Debt-Ridden Equilibria*

– A Simple Theory of Great Depressions –

(Incomplete and Preliminary)

Keiichiro Kobayashi[†] and Masaru Inaba[‡]

June 2007 (First draft: April 2007)

Abstract

The US Great Depression and Japan's lost decade in the 1990s are both characterized as persistent stagnations of economies with debt-ridden corporate sectors subsequent to asset-price collapses. We propose a simple model, in which increases in corporate debt (and/or fluctuations in expectations about the future state of the economy) can account for these episodes. Key ingredients are the assumptions that firms are subject to collateral constraint on liquidity for financing the inputs, and that the firms can hold other firms' stocks as their assets and use them as the collateral. Collateral constraint on inputs interlinks the financial market inefficiency with the factor market inefficiencies; and that the corporate stocks are used as collateral generates an externality of *self-reference* in stock prices and production, that is, higher stock prices loosen the collateral constraint and lead to higher efficiencies in production, which in turn justify the higher stock prices. It is shown that there exists a continuum of steady-state equilibria indexed by the amount of debt that

*This paper is a substantial revision of our earlier paper entitled with "Borrowing constraints and protracted recessions." We are deeply indebted to Tomoyuki Nakajima for valuable discussions and his helpful comments on the earlier version of this manuscript. The views expressed herein are those of the authors, and not necessarily those of the Research Institute of Economy, Trade and Industry.

[†]Research Institute of Economy, Trade and Industry (RIETI). e-mail: kobayashi-keiichiro@rieti.go.jp

[‡]RIETI

the firms owe to the consumers: A steady state with a larger debt can be called a *debt-ridden equilibrium*, since it has more inefficient factor markets, produces less output, and is characterized by lower stock prices. The model provides the policy implication that debt reduction in the corporate sector at the expense of consumers (or taxpayers) may be welfare-improving when the firms are debt-ridden.

Keywords: Great depressions; collateral constraint; corporate debt; self-reference externality.

JEL Classification: E22, E32, E37, G12.

I recognized this kind of paralysis from my Goldman Sachs days. The attitude of much of Japan's political establishment seemed to be that of a trader praying over his weakening positions, when what he needed to do was to reevaluate them unsentimentally and make whatever changes made sense.
[Robert E. Rubin, *In an Uncertain World* (New York: Random House, 2003), chap. 8]

1 Introduction

The 1930s in the United States and the 1990s in Japan are both characterized as persistent stagnations of economies with debt-ridden corporate and financial sectors subsequent to asset-price collapses.[1] This paper shows that a simple variant of a neoclassical growth model with collateral constraints can account for key features of the US Great Depression and the 1990s in Japan. Pioneered by Cole and Ohanian (1999), there has been growing literature in which the neoclassical growth models are used to account for great depressions.[2] Literature includes, among others, Hayashi and Prescott (2002), Bergoeing, Kehoe, Kehoe, and Soto (2002), Fisher and Hornstein (2002), and Chari, Kehoe, and McGrattan (2004).

[1] See Fisher (1933) for description of debt-deflation in the US Great Depression.

[2] We use a "great depression" to denote a large and decade-long recession such as the US Great Depressions and the 1990s in Japan. Kehoe and Prescott (2002) define a great depression somewhat narrowly as a time period during which detrended output per working-age population falls at least 20% and the fall at least 15% must occur within the first decade of the period.

In these papers, it is shown that the declines in the total factor productivity (TFP) can explain the observed declines in output and investment during the onset of great depressions. More challenging for the neoclassical models are the protracted slumps for a decade or more subsequent to the economic collapses at the early stage. Mulligan (2002a), Nakajima (2003), and Chari, Kehoe, and McGrattan(2004) show that during the US Great Depression inefficiencies in the factor markets, especially in the labor market, emerged in the early 1930s and had continued for years.[3] The persistent inefficiencies suggest that the steady state to which the US economy tended to converge had shifted during the Great Depression. Cole and Ohanian (2004) and Ebell and Ritschl (2007) try to explain the persistent labor inefficiency and naturally come up with models in which institutional changes in the labor market in favor of labor unions caused the persistent inefficiency in wage bargaining.

In this paper, we propose a different explanation for the persistent inefficiencies. There are two key ingredients in our model: The costs for inputs, e.g., labor and intermediate goods, are subject to collateral constraint; and firms can hold corporate stocks issued by other firms as their assets and they can use the stocks as the collateral for fiancing the inputs. Our main finding is that the amount of debt that firms owe to consumers permanently affects the productive efficiency and the social welfare of the economy, and that there is a continuum of the steady-state equilibria indexed with the amount of the debt. We call them *debt-ridden equilibria*, since a steady state with a larger debt has more inefficient factor markets, produces less output, and is characterized by lower stock prices.

Our model is a variant of Kobayashi, Nakajima, and Inaba's (2007) model. Firms must pay the costs for inputs, such as labor and intermediate goods, in advance of production, and they need external funds to finance them. The amount that they can borrow is limited by the value of the collateral. It is easily shown that the financial inefficiency, i.e., the tightness of the collateral constraint, generates the inefficiencies in

[3]The persistent inefficiency in the labor market is also found in the 1990s in Japan. See Kobayashi and Inaba (2006).

the factor markets, e.g., the wedges between the marginal products of factors and their market prices.

A novel feature of this paper is that the firms can buy and hold corporate stocks issued by other firms as their financial assets and that they can use the stocks as collateral for the input finance. These assumptions seem quite realistic but, to our knowledge, are excluded from the standard growth and business cycle literature.[4] Firms issue risk-free bonds to consumers and buy the corporate stocks of other firms. They do so in equilibrium where the collateral constraint binds because the corporate stocks are more valuable than bonds for firms, since the stocks can be used as collateral for finanicing the inputs.

In this setting, the model shows the following externality of *self-reference* in stock prices and productive efficiency: A firm enjoys looser collateral constraint when the level of stock prices of other firms are higher; the firm can produce output more efficiently; and the stock price of the firm becomes higher and thus it loosens the collateral constraints of other firms in turn.[5] The self-reference externality, or the interactions between stock prices and productive efficiency, is caused by the setting that the corporate stocks are used as collateral for financing inputs. This externality causes multiplicity of equilibria: There exists a continuum of steady-state equilibria which are indexed with the level of stock price or the amount of corporate debt. It is shown that in equilibrium where the amount of corporate debt is larger, the factor markets are more inefficient, firms produce less output, and the stock prices are lower. We show numerically that this model can replicate the key features of great depressions.

The organization of the paper is as follows. In the next section, we describe our model and analyze the steady-state equilibria. In Section 3, we show the simulation results of the debt-shock experiments. Section 4 provides policy implications and concluding remarks.

[4]In a previous version of this paper, Kobayashi and Inaba (2006), we analyzed a version of the model with the similar assumptions.

[5]Kobayashi (2007) argues a version of the self-reference externality in a simple model without capital accumulation.

2 Model

In this section, we describe our model, which is a variant of the standard growth model with collateral constraint. It is shown that there exists a continuum of steady-state equilibria.

Our model economy is a closed economy with discrete time, that consists of continua of identical consumers and firms, whose measures are both normalized to one. There are also identical banks with unit mass, which only play a role of passive liquidity suppliers. Firms issue stocks and risk-free bonds, and act on behalf of their stock holders. The total supply of corporate stocks issued by one firm is normalized to one. Both stocks and bonds can be traded, and the firms can own stocks issued by other firms as their finanical assets. We assume without loss of generality that only consumers can hold corporate bonds and that firms do not own bonds issued by other firms.[6]

2.1 Consumer

A representative consumer maximizes her lifetime utility, U, defined over sequences of consumption, c_t, and leisure, $1 - n_t$, where n_t is labor supply. To ensure the existence of a balanced growth path, we assume the following class of utility functions:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [c_t (1-n_t)^\gamma]^{1-\sigma}, \tag{1}$$

where E_0 denotes the expectation conditional on the information available in period 0. In period t, the consumer sells labor, n_t, at wage rate w_t, receives the gross returns of corporate bonds, $(1 + r_t)b_t$, and of corporate stocks, $(\overline{\pi}_t + q_t)s_t$, where r_t is the interest rate, b_t the amount of bonds bought in period $t-1$, $\overline{\pi}_t$ the dividend of a corporate stock, q_t the stock price, and s_t the amount of stocks bought in period $t - 1$. She purchases consumption goods, c_t, bonds, b_{t+1}, and stocks, s_{t+1} at the end of period t. Therefore, the consumer's problem is to maximize (1) subject to the following flow budget constraint:

$$c_t + b_{t+1} + q_t s_{t+1} \leq w_t n_t + (1 + r_t) b_t + (\overline{\pi}_t + q_t) s_t. \tag{2}$$

[6] Allowing firms to hold corporate bonds as their assets do not change our results.

2.2 Firm

A representative firm maximizes the discounted sum of dividend flows on behalf of the stock holders. Since the dividend is given to the stock holders in the form of consumer goods, it should be discounted by the market price of the consumer goods at each date (and state). Therefore, the discount factor should be λ_t, the Lagrange multiplier for (2) in the consumer's problem.[7] The firm's objective is to maximize the market value of itself:

$$E_0 \sum_{t=0}^{\infty} \lambda_t \pi_t, \tag{3}$$

where π_t is the dividend in period t. In period t, she employs labor, n_t, buys intermediate inputs, m_t, and produce the (gross) output, y_t, using the following production technology:

$$y_t = A_t^{(1-\eta)(1-\alpha)} m_t^{\eta} k_t^{(1-\eta)\alpha} n_t^{(1-\eta)(1-\alpha)}, \tag{4}$$

where A_t represents the level of productivity. The firm issues risk-free bonds, b_t, and holds corporate stocks issued by other firms, s'_t, as her assets.

We assume that the firm must pay the costs for inputs, $w_t n_t + m_t$, in advance of production. We also assume that a bank can issue bank notes that can be circulated in the economy as payment instruments. The firm needs to borrow bank notes, d_t, to pay input costs. Given d_t, the firm's choice of n_t and m_t is constrained by

$$w_t n_t + m_t \leq d_t. \tag{5}$$

Bank borrowing is intra-period; if R_t is the gross rate of bank loans, the firm is supposed to repay $R_t b_t$ after production. (As discussed below, $R_t = 1$ in equilibrium.) As in Kiyotaki and Moore (1997), however, the firm cannot fully commit herself to repay the bank loan. She can abscond without repayment at the end of period t, and the bank cannot keep track of the absconder's identity from the next period on. Instead, an

[7] As we see later, the corporate stocks are mutually owned and the consumers do not own stocks in equilibrium where the collateral constraint binds. Therefore, it seems to be possible that the firms have a different discount factor. It is, however, easily shown that if the discount factor for the firms is not proportional to λ_t, there exists no steady-state equilibria.

imperfect commitment technology is available for the firm and the bank: The firm can put up a part of the corporate stocks that she owns as collateral, and the bank can seize the collateral when the borrower absconds. Therefore, the value of collateral gives the upper limit of bank loan:

$$d_t \leq \theta q_t s'_t, \tag{6}$$

where θ $(0 \leq \theta \leq 1)$ is the ratio of corporate stocks that can be put up as collateral. The bank's problem is to maximize the return on the loan, $(R_t - 1)d_t$. Since the bank faces no risk of default if the intra-period loan d_t satisfies (6), competition among banks implies that the return on the loan should be zero $(R_t - 1 = 0)$ in equilibrium. Therfore, in equilibrium, the banks become indifferent to the amount of d_t, and work as passive liquidity suppliers to the firms. So we can neglect the banks' decision-making, since it has no effect on the equilibrium dynamics of this economy. Conditions (5) and (6) together imply the following collateral constraint for the firm:

$$w_t n_t + m_t \leq \theta q_t s'_t. \tag{7}$$

At the end of period t, after production, the firm sells y_t, repay $R_t d_t$, determines dividend, π_t, makes investment in capital stocks, $k_{t+1} - (1-\delta)k_t$, receives gross return from corporate stocks, $(\overline{\pi}_t + q_t)s'_t$, buys new stocks, s'_{t+1}, repays the old bonds, $(1+r_t)b_t$, and issues new bonds, b_{t+1}, subject to the flow budget constraint:

$$\pi_t + q_t s'_{t+1} + k_{t+1} - (1-\delta)k_t - b_{t+1} = (\overline{\pi}_t + q_t)s'_t + y_t - R_t d_t - (1+r_t)b_t, \tag{8}$$

where $R_t = 1$ in the equilibrium. The reduced form of the budget constraint is

$$\pi_t = y_t - m_t - k_{t+1} + (1-\delta)k_t - w_t n_t + b_{t+1} - q_t s'_{t+1} - (1+r_t)b_t + (\overline{\pi}_t + q_t)s'_t. \tag{9}$$

Therefore, the entrepreneur's problem is to maximize (3) subject to (4), (7) and (9).

2.3 Dynamics

The dynamics of this economy are determined by the solutions to the consumer's problem and the firm's problem, with the following market clearing conditions:

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + m_t, \qquad s_t + s'_t = 1, \qquad \pi_t = \overline{\pi}_t. \tag{10}$$

If the collateral constraint, (7), does not bind, our model would virtually reduce to the standard business cycle model. Throughout this paper, we focus on the case where the collateral constraint always binds. The first order conditions (FOCs) for the consumer are

$$\lambda_t \geq E_t[(1+r_{t+1})\lambda_{t+1}], \qquad (> \text{ if } b_{t+1}=0, \qquad = \text{ if } b_{t+1}>0), \tag{11}$$

$$\lambda_t q_t \geq E_t[\lambda_{t+1}(\overline{\pi}_{t+1}+q_{t+1})], \qquad (> \text{ if } s_{t+1}=0, \qquad = \text{ if } s_{t+1}>0), \tag{12}$$

$$w_t = \frac{\gamma c_t}{1-n_t}. \tag{13}$$

The FOCs for the firm are

$$\lambda_t \leq E_t[(1+r_{t+1})\lambda_{t+1}], \qquad (< \text{ if } b_{t+1}=0, \qquad = \text{ if } b_{t+1}>0), \tag{14}$$

$$\lambda_t q_t \geq E_t[\lambda_{t+1}(\overline{\pi}_{t+1}+q_{t+1})+\mu_{t+1}\theta q_{t+1}], \qquad (> \text{ if } s'_{t+1}=0, \qquad = \text{ if } s'_{t+1}>0), \tag{15}$$

$$(\lambda_t+\mu_t)w_t = (1-\eta)(1-\alpha)\frac{y_t}{n_t}\lambda_t, \tag{16}$$

$$\lambda_t = E_t[\lambda_{t+1}\{(1-\eta)\alpha y_{t+1}/k_{t+1}+1-\delta\}], \tag{17}$$

$$(\lambda_t+\mu_t)m_t = \eta y_t \lambda_t, \tag{18}$$

where μ_t is the Lagrange multiplier for (7) in the firm's problem.

Since both (11) and (14) should hold, they hold with equality. Since both (12) and (15) should hold, in equilibrium (12) holds with inequality and (15) holds with equality. Therefore, (12) becomes slack. It follows that in equilibrium where collateral constraint binds, $s_t = 0$ and $s'_t = 1$ for all t: That is, all corporate stocks are mutually owned by firms and the consumers never hold corporate stocks. Consumers hold only corporate debt, b_t, in equilibrium. The reason why $s_t = 0$ and $s'_t = 1$ is that the corporate stocks are more valuable for the firms than for the consumers because they work as collateral only for the firms in financing inputs.

Since (11) and (14) are redundant, the system of equations that describes the dynamics reduces to nine equations for ten unknowns (y_t, c_t, n_t, k_t, m_t, x_t, q_t, r_{t+1}, $(1+r_t)b_t$,

$\overline{\pi}_{t+1})$,[8] where $x_t \equiv \mu_t/\lambda_t$ measures the tightness of the collateral constraint:

$$1 = E_t \left[\beta \frac{c_t^{\sigma}(1-n_{t+1})^{\gamma(1-\sigma)}}{c_{t+1}^{\sigma}(1-n_t)^{\gamma(1-\sigma)}}(1+r_{t+1}) \right], \tag{19}$$

$$q_t = E_t \left[\beta \frac{c_t^{\sigma}(1-n_{t+1})^{\gamma(1-\sigma)}}{c_{t+1}^{\sigma}(1-n_t)^{\gamma(1-\sigma)}} \{\overline{\pi}_{t+1} + q_{t+1} + x_{t+1}\theta q_{t+1}\} \right], \tag{20}$$

$$\frac{\gamma c_t}{1-n_t} = \frac{(1-\eta)(1-\alpha)}{1+x_t}\frac{y_t}{n_t}, \tag{21}$$

$$1 = E_t \left[\beta \frac{c_t^{\sigma}(1-n_{t+1})^{\gamma(1-\sigma)}}{c_{t+1}^{\sigma}(1-n_t)^{\gamma(1-\sigma)}} \left\{ (1-\eta)\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right\} \right], \tag{22}$$

$$m_t = \frac{\eta}{1+x_t} y_t, \tag{23}$$

$$\frac{\gamma c_t}{1-n_t} n_t + m_t = \theta q_t, \tag{24}$$

$$(1+r_t)b_t - b_{t+1} = c_t - \frac{\gamma c_t}{1-n_t} n_t, \tag{25}$$

$$c_t + m_t + k_{t+1} - (1-\delta)k_t = y_t. \tag{26}$$

$$y_t = A_t^{(1-\eta)(1-\alpha)} m_t^{\eta} k_t^{(1-\eta)\alpha} n_t^{(1-\eta)(1-\alpha)}, \tag{27}$$

This system of equations cannot specify the equilibrium path uniquely. If this system consisted of ten equations, the equilibrium path would have been determined uniquely for the initial values of the two state variables, k_0 and $(1 + r_0)b_0$, by choosing the initial values of the two control variables, c_0 and x_0.

Note that in the case where the collateral constraint does not bind, the variables b_t, s_t, and s'_t become indeterminate because of the similar reason, i.e., the redundancy of (11) and (14) and that of (12) and (15). In this case, however, the equilibrium allication of goods, labor, and capital is uniquely determined. Therefore, the indeterminacy is innocuous if the collateral constraint does not bind. On the other hand, in the case where the collateral constraint binds, the redundancy of (11) and (14) is not innocuous, since the equilibrium allocation of goods, labor, and capital becomes indeterminate. We analyze this case in this paper.

Adding exogenous expectations: We need to put one exogenous condition for each t for the ten variables to close the model. For example, suppose that $x_t = x_{t+1}$ for

[8] We solve the system of equations by backward shooting.

all t. With this condition, the economy has a unique equilibrium path. The additional condition, which may be a condition on x_t as above or may be that on other variables, can be interpreted as *exogenous expectations* at the initial period $t = 0$ on future variables for $t \geq 0$. If the exogenous expectations change for some reason, the equilibrium path and the resource allocation along it also change. Therefore, the dynamics of this model crucially depend on the exogenous expectations on future values of x_t and/or other variables. (Since x_t is not observable variable, we do not use the condition, $x_t = x_{t+1}$, but use a different specification of the exogenous expectations in the experiments in Section 3.)

The exogenous expectations in this model may be translated into various expectations in reality on wealth distribution in the future between household sector and corporate sector. If agents believe that the corporate debt, b_t, will become much larger than the stock price, q_t, eventually in the future, the tightness of the collateral constraint, x_t, will be large eventually. The exogenous expectations may be those on the government policy in the future: If agents believe that in the future the government will give a lump-sum subsidy to the corporate sector, which is financed by a lump-sum tax on consumers, b_t and x_t will become small eventually.

It can be said that in our model the exogenous expectations (on, e.g., wealth distribution and/or the government policy in the future) drive the business fluctuations and significantly affect productive efficiencies and the resource allocation in the equilibrium path of the economy. It could be said that this feature of our model formalizes Keynes' view that the long-term expectations affect today's economic activities (see Chapter 12 of Keynes [1936]).

Before proceeding to numerical analysis of the dynamics in Section 3, we first specify the nature of the collateral constraint and the steady-state equilibria.

2.4 Collateral constraint and productive inefficiencies

A key variable in our model is x_t, which represents the tightness of the collateral constraint: If (7) does not bind, $x_t = 0$, and if it binds, $x_t > 0$; and the larger the value of x_t, the tighter the collateral constraint. Therefore, x_t can be viewed as a measure of the finanical market inefficiency. At the same time, (21) implies that x_t works as a wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor. In other words, the financial market inefficiency generates the inefficiency in the labor market. Therefore, if x_t is lowered for some reason, the economy experiences a boom, since a reduction in x_t causes an increase in the labor demand (see Kobayashi, Nakajima and Inaba [2007]). Introduction of intermediate inputs, m_t, in the production technology (4) amplifies the business cycles by generating procyclical movements in the "observed" TFP in the production of value added, $y_t - m_t$. Using (23), the production function for value added can be written as

$$y_t - m_t = \left(1 - \frac{\eta}{1+x_t}\right)\left(\frac{\eta}{1+x_t}\right)^{\frac{\eta}{1-\eta}} A_t^{1-\alpha} k_t^{\alpha} n_t^{1-\alpha}. \tag{28}$$

The TFP for production of value added, $\tilde{A}_t$, is defined by $y_t - m_t = \tilde{A}_t^{1-\alpha} k_t^{\alpha} n_t^{1-\alpha}$. Therefore,

$$\tilde{A}_t \equiv \left(1 - \frac{\eta}{1+x_t}\right)^{\frac{1}{1-\alpha}} \left(\frac{\eta}{1+x_t}\right)^{\frac{\eta}{(1-\eta)(1-\alpha)}} A_t, \tag{29}$$

where $\partial \tilde{A}/\partial x < 0$ if $\eta, x_t > 0$. Thus, a fall in the financial market inefficiency increases the TFP in the production of value added. Chari, Kehoe, and McGrattan (2004) also describe the similar mechanism of amplification due to frictions in financing intermediate inputs.

2.5 Continuum of Steady-State Equilibria:

Solving equations (19)–(27) analytically for a steady state where the variables are invariant over time, we obtain the equilibrium values of variables, indexed with x:

$$n(x) = \frac{1}{1+\gamma\Phi(x)}, \tag{30}$$

$$k(x) = \left[\frac{(1-\eta)\eta^{\frac{\eta}{1-\eta}}\alpha}{(1+x)^{\frac{\eta}{1-\eta}} r_k}\right]^{\frac{1}{1-\alpha}} A n(x), \tag{31}$$

$$c(x) = \left[\left(1 - \frac{\eta}{1+x}\right)\frac{r_k}{(1-\eta)\alpha} - \delta\right] k(x), \tag{32}$$

$$y(x) = \frac{r_k}{(1-\eta)\alpha} k(x), \tag{33}$$

$$m(x) = \frac{\eta}{(1+x)(1-\eta)\alpha} r_k k(x), \tag{34}$$

$$rb(x) = (1 - \Phi(x)^{-1}) c(x) \tag{35}$$

$$q(x) = \frac{1-(1-\eta)\alpha}{(1+x)(1-\eta)\alpha\theta} r_k k(x), \tag{36}$$

where $r = \beta^{-1} - 1$, $r_k = \beta^{-1} - 1 + \delta$, and

$$\Phi(x) = \frac{1+x}{(1-\eta)(1-\alpha)} \left[1 - \frac{\eta}{1+x} - \frac{(1-\eta)\alpha\delta}{r_k} \right]. \tag{37}$$

It is easily confirmed from the above solutions that gross output, $y(x)$, capital, $k(x)$, labor, $n(x)$, intermediate inputs, $m(x)$, and stock price, $q(x)$, are all decreasing in x. All these variables are smaller in a steady-state equilibrium where the financial market inefficiency, x, is larger. Whether corporate debt, $b(x)$, and consumption, $c(x)$, are larger or smaller for a larger x is ambiguous. As we see numerically in the next section, $b(x)$ tends to increase and $c(x)$ tends to decrease as x increases for plausible parameters: Through messy algebra, we have

$$b(x) \propto \frac{1-(1-\eta)\alpha + (1-\delta k(x)/y(x))(1+x)}{1 - \frac{\gamma\eta}{(1-\eta)(1-\alpha)} + (1-\delta k(x)/y(x))\frac{\gamma(1+x)}{(1-\eta)(1-\alpha)}} (1+x)^{-\frac{\eta}{(1-\eta)(1-\alpha)}-1}. \tag{38}$$

Since $k(x)/y(x)$ is constant, it is easily shown that the left-hand side is decreasing in x in the case where $\eta = .5$, $\alpha = .33$, $\beta = .99$, $\delta = .02$ and $\gamma \geq .67$. These parameter values are standard. A steady-state equilibrium with a large b can be called a *debt-ridden equilibrium* in this case: A large debt induces a large financial inefficiency, and lowers output, labor, investment, consumption, and stock price.

Self-reference externality of stock price: We can explain the multiplicity of equilibria from the self-reference externality in stock price and productive efficiency. Since the corporate stocks are used as collateral, a higher stock price lowers x_t and enhances the productive efficiency; as a result, the firm can produce output more efficiently, and the stock price of the firm rises, making the collateral constraint of other firms looser. This self-reference externality, or the interaction between stock price and productive efficiency, causes multiplicity of equilibria.

3 Numerical Experiments for Great Depressions

In this section we report the results of numerical experiments. The parameter values are set as follows: $\beta = .99$; $\gamma = 1.6$; $\sigma = .8$; $\delta = .02$; $\eta = .5$; $\alpha = .33$; $\theta = .15$. Most of these values seem standard. We set the values of β and δ so that the unit of time is a quarter; the value of γ is chosen so that the steady-state value of n becomes about 0.3; the vaule of θ is chosen so that the collateral constraint binds tightly enough. As Kobayashi, Nakajima, and Inaba (2007) argue in detail, the value of σ is chosen so that (the expectation of) a higher productivity tends to increase the value of stocks relative to output. This requirement necessitates the elasticity of intertemporal substitution (EIS), σ^{-1}, to be greater than one. Setting the EIS greater than one appears consistent with the empirical evidence: see, for example, Mulligan (2002b), Gruber (2006), and Vissing-Jørgensen and Attanasio (2003).

Figure 1 shows the steady-state allocations and prices as functions of x. We show $\tilde{A}_t$ defined in (29) as the TFP and value added, $y_t - m_t$, as the output. The debt, b, increases as x increases; and TFP, value added, consumption, investment, labor, and stock price are all decreasing in x.

3.1 Debt-shock experiment

Figure 2 shows the response of the model to an exogenous increase in the amount of corporate debt: The economy is initially in a steady state equilibrium, where the corporate asset and liability balance, that is, $(1 + r_s)b_s = (\overline{\pi}_s + q_s)s'_s$, where the variables with subscript s denotes the respective values in the steady state; and at $t = 1$ the debt shock hits the economy and the amount of debt jumps up to $(1 + r_s)b_s + \Delta$, where $\Delta = 0.9571$ is the debt shock.

This exogenous increase in corporate debt may be a good approximation of the economic turmoil caused by the emergence and collapse of the asset-price bubble at the onset of the US Great Depression and the lost decade in Japan in the 1990s. It is shown in Figure 2 that after the debt shock the economy stagnates persistently and converges to a steady state where output, labor, investment, consumption, and stock price are all lower and the corporate debt is larger than their respective values in the initial state.

As we argue in Section 2.3, in order to derive the equilibrium path in this experiment, we needed to add one condition on the exogenous expectations to the system of equations (19)–(27). We assume that all agents in this economy believe that

$$e_{t+1} = e_t \qquad \text{for all } t \geq 1, \tag{39}$$

where e_t is defined by

$$e_t \equiv (1 + r_t)b_t - (\overline{\pi}_t + q_t)s'_t, \tag{40}$$

which can be interpreted as a measure of the debt-burden for the firms. Therefore, people believe that the debt-burden for the corporate sector is invariant over time after the debt shock. Under this expectation, e_t jumps up at $t = 1$ from the initial value to the new steady-state value, which is $e_{ss} = (1 + r_{ss})b_{ss} - (\overline{\pi}_{ss} + q_{ss})s'_{ss}$, where the variables with subscript ss is the values in the steady state where the economy eventually converges after the debt shock. The new steady state where the economy converges and the value of e_{ss} are uniquely determined corresponding to the value of Δ. We calculated the equilibrium path after the debt shock by solving the system of equations (19)–(27) and (39) by the backward shooting method, in which we choose the initial values of consumption, c_1, and the debt-burden, $e_1 = e_{ss}$, such that the initial value of the capital stock is k_s and that of the corporate debt is $(1 + r_s)b_s + \Delta$.

3.2 Debt reduction policy

How can we model the policy responses to the great depressions such as the Bank Holiday in March 1933 during the US Great Depression and the (gradual) disposal of nonperforming loans in the 1990s in Japan? In our model, these policy responses may be modeled as an exogenous decrease in the corporate debt, b_t, by a lump-sum transfer from the consumers to the firms. Figure 3 shows the response of the economy to an exogenous and unexpected debt reduction at $t = 21$: The corporate debt changes to $(1 + r_{21})b_{21} - \Delta'$, where $\Delta' = 0.6894$. We still assume (39) for $t \geq 21$ in this experiment, too. The economy picks up when debt-reduction policy is implemented and converges to another steady state, which is more inefficient than the initial steady state but more efficient than the steady state where the economy converges in the case of no debt reduction in Figure 2. Figure 3 shows the behaviors of the macroeconomic variables that seem similar to those in the US Great Depression (see, for example, Chari, Kehoe, and McGrattan [2004]).

4 Conclusion

The US Great Depression in the 1930s and Japan's lost decade in the 1990s are both characterized as persistent recessions with debt-ridden corporate sectors subsequent to asset-price booms and their collapses. Recent literature shows that the persistent stagnations were associated with persistent inefficiencies in the factor markets, especially in the labor market. In this paper, we propose a simple explanation that financial frictions may generate the persistent inefficiencies: Collateral constraint on input costs interlinks the inefficiency in the financial market with that in the labor market; and the persistence of inefficiencies may be generated by the self-reference externality in stock prices and productive efficiency, that is, lower stock prices tighten the collateral constraint and lead to lower efficiencies in production, which in turn justify the lower stock prices.

It was easily shown that there exists a continuum of steady-state equilibria, which are indexed by the amount of debt: In a steady state with a larger debt, the factor markets are more inefficient, stock prices are lower, and output is smaller. Therefore, a steady-state equilibrium that corresponds to a large amount of debt may be called a debt-ridden equilibrium.

This model provides us with straightforward but surprising implications for economic policy: Debt reduction in the corporate sector at the expense of the consumers (or taxpayers) may improve the efficiency and the social welfare, when the firms are debt-ridden. That debt reduction is welfare improving is easily confirmed by reducing the initial value of b_t by a lump-sum transfer from the consumers to the firms (Figure 3). If our model is a precise description of the decade-long stagnation associated with the persistent nonperforming loan problem in Japan, the policy implications above may be a theoretical translation of what Robert E. Rubin, the seventieth US Securetary of the Treasury, said about Japan in the 1990s (see the epigraph). In fact, the Japanese economy has been picking up since 2002, when the government changed its policy stance toward aggressive disposal of nonperforming loans. The Bank Holiday in March 1933 in the United States may also be an example of debt reduction policy in a debt-ridden equilibrium: As is known well, the US economy picked up since March 1933 (see, for example, Cole and Ohanian [1999] and Chari, Kehoe, and McGrattan [2004]).

This model may also provide a new interpretation of the Keynesian prescriptions for recessions. Keynesian fiscal and monetary policies may be effective in our model only if these policies reduce corporate debt, b_t: For example, an expansionary fiscal policy is interpreted as reduction of b_t by a lump-sum transfer from the taxpayers (the consumers) to the firms. Therefore, the Keynesian notion of "stimulating demand" may be interpreted as reducing the corporate debt or changing the exogenous expectations on x_t and/or other variables in the future in our model.

Note that our model of debt-ridden equilibria is purely real: Nominal factors, e.g., deflation in nominal prices, may be relevant only if they affect the amount of debt by redistributing wealth between consumers and firms. This implication seems consistent with Fisher's (1933) debt-deflation theory. Our model also suggests that the decade-long deflation in Japan since the late 1990s was not a direct cause of the persistent recession: On the contrary, the deflation in Japan, which still continues in 2007, may be a natural response of nominal prices to the zero-nominal-interest-rate policy adopted by the Bank of Japan in a debt-ridden equilibrium.[9]

5 References

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2002). "Avoiding Liquidity Traps." *Journal of Political Economy* 110(3): 535–63.

[9]Since the real interest rate takes on a positive value in the equilibrium, the credible commitment of the Bank to set the nominal interest rate at zero for a long period may generate the expectation that the price deflation continues. This mechanism is similar to the one that generates a deflationary equilibrium in Benhabib, Schmitt-Grohé, and Uribe (2002).

Bergoeing, R., P. J. Kehoe, T. J. Kehoe, and R. Soto (2002). "A decade lost and found: Mexico and Chile in the 1980s," *Review of Economic Dynamics* 5 (1): 166–205.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2004). "Business cycle accounting." Federal Reserve Bank of Minneapolis Staff Report 328.

Cole, H. L., and L. E. Ohanian (1999). "The Great Depression in the United States from a neoclassical perspective." Federal Reserve Bank of Minneapolis *Quarterly Review* 23 (1): 2–24.

—— (2004). "New Deal policies and the persistence of the Great Depression: A general equilibrium analysis." *Journal of Political Economy* 112 (4): 779–816.

Ebell, M. and A. Ritschl (2007)."Real origins of the Great Depression: Monopolistic competition, union power, and the American business cycle in the 1920s." CEPR Discussion Paper Series No.6146.

Fisher, I. (1933). "The debt-deflation theory of great depressions." *Econometrica* 1 (4): 337–57.

Fisher, J. D. M., and A. Hornstein (2002). "The role of real wages, productivity, and fiscal policy in Germany's Great Depression 1928–1937." *Review of Economic Dynamics* 5 (1): 19–44.

Gruber, J. (2006). "A tax-based estimate of the elasticity of intertemporal substitution." NBER Working Paper 11945.

Hayashi, F., and E. C. Prescott (2002). "The 1990s in Japan: A lost decade." *Review* *of Economic Dynamics* 5 (1): 206–35.

Kehoe, T. J., and E. C. Prescott (2002). "Great depressions of the 20th century." *Review of Economic Dynamics* 5 (1): 1–18.

Keynes, J. M. (1936). *General Theory of Employment, Interest, and Money.* (Reprint, 1997, Prometheus Books: New York).

Kiyotaki, N., and J. Moore (1997). "Credit cycles." *Journal of Political Economy* 105 (2): 211–48.

Kobayashi, K. (2007). "Transaction services and asset-price bubbles." *Macroeconomic Dynamics,* forthcoming.

Kobayashi, K. and M. Inaba (2006). "Borrowing constraints and protracted recessions." RIETI Discussion Paper Series 06-E-011.

Kobayashi, K. and M. Inaba (2006). "Business cycle accounting for the Japanese economy." *Japan and the World Economy* 18 (4): 418-40.

Kobayashi, K., T. Nakajima, and M. Inaba (2007). "Collateral constraint and news-driven cycles." RIETI Discussion Paper Series 07-E-013.

Mulligan, C. B. (2002a). "A dual method of empirically evaluating dynamic competitive equilibrium models with market distortions, applied to the Great Depression and World War II." NBER Working Paper 8775.

—— (2002b). "Capital, interest, and aggregate intertemporal substitution." NBER Working Paper 9373.

Nakajima, T. (2003). "Decline in the capital-output ratios and the Great Depression." Mimeo.

Vissing-Jørgensen, A., and O. Attanasio (2003). "Stock-market participation, intertemporal substitution, and risk aversion." *American Economic Review* 93(2):382–391.

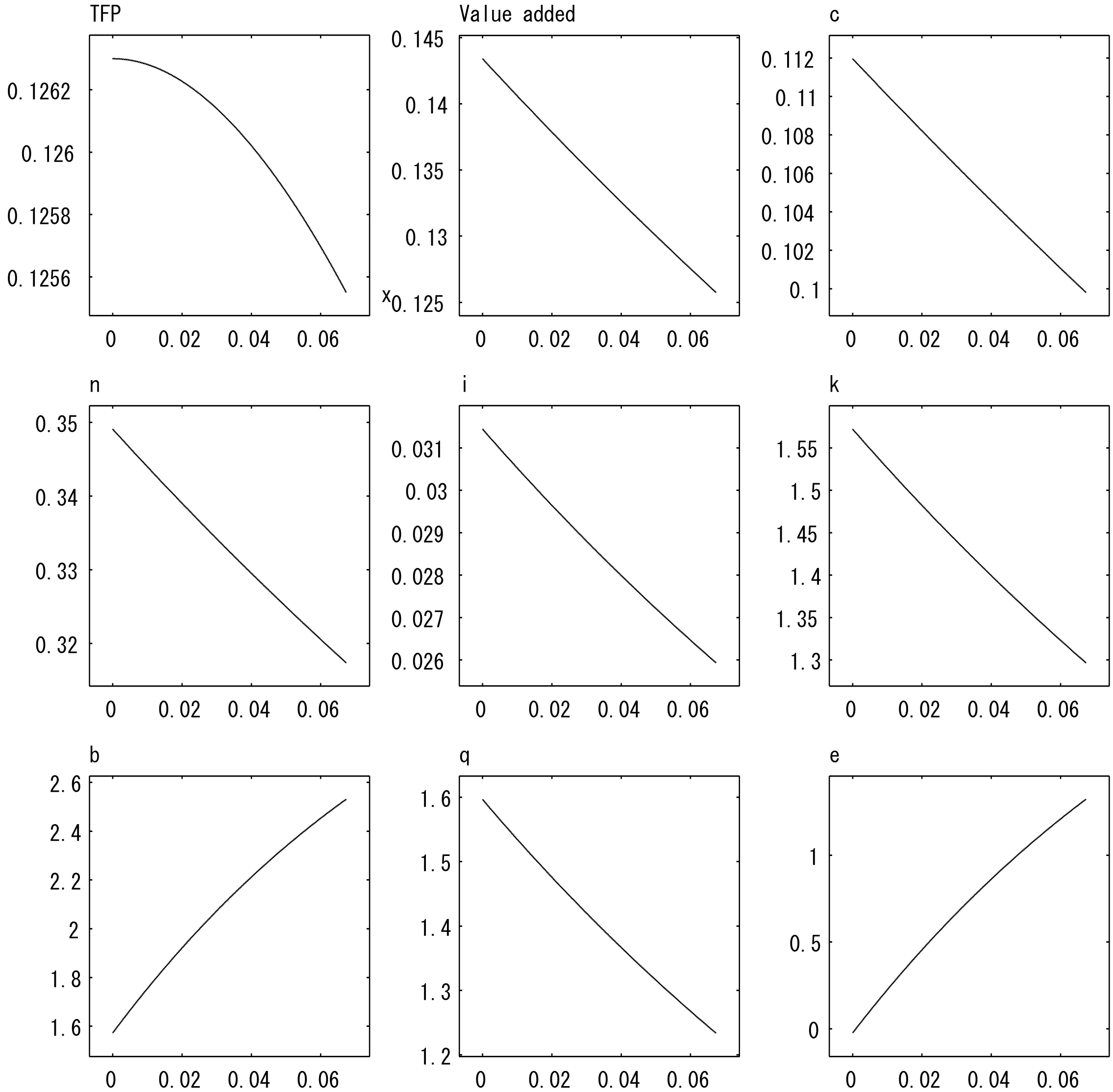

Figure 1: Continuum of steady-state equilibria indexed by x.

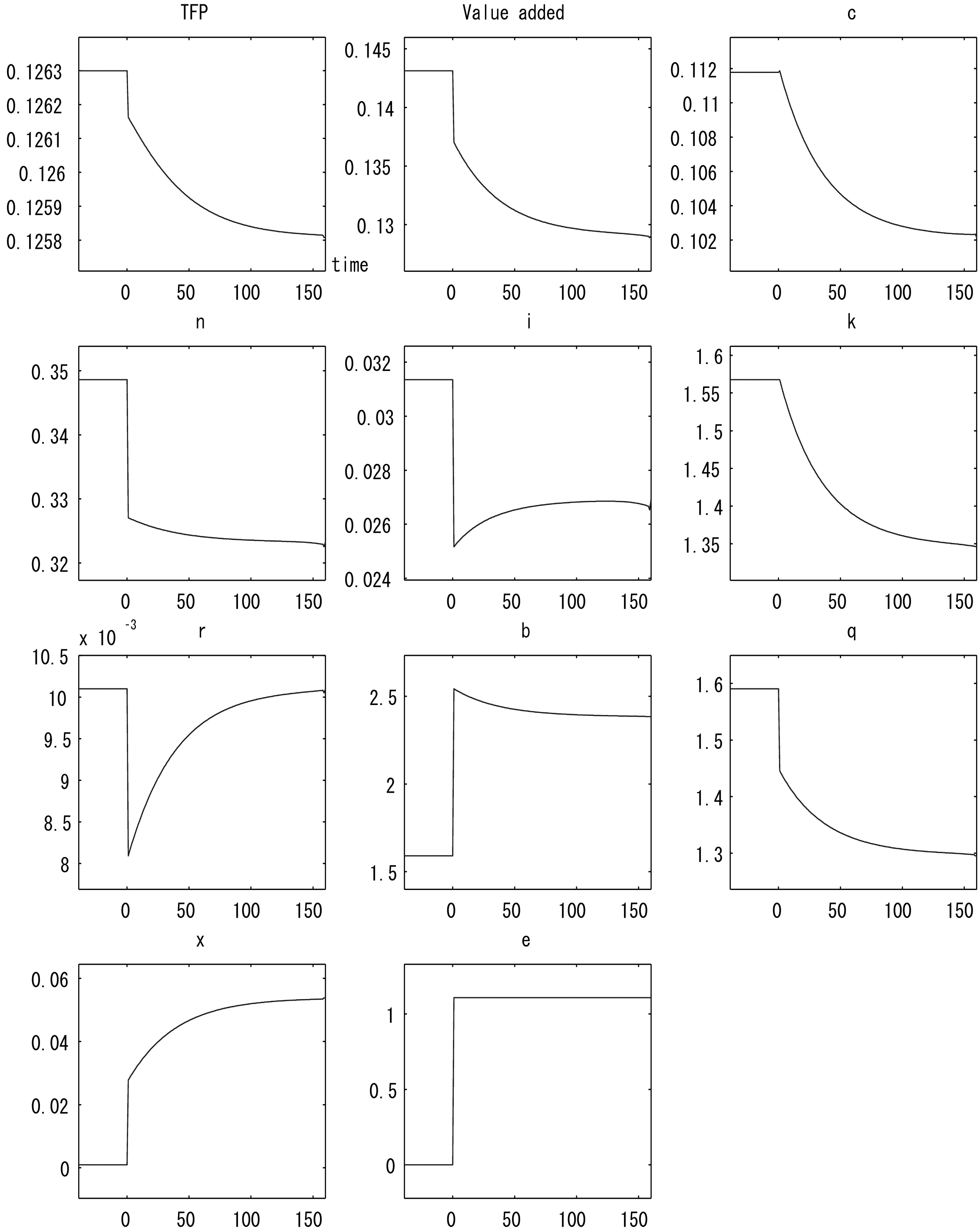

Figure 2: The debt-shock experiment (Debt increases unexpectedly at $t = 1$).

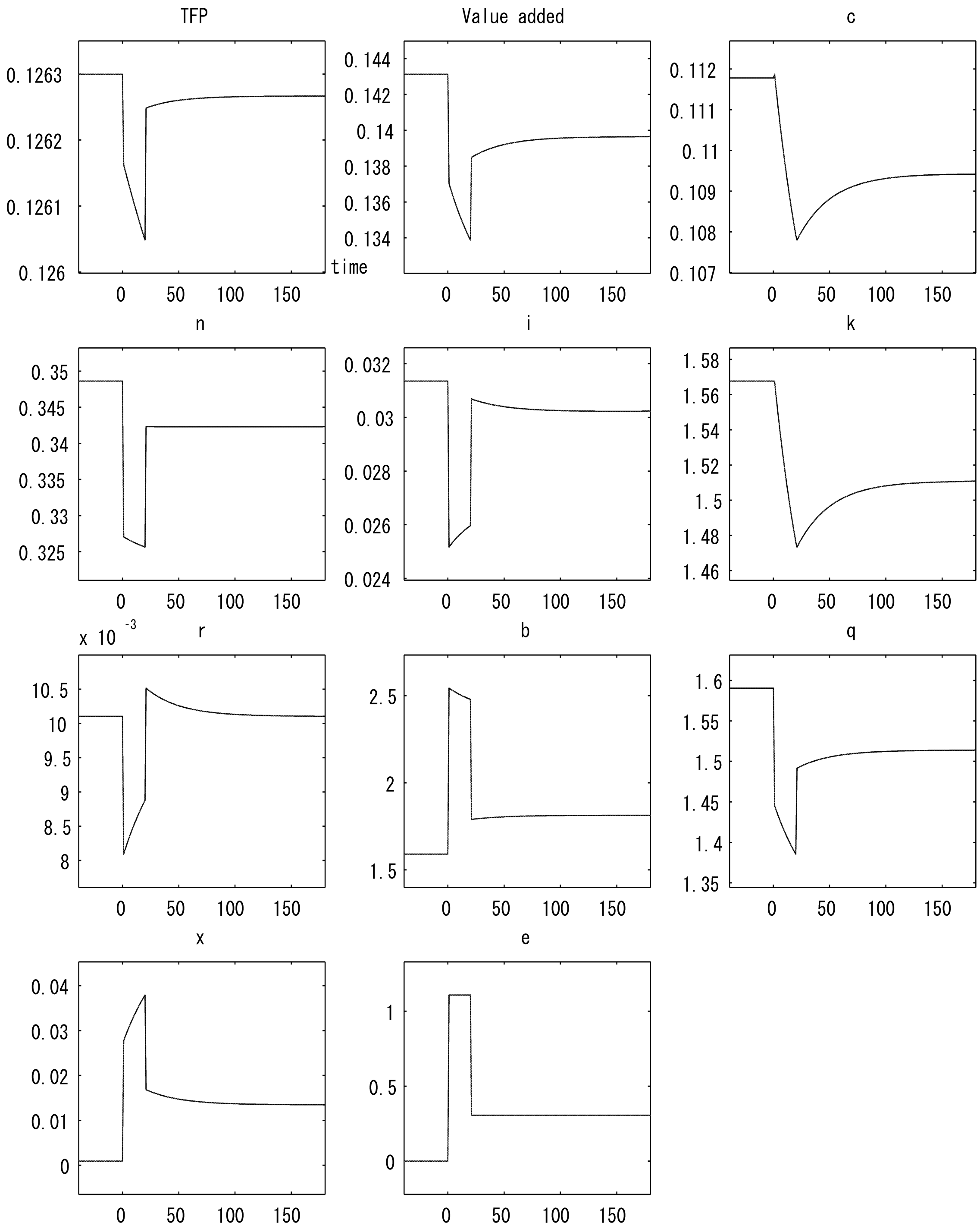

Figure 3: The debt-reduction policy (Debt is unexpectedly reduced at $t = 21$).