mathcal{G}\left(\mathcal{Q}\right) \equiv \left\{Q \cap \left(A \times \Gamma_L^*\right)^2 \mid Q \in \mathcal{F}\right\}$. Then, $\mathcal{G}(\mathcal{Q}) = \mathcal{Q}_{A \times \Gamma_L^*}$. This implies $(\mathcal{F}, \mathcal{Q}_{-i})$ is essentially a universal domain within the restricted set of extended alternatives $A \times \Gamma_L^*$. Thus, if $\Delta_\Psi \supseteq \mathcal{F}^n$, then there exists a local dictator $d \in N$ for the choice problem over the restricted set of extended alternatives $A \times \Gamma_L^*$. In this case, if $\{j\} \notin \cap_{\mathbf{Q} \in \Delta_\Psi} \mathcal{L}(\mathbf{Q})$ for any $j \in N$, then for some $\mathbf{Q} \in \Delta_\Psi$, Q_d does not satisfy the condition (**n-c**) over the restricted set $A \times \Gamma_L^*$. This implies that, for some $\mathbf{Q} \in \Delta_\Psi$, the associated social ordering function $Q = \Psi(\mathbf{Q})$ cannot satisfy the condition (**n-c**). ■

Proof of Theorem 5: By **Theorem 4**, our remaining task is to examine the uniform rationalizability by means of $Q = \Psi(\mathbf{Q})$. A similar method used in the proof of **Theorem 3** can be applied to establish this result. ■

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