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# Multi-Sector Menu Cost Model, Decreasing Hazard, and Phillips Curve

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# Multi-Sector Menu Cost Model, Decreasing Hazard, and Phillips Curve †

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**Abstract**

This paper generalizes the Golosov-Lucas model (GL model), a single sector menu cost model with idiosyncratic productivity shocks, to multi-sector setting. While the GL model matches some empirical facts, it cannot mimic decreasing hazard functions for price changes, which are observed in many countries. With realistic parameters, the simulation results of the generalized GL model show many features observed in empirical evidences such as decreasing hazard rates. In addition, the simulation results with monetary shocks show flattening of the Phillips curve in a low inflation environment.

Keywords: Menu Cost Models, Hazard Functions, Phillips Curve
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# 1 Introduction

Price setting behavior is one of the most important research areas in macroeconomics. There are two major categories: time-dependent pricing and state-dependent pricing. In the New Keynesian context, time-dependent pricing such as Calvo (1983) and Taylor (1979, 1980) is popular. While time-dependent pricing is convenient for monetary policy analysis, there is no micro foundation in it and therefore it is subject to Lucas critique.

State-dependent pricing is more appealing since it assumes that firms choose not only the size of price changes, but also the timing of price changes. State-dependent pricing can be split into two categories: menu cost models with idiosyncratic productivity shocks such as Golosov and Lucas (forthcoming) (GL model), and models without them such as Caplin and Spulber (1987) and Dotsey, King and Wolman (1999). Note that, in menu cost models without idiosyncratic productivity shocks, the only source of price changes is monetary shock. Such models are inconsistent with the empirical fact that price changes occur frequently even when inflation is zero.

Golosov and Lucas (forthcoming) introduce idiosyncratic productivity shocks into a menu cost model so that price changes occur even under a zero inflation environment. Even the GL model, however, can't explain decreasing hazard.[1] Decreasing hazard is observed in many countries such as Japan (Saita et al. (2006)), United States (Nakamura and Steinsson (2006a), Klenow and Kryvtsov (2005)), and Euro Area (Álvarez, Burriel, and Hernando (2005)).[2]

To produce decreasing hazard, the model needs to generate time series of individual prices so that a substantial fraction of price changes occur in the first few months after a price change while a considerable fraction of individual prices remain constant in two or three years after a price change. Since the GL model includes only one sector, the model can't generate time series with such heterogeneity.

[1] Decreasing hazard rates imply that a firm will have lower probability of changing its price the longer it has kept it unchanged. By the term "decreasing hazard," people often mean two different things: decreasing hazard across products and decreasing hazard for individual products. Throughout this paper, I use the term "decreasing hazard" to mean a decreasing hazard across products.

[2] The decreasing hazard for individual products found by Nakamura and Steinsson (2006a) is also interesting but I will not focus on this fact. This fact may not be robust since (1) the slope of hazard they estimate is "slightly decreasing or almost flat," and (2) their estimate is subject to a downward bias due to "heterogeneity in frequency of price changes.": Note that Nakamura and Steinsson (2006a) also found that the frequency of price changes is increasing in inflation rates. In addition, note that the database they use to estimate hazard contains prices under the different inflation rates. These two things imply that there is "heterogeneity in frequency of price changes," which can be the source of a downward bias.

Explaining decreasing hazard is important to understand the effects of monetary policy on real economy such as level of production: In the calibrated GL model, money is almost neutral, which may be caused by the model's failure to generate time series of prices with a considerable fraction constant in long-term as seen in Figure 1.

This paper generalizes the GL model to multi-sector setting, where each sector has different menu cost, variance of productivity shocks, and average productivity level.[3]

Simulation results show many desirable features: First, the calibrated model can generate decreasing hazard.[4] Second, the calibrated model predicts that the frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not. They are consistent with the empirical facts established by Nakamura and Steinsson (2006a), on individual price data. Third, the relationship between inflation and GDP predicted by the model has some interesting features, such as slope of Phillips curve increasing in inflation rates, which are consistent with empirical facts established by Benati (forthcoming).[5]

This paper is organized as follows. Section 2 summarizes the stylized facts this paper tries to explain. Section 3 presents a two-sector menu cost model (Generalized GL Model) where the differences of sectors consist of menu cost, the variance of productivity shocks, and the average productivity level. Section 4 describes the calibration procedure. I also show the predictions of the calibrated model for price changes. Section 5 shows the model's predictions on the relationship between inflation and production. Section 6 concludes.

## 2 Stylized Facts

The main purpose of this paper is to show that the generalization of the GL model significantly improves the model's ability to mimic the empirical facts about the individual prices and the relationship between inflation and GDP. Here, I summarize the stylized facts with which the predictions of the generalized GL model are consistent.

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[3]Although I show only a two-sector model in this paper, a model with finitely many sectors can be constructed in the same way technically.

[4]For some readers, this accomplishment might seem not to be significant since I calibrate this model to sample moments including the information of decreasing hazard. I can, however, assert that this is good job since previous state-dependent pricing models including the GL model can't be calibrated so as to replicate decreasing hazard.

[5]By the term "Phillips Curve," I mean the relationship between inflation and GDP generated by the model (or observed in data), as seen in Figure 12, throughout this paper.

The first three facts are about the individual retail prices in Japan due to Saita et al. (2006). In 1999-2003 when Japanese economy was in moderate deflation, (1) the average frequency of price changes is 23.1 percent per month, (2) almost half of price changes are price decreases, and (3) the hazard function of price changes is decreasing. The decreasing hazard is also observed in United States (Nakamura and Steinsson (2006a), Klenow and Kryvtsov (2005)), and Euro Area (Álvarez, Burriel, and Hernando (2005)).

The fourth fact is about the individual prices in U.S. due to Nakamura and Steinsson (2006a): (4) The frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not.[6]

The fifth fact is about the relationship between inflation and GDP. (5) The slope of Phillips curve is increasing in inflation rates according to Benati (forthcoming).[7]

As for these facts, the GL model can't explain (3) and (5) while the generalized GL model can explain all five facts.[8]

# 3 Model

## 3.1 Outline of the Model

In this section, I present a two-sector menu cost model with idiosyncratic productivity shocks. I construct the model modifying the GL model. Recall that the GL model can't generate decreasing hazard since decreasing hazard is consistent with heterogeneity in frequency of price changes while the GL model includes only one sector.

In the GL model, frequency of price changes is affected by menu cost, the variance of idiosyncratic productivity shocks, and the level of productivity. Therefore, I present a two-sector menu cost model where the differences of sectors consist of menu cost, the variance of productivity shocks, and the average productivity level.

Outline of the model is as follows: There is a continuum of infinitely lived households, each of which consumes a continuum of goods and services. Each

[6]Golosov and Lucas (forthcoming) provide the international evidence showing that the frequency of price changes is increasing in inflation rates.

[7]As for the relationship between inflation and GDP, the generalized GL model also predicts that high inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies high and stable GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005).

[8]The generalized GL model can't explain the seasonality of frequency of price changes pointed out by Nakamura and Steinsson (2006a). This feature, however, seems to be just the out of the scope of this model.

household supplies labor on a competitive labor market. Households are assumed to obtain utility from real cash holdings. Money supply follows a monetary shock process specified later.

There is a continuum of firms, subject to idiosyncratic productivity shocks: 50 percent of firms are "g sector" firms, associated with relatively high frequency of price changes, and the rest of firms are "s sector" firms, associated with relatively low frequency of price changes.[9] Firms in a different sector are subject to idiosyncratic productivity shocks generated by a different shock process. Each firm produces only one of the continuum of consumption goods or services. Each firm sets price of the good subject to a menu cost of re-pricing. Menu cost is measured by labor hour. The length of the labor hour needed to change a nominal price is different between two sectors. Firms use labor to produce the good or service and to re-set nominal prices.

## 3.2 Two Shocks in the Economy

There are two types of shocks in this economy: a monetary shock and a firm-specific productivity shock. A different sector is subject to firm-specific productivity shocks generated by a different shock process. The log of the money supply is assumed to follow a Brownian motion with drift parameter $\mu$ and variance $\sigma_m^2$,

$$d\log(m) = \mu dt + \sigma_m dZ_m \tag{1}$$

where $Z_m$ denotes a standard Brownian motion with zero drift and unit variance.

There are firm-specific productivity shocks for "g sector" firms denoted by $v_g$, and those for "s sector" firms $v_s$, which are assumed to be independent across firms. The log of a firm-specific productivity shock follows the mean-reverting process:

$$d\log(v_j) = -\eta(\log(v_j) - \log(1 + e_j))dt + \sigma_{v_j} dZ_{v_j} \qquad j = g \text{ or } s \tag{2}$$

where $(1 + e_j)$ is the average productivity level of "j sector" firms, and $Z_{v_j}$ is a standard Brownian motion with zero drift and unit variance, distributed independently of $Z_m$.

[9] I associate "g sector" and "s sector" with data in the way described in the subsection 4.2. Given this, the weights of consumer price index adjusted by Saita et al. (2006) suggest that the fraction of "g sector" firms is about 50 percent.

## 3.3 State of the Economy and of an Individual Firm

The state of this economy at date $t$ includes money stock $m_t$, a nominal wage rate $w_t$ and the joint distribution of firms with $(p_t, v_t)$, denoted by $\phi_t(p, v)$, where a firm with $(p_t, v_t)$ denotes a firm with its nominal price $p_t$ and its current productivity shock $v_t$. The state of an individual "j sector" firm includes $p_t$ and $v_{jt}$ additionally.

## 3.4 Markets

There is a labor market where firms hire labor from households with $w_t$. There is also a capital market where claims to the monetary unit are traded. As in Golosov and Lucas (forthcoming), I adopt the convention that

$$\mathrm{E}\left[\int_0^\infty Q_t y_t dt\right] \tag{3}$$

is the value at date 0 of a dollar earnings stream $\{y_t\}_{t=0}^\infty$, a stochastic process defined in terms of $m_t$.

In equilibrium, the market clearing conditions for consumption, labor, and money are satisfied.

## 3.5 Consumer

At each date $t$, each household buys goods and services from every firm distributed according to $\phi_t(p, v)$. The household chooses a buying strategy $\{C_t(\cdot)\}$, where $C_t(p)$ is the number of units of the consumption good that it buys from a firm charging nominal price $p$ at date $t$. Additionally, the household chooses a labor supply strategy $\{l_t\}$ and a money holding strategy $\{\hat{m}_t\}$, where $l_t$ is the units of labor supplied and $\hat{m}_t$ is dollar balances held at date $t$.

Let $c_t$ denote Spence-Dixit-Stiglitz consumption aggregate

$$c_t = \left[\int C_t(p)^{1-\frac{1}{\epsilon}} \phi_t(dp, dv)\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

A price index $P_t$ is defined as follows:

$$P_t = \left[\int p^{1-\epsilon} \phi_t(dp, dv)\right]^{\frac{1}{1-\epsilon}}. \tag{5}$$

The expected utility of the household over time is expressed as

$$
\mathrm{E}\left[\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\gamma}c_t^{1-\gamma} - \alpha l_t + \log\left(\frac{\hat{m}_t}{P_t}\right)\right)dt\right] \tag{6}
$$

where the operator $\mathrm{E}(\cdot)$ is defined by the shock processes (1) and (2).[10]

The budget constraint of the household is expressed as

$$
\mathrm{E}\left[\int_0^\infty Q_t\left(\int pC_t(p)\phi_t(dp,dv) + R_t\hat{m}_t - w_t l_t - \Pi_t\right)dt\right] \leq m_0 \tag{7}
$$

where $\Pi_t$ is profit income, obtained from the household's holdings of a fully diversified portfolio of claims on the individual firms, plus any lump sum cash transfers. $R_t$ is the nominal interest rate.

The household chooses buying strategy $\{C_t(\cdot)\}$, labor supply strategy $\{l_t\}$, and money holding strategy $\{\hat{m}_t\}$ so as to maximize (6) subject to (7), taking $\{Q_t\}$, $\{R_t\}$, $\{w_t\}$, $\{\Pi_t\}$, $\{\phi_t\}$ as given. The first-order condition for money holdings is expressed as

$$
e^{-\rho t}\frac{1}{m_t} = \lambda Q_t R_t, \tag{8}
$$

where $\lambda$ is the Lagrange multiplier independent of time. Note that the equilibrium condition $\hat{m}_t = m_t$ is imposed in the above equation. The first-order condition for consumption is described as

$$
e^{-\rho t}c_t^{-\gamma}c_t^{1/\epsilon}C_t(p)^{-1/\epsilon} = \lambda Q_t p,. \tag{9}
$$

The first-order condition for labor supply is

$$
e^{-\rho t}\alpha = \lambda Q_t w_t,. \tag{10}
$$

It can be shown that there is an equilibrium where the nominal interest rate is constant at the level

$$
R_t = R = \rho + \mu \tag{11}
$$

[10]The linear disutility of labor in (6) can be interpreted as a reflection of the indivisible labor setting proposed by Hansen (1985).

for all realizations of the two shock processes. As Golosov and Lucas (forthcoming) did, I focus on the equilibrium where (11) holds throughout this paper. Then, (8), (10), and (11) imply

$$w_t = \alpha R m_t. \tag{12}$$

(12) implies that $\log(w_t)$ follows the same Brownian motion as the one for monetary shocks. Note that this structure of the model associated with (12) simplifies the analysis significantly.

## 3.6 Firms

A "j sector" firm faces the demand for the good it produces $C_t(\cdot)$, nominal wage $w_t$, and a productivity parameter $v_{jt}$. The production function of "j sector" firms is assumed to be

$$C_t(p) = v_{jt} l_t^f \tag{13}$$

where $l_t^f$ denotes labor used to produce the good.[11] Suppose a firm enters the period with a nominal price $p$ carried over from the past. Then, if this firm leaves price unchanged, its current profit is

$$C_t(p)(p - w_t/v_{jt}). \tag{14}$$

If this firm chooses any price $q \neq p$, its current profit becomes

$$C_t(q)(q - w_t/v_{jt}) - k_j w_t \tag{15}$$

where the parameter $k_j$ is the hours of labor needed for a "j sector" firm to change its nominal price.

Let's think about the present value of a "j sector" firm with its state $(p, v_j, w, \phi_t)$. I express this present value by $\varphi_j(p, v_j, w, \phi_t)$. This firm chooses a shock-contingent repricing time $T \geq 0$, and a shock-contingent nominal price $q$ to be chosen at $T$ so as to solve

[11] Here, as Golosov and Lucas (forthcoming) and many New Keynesians did in their research, I assume each firm must satisfy household's demand for the good the firm produces.

$$\varphi_j(p, v_j, w, \phi) = \max_T \mathrm{E}\bigg[\int_0^T Q_t C_t(p)(p - w_t/v_{jt})dt + Q_T \cdot \max_q [\varphi_j(q, v_{jT}, w_T, \phi_T) - k_j w_T]\bigg]. \quad (16)$$

Note that, from (4), (9), and (10), the demand function of goods and services is

$$C_t(p) = c_t^{1-\epsilon\gamma}\left(\frac{\alpha p}{w_t}\right)^{-\epsilon}. \quad (17)$$

From the natural normalization $Q_0 = 1$ and (10), I obtain

$$Q_t = e^{-\rho t}\frac{w_0}{w_t}. \quad (18)$$

Then, using (17) and (18), (16) can be expressed as

$$\varphi_j(p, v_j, w, \phi) = \max_T \mathrm{E}\bigg[\int_0^T e^{-\rho t}\frac{w}{w_t}c_t^{1-\epsilon\gamma}\left(\frac{\alpha p}{w_t}\right)^{-\epsilon}(p - w_t/v_{jt})dt + e^{-\rho T}\frac{w}{w_T}\cdot \max_q [\varphi_j(q, v_{jT}, w_T, \phi_T) - k_j w_T]\bigg]. \quad (19)$$

The choice of stopping times $T$ and nominal prices $q$ that attain the right side of (19) is a pricing strategy of a “j sector” firm. In this paper, I analyze a Nash equilibrium of pricing strategies over a continuum of monopolistically competitive firms. In the rest of this section, I will describe how (19) can be analyzed and how this pricing strategy is determined.

Finally, I define $\{\Gamma_{jt}\}$ so that $\Gamma_{jt}dt$ is the fraction of the firms that belong to “j sector” and reprice during the time interval $(t, t + dt)$ in equilibrium. Then, the labor market clearing condition is expressed as

$$l_t = \int \frac{C_t(p)}{v}\phi_t(dp, dv) + k_g\Gamma_{gt} + k_s\Gamma_{st}. \quad (20)$$

The market clearing conditions for consumption goods and services have been incorporated in (19).

## 3.7 Restatement of Firm's Bellman Equation as a Recursive Form

Note that (19) is not recursive, including the joint distribution $\phi_t$ implicitly in $c_t$. This makes (19) difficult to analyze. In addition, to solve this problem numerically, I need to discretize this continuous time model in advance. Therefore, I make two-step approximation as Golosov and Lucas (forthcoming) did: The first approximation is made to keep problem recursive even if $\sigma_m$ is positive so that the time-invariant Bellman equation is obtained. The details will be discussed in this subsection. The second approximation is made to discretize the continuous time model, which will be discussed in the next subsection. I will analyze the easier case, $\sigma_m = 0$, first, then the harder one, $\sigma_m > 0$.

[The Case of $\sigma_m = 0$]

From (4) and (17), the consumption aggregate can be expressed as

$$c_t = \left[ \int \left( \frac{\alpha p}{w_t} \right)^{1-\epsilon} \phi_t(dp, dv) \right]^{1/(\gamma(\epsilon-1))}. \tag{21}$$

Using the change of variable $x = p/w_t$, (21) is rewritten as

$$c_t = \left[ \alpha^{1-\epsilon} \int x^{1-\epsilon} \tilde{\phi}_t(dx, dv) \right]^{1/(\gamma(\epsilon-1))}. \tag{22}$$

Now, I assume that there exists an invariant measure $\tilde{\phi}$ and express the corresponding consumption aggregate, given by (22), as $\bar{c}$.[12] Then, I can restate (19) as

$$\begin{aligned} \varphi_j(p, v_j, w) = \max_T \mathrm{E}\Bigg[ \int_0^T e^{-\rho s} \frac{w}{w_s} \bar{c}^{1-\epsilon\gamma} \left( \frac{\alpha p}{w_s} \right)^{-\epsilon} (p - \frac{w_s}{v_{js}}) ds \\ + e^{-\rho T} \frac{w}{w_T} \cdot \max_q [\varphi_j(q, v_{jT}, w_T) - k_j w_T] \Bigg]. \end{aligned} \tag{23}$$

Again, using the change of variable $x = p/w$, (23) can be expressed as

$$\begin{aligned} \frac{1}{w} \varphi_j(wx, v_j, w) = \max_T \mathrm{E}\Bigg[ \int_0^T e^{-\rho s} \bar{c}^{1-\epsilon\gamma} (\alpha x_s)^{-\epsilon} (x_s - \frac{1}{v_{js}}) ds \\ + e^{-\rho T} \frac{1}{w_T} \cdot \max_{x'} [\varphi_j(w_T x', v_{jT}, w_T) - k_j w_T] \Bigg]. \end{aligned} \tag{24}$$

[12] The existence of this invariant measure can be confirmed numerically in the later calculations.

Finally, setting $\varphi_j(p, v, w) = w\psi_j(x, v)$, firm's Bellman equation becomes a recursive form as follows:

$$\psi_j(x, v_j) = \max_T \mathrm{E}\bigg[ \int_0^T e^{-\rho t}\overline{c}^{1-\epsilon\gamma}(\alpha x_t)^{-\epsilon}(x_t - \frac{1}{v_{jt}})dt + e^{-\rho T} \cdot \max_{x'}[\psi_j(x', v_{jT}) - k_j]\bigg]. \quad (25)$$

[The Case of $\sigma_m > 0$]

A two-shock case is hard to analyze since there must be no invariant measure in this case, implying that the actual policy function is dependent on $\tilde{\phi}_t$. Here, I compute the pseudo-equilibrium proposed by Golosov and Lucas (forthcoming) as an approximation, where each firm is assumed to observe the mean level of $c_t$ correctly but ignore all the fluctuations around this mean level. Under this assumption, the invariant measure $\tilde{\phi}$ and $\overline{c}$ can be obtained using (22) and (25) as in the case of $\sigma_m = 0$, while $\overline{c}$ reflects the effects of $\sigma_m$.

Golosov and Lucas (forthcoming) pointed out that the model's property that money is almost neutral keeps the loss in accuracy caused by this approximation little. As seen in the subsequent sections, however, money has more effects on real consumption in the calibrated generalized GL model than in the calibrated GL model. These two things suggest that, if I adopt this pseudo-equilibrium as an approximation, there seems to be more loss in accuracy in the calibrated generalized GL model than in the calibrated GL model. Given the current level of computation ability, there is no feasible alternative method available now.

## 3.8 Approximating Markov Chains

Here, I show the construction of approximating Markov chains. Define $\tilde{x} = \log(p/w)$ and $\tilde{v} = \log(v)$. Choose some value $h$ as the grid size and define the state space $S = X \times V$. In particular, I take $h = 0.025$ in this paper. Smaller $h$ means higher accuracy of approximation while more computer memory is necessary to take smaller value as $h$. In addition, I take $\overline{v} = 0.6$ as the common upper bound of $\tilde{x}$ and $\tilde{v}$ and assume that $-\overline{v}$ is the common lower bound of $\tilde{x}$ and $\tilde{v}$.

Based on the description of finite-element methods of Kushner and Dupuis (2001), I can obtain a discrete time and state approximation of the problem (25) as seen in below:

$$\psi_j(\tilde{x},\tilde{v}_j) = \max\left\{\Pi(\tilde{x},\tilde{v}_j)\Delta t + e^{-r\Delta t}\sum_{\tilde{x}',\tilde{v}'_j}\pi_j(\tilde{x}',\tilde{v}'_j|\tilde{x},\tilde{v}_j)\psi_j(\tilde{x}',\tilde{v}'_j),\right.$$

$$\left.\max_{\xi_j}\left[\Pi(\xi_j,\tilde{v}_j)\Delta t + e^{-r\Delta t}\sum_{\tilde{x}',\tilde{v}'_j}\pi_j(\tilde{x}',\tilde{v}'_j|\xi_j,\tilde{v}_j)\psi_j(\tilde{x}',\tilde{v}'_j)\right] - k_j\right\}, \quad (26)$$

where

$$\Pi(\tilde{x},\tilde{v}_j) = \overline{c}^{1-\epsilon\gamma}(\alpha)^{-\epsilon}e^{-\epsilon\tilde{x}}[e^{\tilde{x}} - e^{-\tilde{v}_j}] \quad (27)$$

and where $\pi_j$ is a transition function, defined on $S \times S$, that I define later. The time interval $\Delta t$ is set by

$$\Delta t = \frac{h^2}{Q} \quad (28)$$

where

$$Q = \sigma_m^2 + \mu h + \sigma_{v_g}^2 + \eta\overline{v}h. \quad (29)$$

If I take time interval $\Delta t$ small enough, at most one of the variables $\tilde{x}$ and $\tilde{v}$ changes. More specifically, if I take $\Delta t$ satisfying (28) and if neither $\tilde{x}$ nor $\tilde{v}$ is at its upper bound or lower bound, then $\pi_j$ of all transitions is zero except for the following transitions:

$$\pi_j(\tilde{x}+h,\tilde{v}_j|\tilde{x},\tilde{v}_j) = \frac{\sigma_m^2/2}{Q}, \quad (30)$$

$$\pi_j(\tilde{x}-h,\tilde{v}_j|\tilde{x},\tilde{v}_j) = \frac{\sigma_m^2/2 + \mu h}{Q}, \quad (31)$$

$$\pi_j(\tilde{x},\tilde{v}_j+h|\tilde{x},\tilde{v}_j) = \frac{\sigma_{v_j}^2/2}{Q}, \quad (32)$$

$$\pi_j(\tilde{x},\tilde{v}_j-h|\tilde{x},\tilde{v}_j) = \frac{\sigma_{v_j}^2/2 + \eta(\tilde{v}_j - e_j)h}{Q}, \quad (33)$$

and

$$\pi_j(\tilde{x}, \tilde{v}_j|\tilde{x}, \tilde{v}_j) = 1 - \frac{\sigma_m^2 + \sigma_{v_j}^2 + \eta(\tilde{v}_j - e_j)h + \mu h}{Q}, \tag{34}$$

if $\sigma_{v_g}^2 \geq \sigma_{v_s}^2, \mu \geq 0$ and $\tilde{v}_j \geq e_j$ are satisfied.[13] At the boundaries, I assume the probability of staying at the current state is increased by the probability of moving out of the boundaries in the next period $(t + \Delta t)$ if $\bar{v}$ were huge. The adaptations of (30) – (34) for the other cases are obvious. I omit them for the brevity.

## 3.9 Decision Making on Price in Details

As a result of these approximations, firm's problem becomes tractable. If you follow the procedure of the value function iteration as seen in the Appendix A, you can obtain the firm's pricing strategy for each sector.

Note that, by construction, the firm's pricing strategies obtained as a result of calculation in the Appendix A are a Nash equilibrium over a continuum of monopolistically competitive firms: For given joint distributions $\{\tilde{\phi}_t\}$ of prices and productivity levels at current and future dates, each firm's pricing strategy is determined by (26). Conversely, the pricing strategies adopted by all firms define the distributions $\{\tilde{\phi}_t\}$ at future dates, given the initial distribution $\tilde{\phi}_0$.

Here, I explain the basics about firm's price setting behavior under some menu costs using (26). Figure 2 illustrates when each firm changes the nominal price of the good the firm produces if $\mu$ is positive and the variance of monetary shocks and that of productivity shocks are both zero. Note that

$$\Pi(\tilde{x}, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}_j'} \pi_j(\tilde{x}', \tilde{v}_j'|\tilde{x}, \tilde{v}_j)\psi_j(\tilde{x}', \tilde{v}_j')$$

and

$$\Pi(\tilde{x}, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}_j'} \pi_j(\tilde{x}', \tilde{v}_j'|\tilde{x}, \tilde{v}_j)\psi_j(\tilde{x}', \tilde{v}_j') - k_j$$

are expressed by bold curve and normal curve respectively. In addition, $\hat{\xi}_j$ defined by

$$\hat{\xi}_j(\tilde{v}_j) = \arg\max_{\xi_j} \left[ \Pi(\xi_j, \tilde{v}_j)\Delta t + e^{-r\Delta t} \sum_{\tilde{x}', \tilde{v}_j'} \pi_j(\tilde{x}', \tilde{v}_j'|\xi_j, \tilde{v}_j)\psi_j(\tilde{x}', \tilde{v}_j') \right] \tag{35}$$

[13] Note that $x = p/w$. Therefore, $x$ is expected to decrease after the nominal price is determined if $\mu$ is positive.

is expressed by the vertical line. As seen in this figure, each firm changes the nominal price and chooses $\hat{\xi}_j(\tilde{v}_j)$ as a new nominal price when the increase of profit caused by the price change exceeds the size of menu cost ((a), (c) in Figure 2). In other words, there is a "region of inaction" where the firm leaves its nominal price unchanged ((b) in Figure 2).

Figure 3 illustrates how $\hat{\xi}_j$ and "region of inaction" are changed by a negative productivity shock. $\hat{\xi}_j$, the upper bound and the lower bound of "region of inaction" are increased when a negative productivity shock occurs since it induces the increase of marginal cost. Note that "region of inaction" is usually enlarged by the negative productivity shock since the menu cost is assumed to be independent of the quantity sold by the firm, which implies that the effective burden of the menu cost is relatively heavy for firms with low productivity.

# 4 Predictions of the Calibrated Model for Price Changes

In this section, I calibrate the model and run simulations of the calibrated model for price changes. The predictions can be used to evaluate this model.

Outline of this section is as follows: First, I explain why some specific sample moments are used to calibrate this model. Then, I calibrate this model. To do calibration, I did some simulations to obtain survival rates or hazard functions for price changes. The method of this simulation is straightforward. I describe this method in Appendix B. It is shown that the calibrated model can generate decreasing hazard.[14] Then, I test this model checking whether this model can predict some other empirical facts on individual price data.

## 4.1 Sample Moments for Calibration

I choose the values of $\eta, \sigma_{v_j}$, and $k_j$ so that the model's predictions on the following sample moments fit the data best: survival rates, the share of price increases in price changes, and the average size of price changes.[15] These sample moments are obtained from the empirical research on individual retail price data in Japan due to Saita et al. (2006). Specifically, in the next subsection, I use the sample moments based on the data from 1999-2003 when Japanese economy was in moderate deflation.

[14]While I also perform sensitivity analysis, I omit the results of them for brevity. If you want to see the results, please send me email. Then, I may provide them to you.

[15]The survival rate of period $t$ is defined as the share of nominal prices unchanged in first $t$ periods after a price change. The hazard rate of period $t$ is defined as the conditional probability that a nominal price is changed in period $t$ given that the price is unchanged in the first $(t-1)$ periods after a price change.

Here, I explain why I use these sample moments to calibrate $\eta, \sigma_{v_j}$, and $k_j$. First, survival rate is necessary to calibrate the two-sector model since this includes the information about heterogeneity in frequency of price changes across goods.[16]

Second, the share of price increases in price changes is also important because this information prevents overestimates and underestimates of $\sigma_{v_j}$. Note that price changes occur due to nominal wage changes or productivity shocks. Since the productivity shocks are assumed to be symmetry, larger $\sigma_{v_j}$ implies higher degree of symmetry in price changes given the value of $\mu$ in the sense that the share of price increases becomes closer to 50 percent. Therefore, if I overestimate the value of $\sigma_{v_j}$, then the share of price increases predicted by the model becomes too close to 50 percent.

Third, the average size of price changes is also useful. Suppose that you observe low frequency of price changes. To replicate low frequency, there are two choices in the general case: High $k_j$ or low $\sigma_{v_j}$. On the one hand, higher $k_j$ implies larger average size of price changes. On the other hand, lower $\sigma_{v_j}$ generally implies smaller average size of price changes because low $\sigma_{v_j}$ implies relatively large effects of price on firm's profit. Intuitive explanation is given in Figure 4.[17] Thus, the average size of price changes suggests why price changes are so frequent (or infrequent).

## 4.2 Calibration

Here, I specify values of all parameters in the model. As for preference parameters $\rho$ (subjective discount rate), $\gamma$ (relative risk aversion), and $\epsilon$ (elasticity of substitution), I draw on the existing research about these parameters in Japan. Coincidentally, the existing research seems to suggest that the values Golosov and Lucas (forthcoming) used for those parameters in the United States might be applicable in Japan as summarized in Table 1. Therefore, I set $(\rho, \gamma, \epsilon) = (0.01, 2, 7)$. As for $\alpha$, I draw on Golosov and Lucas (forthcoming) and set $\alpha = 6$, implying that about 37 percent of the unit time endowment is allocated to work.[18]

[16] As seen in Figure 8, the empirical survival rate decreases sharply in the first few months after a price change while it decreases slowly otherwise. This suggests that a substantial fraction of price changes occur in the first few months after a price change while a considerable fraction of individual prices remain constant in two or three years after a price change.

[17] If the variance of productivity shocks is high, then the possibility of changes of productivity level is high. Therefore, the discounted value of the firm's profits reflects the profits consistent with various states of productivity. As a result, the discounted value of profits is not sensitive to the real price ($p/w$). Given the level of menu cost, this implies that "the region of inaction" is wide and the size of price changes is large.

[18] As a result, $(\rho, \gamma, \epsilon, \alpha)$ in this paper is the same as in Golosov and Lucas (forthcoming). This eases the comparison.

I set the values of parameters of monetary shocks, $\mu$ and $\sigma_m$, based on the consumer price index from 1999-2003 (Table 2). This is because money was almost neutral in the GL model. In the next section, you can confirm that money is almost neutral in the generalized GL model although the neutrality of money in the generalized GL model is weaker than that in the GL model.

The difference of average productivity between "g sector" and "s sector" is based on SNA data from 1999-2003 (Table 2 and Figure 5). Using the weights of consumer price index adjusted by Saita et al. (2006), I calculate the labor productivity (per hour basis) of goods sector including "agriculture, forestry and fishing," "food products and beverages," and "textiles," and that of service sector including "service activities," "transport and communications" and "electricity, gas and water supply" industries.[19] The definition of each sector is determined so that industries characterized by low frequency of price changes are categorized into service sector. Thus, I can associate goods sector with "g sector" and service sector with "s sector." As a result, the average productivity of "s sector" is assumed to be 10 percent higher than that of "g sector."

After setting these parameters, I perform value function iteration following steps described in Appendix A and obtain hazard rates through simulations following steps described in Appendix B for each set of values $(\eta, \sigma_{v_j}, k_j)$. Note that the share of price increases in price changes, the average size of price increases and that of price decreases for each set of values $(\eta, \sigma_{v_j}, k_j)$ can be calculated based on the invariant joint distributions, which is obtained as a result of value function iteration.[20] Now, I also know survival rates for each set of values $(\eta, \sigma_{v_j}, k_j)$ since I've already known hazard rates. Thus, I can choose the values of $\eta, \sigma_{v_j}$, and $k_j$ so that the model's predictions on these sample moments are fitted best.

The selected values of $\eta, \sigma_{v_j}$, and $k_j$ can be seen in Table 2. As a result of calibration, the variance of productivity shocks of "g sector" is assumed to be larger than that of "s sector." The menu cost of "g sector" is assumed to be smaller

[19] These industries cover about 77 percent of goods and services based on the weights of consumer price data compiled by Saita et al.(2006). About 23 percent of goods and services are not covered mainly because industries in SNA data are defined too roughly. For example, the labor productivity of "precision instruments" industry should not be associated with the labor productivity of firms producing watch. This is because the "precision instruments" industry includes not only firms producing watch consumers use but also firms producing big machineries consumers never use.

[20] Note that, in this analysis, the minimum size of price change is 2.5 percent since I take $h = 0.025$. Because of this, the size of price change may be overestimated somewhat. Although the best way to improve the estimation is to take smaller value for $h$, it takes long time or may be impossible because of the limitation of the memory capacity. Therefore, I settle for the second best: As the model's predictions on the average size of price changes, I use the values which are the original predictions on the average size minus 1.25 percent.

than that of “s sector.”[21]

The pricing strategies obtained as a result of calibration are summarized in Figure 6 and Figure 7. In these figures, gray grids represent $\hat{\xi}_j$ and the region between two black grids given the productivity level represents “region of inaction.” As long as a firm is in “region of inaction,” then the firm keeps its nominal price level. If a firm with its current productivity level $v_j$ moves in the black grid, then this firm changes its nominal price and chooses $\hat{\xi}_j(\tilde{v}_j)$ as its new nominal price. There are two main features in these two figures: (A) The region of inaction for the firms with low current productivity level is wider than that for the firms with high current productivity level although there are some exceptions .[22] (B) Roughly speaking, the region of inaction for “s sector” firms is wider than that of “g sector” firms.

(A) holds because the effective burden of the menu cost is relatively heavy for firms with low productivity.[23] (B) comes from the assumptions that $k_g < k_s$. It is obvious that larger menu cost implies wider region of inaction.

The sample moments predicted by the calibrated generalized GL model are summarized in Figure 8 and Figure 9. By and large, this calibrated model succeeds in replicating these sample moments: (i) Predicted hazard rates are decreasing in time. (ii) Predicted share of price increases in price changes is about 50 percent. (iii) Both the average size of price increases and that of price decreases predicted by this calibrated model are around 6 percent. (i)-(iii) are consistent with the empirical facts from 1999-2003.[24]

## 4.3 Predictions of the Model

To test this model in more strict way, I also check whether this model can predict some facts on individual price data: (i) The shape of hazard function is robust in the sense that decreasing hazard is observed in United states and Euro area where

[21]The labor required to adjust prices in this calibrated model is equal to 0.2 percent of overall employment while that in the calibrated one sector GL model is equal to 0.5 percent. The menu cost in this calibrated model is about 0.1-0.2 percent of revenues while that in the calibrated one sector GL model is about 0.5 percent of revenues. Levy et al. (1997) estimate that the menu cost in supermarkets is about 0.7 percent of revenues. Note that the frequency of price changes used here, which is obtained from Saita et al (2006), is calculated based on the data which do not reflect price changes due to the promotional sale. This thing may at least partly explain why the menu cost obtained as a result of calibration seems to be small.

[22]One may find the difference in the shape of “region of inaction” between these two figures. I will explain this point in the next section.

[23]See the subsection 3.9 and Figure 3 for more details.

[24]The frequency of price changes predicted by the calibrated model is close to the observed frequency (Data: 23.1 percent per month, Prediction (after excluding double-counting): 24.8 percent). See Figure 11 and its note.

the average inflation rate is higher than in Japan. (ii) The frequency of price increases responds strongly to inflation while the frequency of price decreases and the size of price increases and price decreases do not.

In this test, only the parameters of monetary shocks are changed to generate different average inflation rates. In concrete, I use the following values as $(\mu, \sigma_m)$: CY1999-2003: (-0.0017, 0.0037), CY1986-1990: (0.0039, 0.0078), CY1980-1985: (0.0079, 0.0077), CY1971-1975: (0.0272, 0.0204).[25] These numbers are obtained from CPI data in the corresponding periods as I did in the calibration part. Since I use data under zero or negative inflation to calibrate this model, this is a strict out-sample test of the model.

The model provides good predictions about (i) and (ii) as seen in Figure 10 and Figure 11. The predicted hazard function is decreasing as long as the average inflation is moderate. This is consistent with the empirical facts that the observed hazard function is decreasing in United States and Euro area. The predictions of the model is also consistent with (ii). The predicted frequency of price decreases doesn't respond strongly to inflation because all price decreases are caused by the idiosyncratic productivity shocks when $\mu$ is positive.[26]

# 5 Monetary Policy Experiments

In this section, I use the calibrated generalized GL model to conduct numerical experiments on the economy's response to various shocks as Golosov and Lucas (forthcoming) did. I focus on the model's predictions about the relationship between inflation and GDP.

## 5.1 Procedure of Experiments

Here is how I obtain the relationship between inflation and the level of production for different values of $(\mu, \sigma_m)$.

(1) Assume that the initial distribution is the invariant one consistent with $(\mu, \sigma_m)$.

[25]The average annual inflation rates are as follows: CY1999-2003: -0.7 percent, CY1986-1990: 1.6 percent, CY1980-1985: 3.2 percent, CY1971-1975: 11.3 percent.

[26]Suppose $\mu = 0$. Then, all price increases and decreases are caused by the idiosyncratic productivity shocks, which are horizontal movements in Figure 6 or Figure 7. Now, increase the value of $\mu$. Note that positive $\mu$ implies that $(p/w)$ tends to decrease, which is downward movement in Figure 6 or Figure 7. Because the downward movement and vertical movements are assumed to be independent, this increase in the value of $\mu$ raises strongly the probability of hitting the lower black grids but not changes the probability of hitting the upper black grids in Figure 6 or 7 very much.

(2) Generate the sequences of money with average growth rate $\mu$ and the variance $\sigma_m^2$ and the sequences of productivity shocks with $\eta$ and $\sigma_{v_j}^2$. Based on the pricing strategy consistent with $(\mu, \sigma_m)$, calculate the firm's pricing behaviors.

(3) Aggregating the firm's pricing behavior, calculate the inflation rates and GDP defined as $y_t = \int C_t(p)\phi_t(dp, dv)$.

## 5.2 Results

The typical results of the above experiments can be seen in Figure 12. In this figure, I choose the level of GDP consistent with a stable price environment, i.e. $(\mu, \sigma_m) = (0, 0)$, as a bench mark. The relationship between inflation and GDP predicted by this calibrated model has some interesting features: (i) Roughly speaking, the slope of Phillips curve is increasing in inflation rates, which is consistent with empirical facts established by Benati (forthcoming). (ii) High inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies stable and high average GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005). I explain the reasons of (i) and (ii) below.

The region of inaction for "s sector" firms with current productivity low is relatively wide as seen in Figure 6 and Figure 7. Moreover, you can see that $\hat{\xi}_s$ for firms with low productivity is "distorted" in the sense that this wide region of inaction implies downward nominal price rigidity. Given this downward rigidity, it is easy to explain (i): When $\mu$ is positive but very low or negative, then money must decrease sometimes. Note that the decrease of money has some real effects although the increase of money has almost no real effects since there is a certain downward rigidity while there is not significant upward rigidity. Therefore, resulting relationship between inflation and GDP exhibits the feature summarized as (i).

The remaining problem is why the region of inaction exhibits downward rigidity and does not exhibit upward rigidity. Note that the region of inaction for "g sector" firms is narrow mainly because of the relatively small menu cost. The region of inaction for "s sector" firms with high productivity is also narrow because the effective burden of the menu cost is relatively light for firms with high productivity.[27]

The region of inaction for "s sector" firms with low productivity is wide since the menu cost is relatively large and current productivity is low. Intuitive explanation of the "distortion" for "s sector" firms with low productivity can be seen

[27] See the subsection 3.9 and Figure 3 for more details.

in Figure 13. This "distortion" is a result of the assumption that the productivity is mean-reverting. If the current productivity is low, then the expected future productivity is higher than the current one because of the mean-reversion. The discounted value of the firm's profits reflects the profits consistent with higher productivity. Note that the profit consistent with higher productivity is maximized at lower relative price ($p/w$) since higher productivity implies lower marginal cost. Therefore, the typical relationship between discounted value of the firm's profits and the relative price can be expressed as in the right-hand side of Figure 13.

To explain (ii), I need to show why high inflation implies volatile and low average GDP. Volatile GDP comes from large $\sigma_m$.[28] Because of large $\sigma_m$, the money sometimes decreases. Low average GDP is a result of firm's pricing strategy: As seen in Figure 14, higher $\mu$ generally implies higher $\hat{\xi}_j$. Each firm chooses high $\hat{\xi}_j$ since inflation lowers its real price during the period with its nominal price unchanged. Therefore, higher $\mu$ implies upward shift of the distribution of real prices. This dampens the demand for goods and services.

Golosov and Lucas (forthcoming) show that the slope of Phillips curve of the calibrated one-sector GL model is always steep. This is because the calibrated one-sector GL model generates prices such that almost all prices change in first one year after a price change, implying that money is nearly neutral. Thus, these predictions about the relationship between inflation and GDP shown in this section are the new implications obtained as a result of the generalization of the GL model[29].

# 6 Concluding Remarks

I've presented a two-sector menu cost model with idiosyncratic productivity shocks. I split firms into two sectors since the hazard function for price changes implies heterogeneity in frequency of price changes. I name high frequency sector "g sector" and low frequency sector "s sector." This model includes the GL model as a special case since this model becomes the GL model if and only if each sector is identical. I use the sample moments on individual prices in Japan calculated by Saita et al. (2006) to calibrate the menu cost and the variance and autocorrelation of the idiosyncratic shocks. As a result of calibration, "g sector" is characterized

[28] Usually, $\sigma_m$ is large when $\mu$ is large.

[29] Nakamura and Steinsson (2006b) analyze the menu cost model which allows for intermediate goods. They show that the monetary non-neutrality in their multi-sector model is clearer than that in their one sector model. As for the monetary non-neutrality, the analyses presented here suggest that the difference between one sector model and multi-sector model is larger in deflationary or very low inflationary environments. It is curious if the model of Nakamura and Steinsson (2006b) exhibits the same characteristic.

by relatively small menu cost, large productivity shocks, and low average productivity. The behavior of this economy is studied numerically.

As for price changes, the predictions of the generalized GL model are consistent with almost all facts, including decreasing hazard, found by recent research on microdata of individual prices: The only exception I know is the seasonality of the frequency of price changes. Thus, the performance of the generalized GL model to fit the facts on microdata of individual prices is best among the menu cost models since all previous menu cost models couldn't explain decreasing hazard and seasonality of the frequency of price changes.

Then, I use this calibrated model to conduct numerical experiments on the economy's response to various shocks as Golosov and Lucas (forthcoming) did. The relationship between inflation and GDP predicted by this calibrated model has some interesting features: (1) The slope of Phillips curve is increasing in inflation rates, which is consistent with empirical facts established by Benati (forthcoming). (2) High inflation, near zero inflation and deflation imply volatile and low average GDP while moderate inflation implies high and stable GDP, which is consistent with empirical facts about Japan and US data found by Sakura et al. (2005).

These implications may be important to understand why deflation is bad theoretically.[30] To explain Phillips curve relationship during the recent deflation in Japan, there may be three ways: (A) GDP was lowered by some negative shocks, and deflation was caused by low GDP, (B) deflation was caused by monetary shock, and low GDP was caused by deflation, and (C) deflation and low GDP are independent of each other, implying relationship is observed just by accident. This paper may provide theoretical backbone for (B). Note that, however, this paper doesn't prove that (A) is not true: The views such as (A) are just out of the scope of this paper.

In addition, note that the quantified effects of monetary policy in this paper may not be so precise since I use a very simplified model in this paper. To enhance the precision, further research must be done. I end this paper providing the lists for the future research: (i) Divide firms into more sectors. (ii) Generalize this model so as to deal with the difference in the average productivity growth rate of each sector. (iii) Change the specification of the production function into more realistic form. Especially, include capital into the model. (iv) Change the specification of the utility function into more realistic form.

(i) and (ii) is connected since if you divide firms into multi-sector, then you might need to deal with the difference in the average productivity growth rate of

[30] In this model, the increase in GDP means a welfare improvement. This is because firms have some monopolistic power, implying that the production under the price stability is too small in terms of social welfare.

each sector.[31] There is a possibility that the results in this paper will be changed somewhat if you deal with the difference in the average productivity growth rate of each sector carefully. While this extension is desirable, it is difficult since the problem after this extension is no longer recursive.

(iii) is interesting since investment may amplify the fluctuations caused by monetary policy. It is, however, also difficult because of "curse of dimension" that I include capital into the generalized GL model making state space huge. (iv) is necessary to implement reliable welfare analysis. It is known, however, that any change of the form of the utility function used in this paper makes the problem much harder as suggested by Golosov and Lucas (forthcoming). Thus, I need to leave (i)-(iv) as a future research.

# Appendix

## A. Value Function Iteration

Here, I describe how to obtain the value of $\psi_j(\tilde{x}, \tilde{v}_j)$ using (26).

(1) Guess the value of $\bar{c}$.[32] Guess the joint distribution of firms in "g sector" and that of firms in "s sector" respectively.[33] Guess the value of $\psi_j$.[34]

(2) Solve (26) based on the guess I made and update the values of $\psi_j$. If the values of $\psi_j$ are close enough, in the sense of sup norm, to the previous guess of them, go to the next step. Otherwise, go back to the step (1) and use the values of $\psi_j$ as a new guess.

(3) Given the values of $\psi_j$ obtained in the previous step, obtain the pricing strategy (policy function) of "g sector" firms and that of "s sector" firms. Using these strategies and taking account of the effects of $\pi_j$, change the joint distribution of firms in "g sector" and that of firms in "s sector." Based on the obtained joint distributions, calculate the value of $\bar{c}$. If these joint distributions and the value of $\bar{c}$ are close enough to the previous guess of them, stop. Otherwise, go back to step (1) and use these joint distributions and the value of $\bar{c}$ as a new guess.

[31] In the two-sector case in Japan, there is no difference in the average productivity growth rate of each sector (Figure 5). This might be because of good luck.

[32] If you have a good guess as a result of calculations for different but similar values of parameters, use it as an initial guess. If you don't have a good guess, calculate the value of $\bar{c}$ in the case of zero variance of all shocks and zero menu cost, and use this value as an initial guess.

[33] If you don't have a good guess, calculate the joint distribution of firms in "g sector" and that of firms in "s sector" in the case of zero variance of all shocks and zero menu cost, and use these joint distributions as an initial guess.

[34] If you don't have a good guess, use zeros as an initial guess.

## B. Method of Simulation to Obtain Hazard Functions

Here, I describe how to obtain the hazard rate implied by the generalized GL model with given parameter values.

(1) Generate the sequence of money with average growth rate $\mu$ and the variance $\sigma_m^2$ and the sequences of productivity with $\eta$ and $\sigma_{v_j}^2$ starting with grids $(\hat{\xi}_j(\tilde{v}_j), \tilde{v}_j)$ using random numbers and (30) – (34).[35]

(2) Record the timing of price changes given the pricing strategy consistent with the parameter values, the sequence of money and that of productivity. Based on the invariant joint distribution consistent with the parameter values, obtain the weight of firms with each productivity level which change their nominal prices. Using these weights and information of the timing of changes of each price starting with grids $(\hat{\xi}_j(\tilde{v}_j), \tilde{v}_j)$, calculate the hazard rates.

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[36]The English version of this paper is forthcoming.

# Table 1: Preference Parameters

(1) Subjective Discount Rate (Quarter) $\rho$ [Golosov and Lucas (2006): 0.01]

| Research Paper | Subjective Discount Rate |
|---|---|
| Kitamura and Fujiki (1997) | 0.01 |
| Hayashi and Prescott (2002) | 0.006 |

(2) Relative Risk Aversion $\gamma$ [Golosov and Lucas (2006): 2]

| Research Paper | Relative Risk Aversion |
|---|---|
| Kitamura and Fujiki (1997) | 0.6 - 2.5 |
| Yoshikawa (2001) | 1.36 |
| Moridaira and Kamiya (2001) | 1 |

(3) Elasticity of Substitution $\epsilon$ [Golosov and Lucas (2006): 7]

| Research Paper | Elasticity of Substitution |
|---|---|
| Nishimura, Ohkusa, and Ariga (1999) | 7 |
| Inui and Kwon (2004) | 6 |

Note: Nishimura, Ohkusa, and Ariga (1999) and Inui and Kwon (2004) estimate not the elasticity of substitution but the mark-up. In this model, the mark-up is a function of the elasticity of substitution. Given the relationship, I find the value of the elasticity of substitution which is consistent with the estimated value of the mark-up.

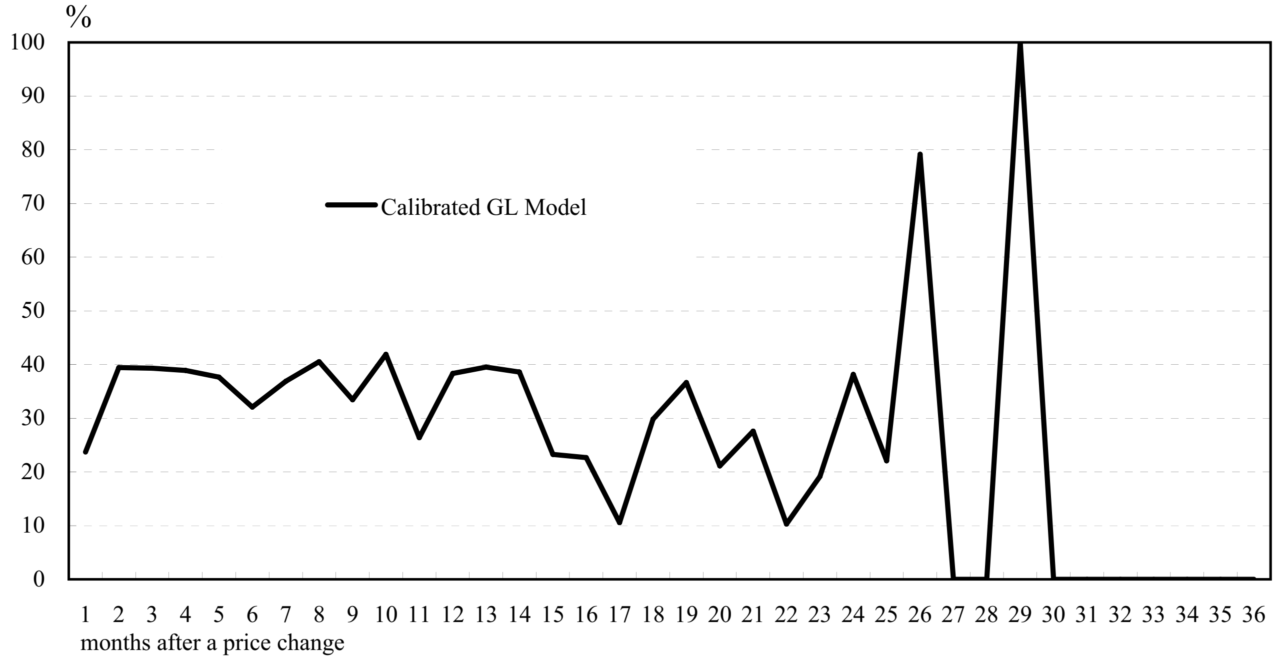
Table 2: Parameters

| | Parameters | Value of Parameter | |
|---|---|---|---|
| $\mu$ | Average Growth Rate of Money (Quarter) | -0.0017 | ( 0.0064) |
| $\sigma_m$ | S.d. of monetary shocks | 0.0037 | ( 0.0062) |
| $k_g$ | menu cost ("g sector") | 0.00055 | ( 0.0025) |
| $k_s$ | menu cost ("s sector") | 0.008 | ( - ) |
| $\eta$ | Rate of Mean Reversion | 0.75 | ( 0.55) |
| $\sigma^2_{V_g}$ | Variance of productivity shocks ("g sector") | 0.011 | ( 0.011) |
| $\sigma^2_{V_s}$ | Variance of productivity shocks ("s sector") | 0.0005 | ( - ) |
| $e_g$ | Average productivity of "g sector" firms | -0.05 | ( 0) |
| $e_s$ | Average productivity of "s sector" firms | 0.05 | ( - ) |
| | Fraction of "g sector" firms | 0.5 | ( 1) |
| | Fraction of "s sector" firms | 0.5 | ( - ) |

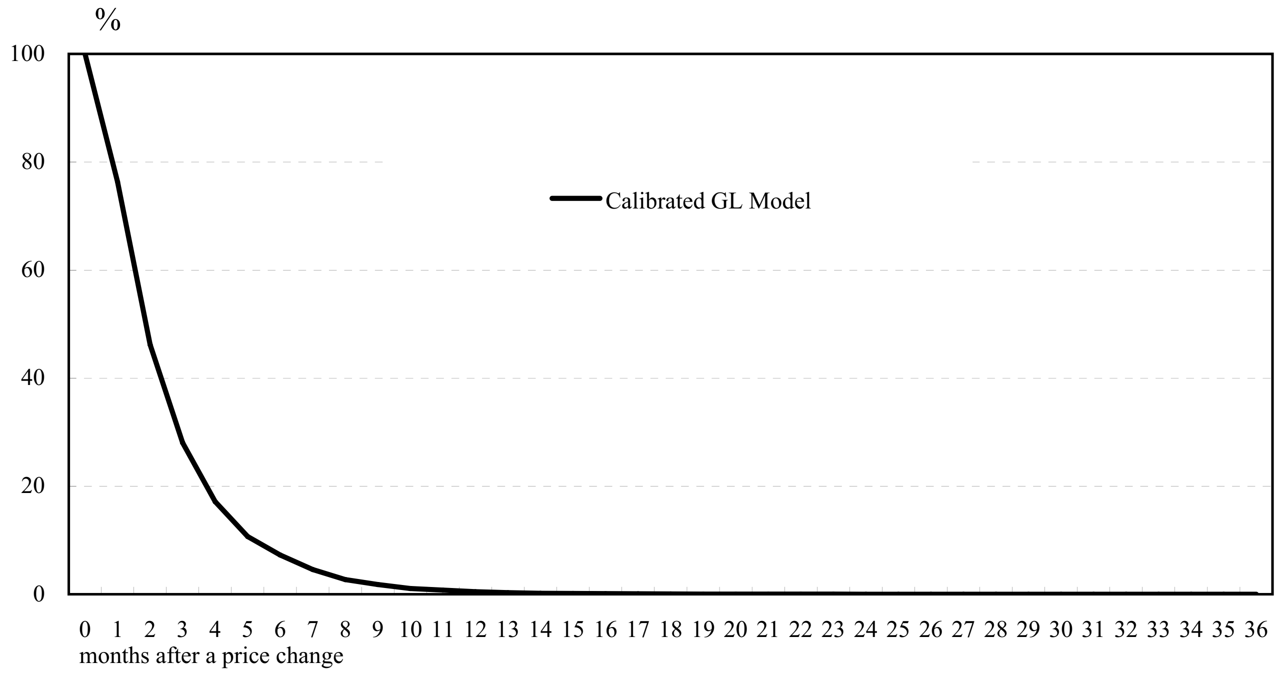
Note: Each value in the parenthesis is the value used by Golosov and Lucas (forthcoming).

Figure 1: Hazard Rates and Survival Rates of Calibrated GL Model

(1) Hazard Rates




(2) Survival Rates




Notes: 1. I use the values of parameters Golosov and Lucas (forthcoming) obtain as a result of their calibration. (I use 0.0062 as $\sigma_m$ .)

2. As for the definition of hazard rates and survival rates, see subsection 4.1.

3. As for the method of simulation to obtain hazard rates, see Appendix B.

Figure 2: Decision Making on Price

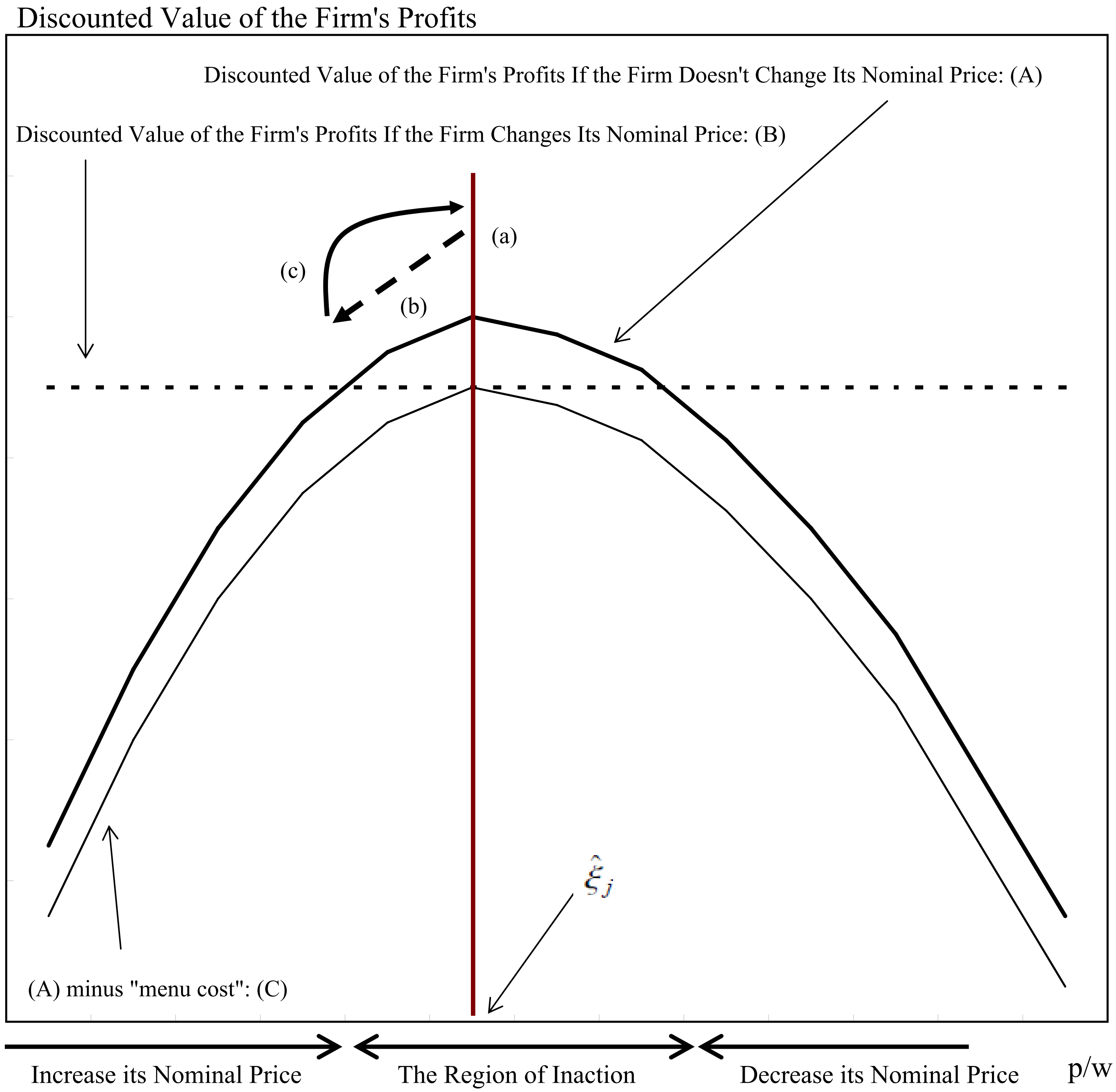


Figure3: The Region of Inaction and Productivity shocks

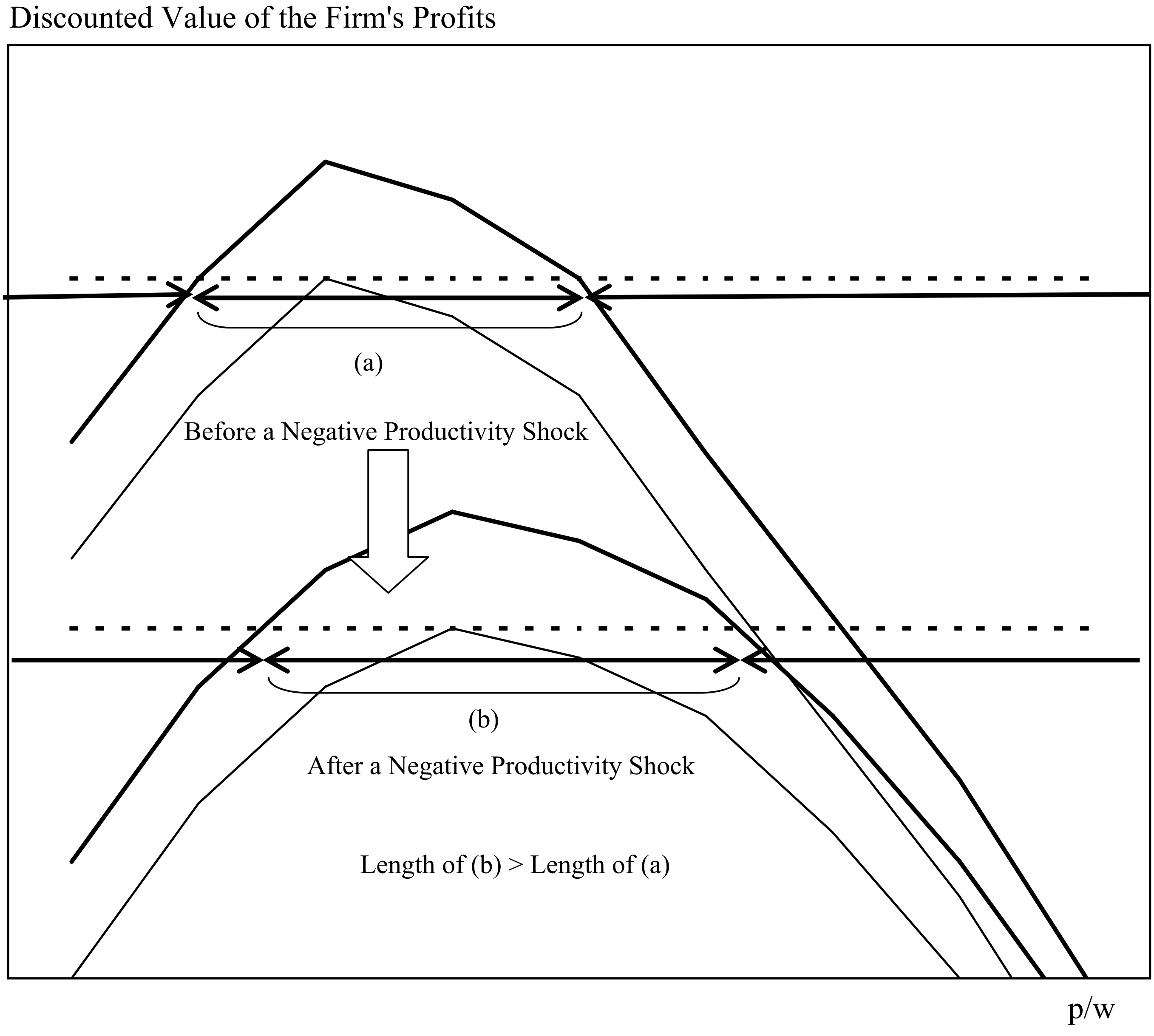


Figure 4: Size of Price Change and Variance of Productivity Shocks

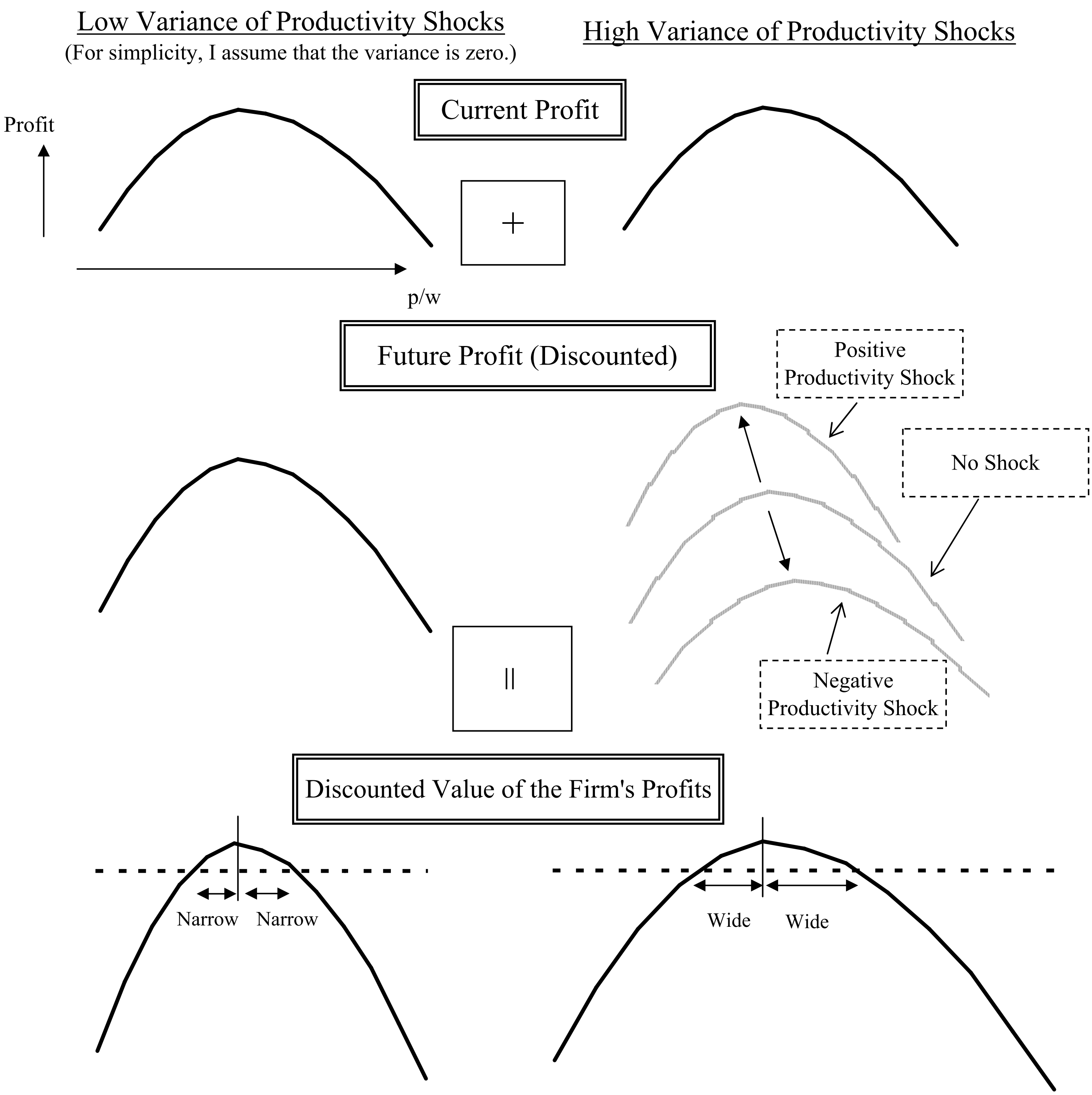


Figure 5: Difference of Average Productivity between goods and services (services/goods)

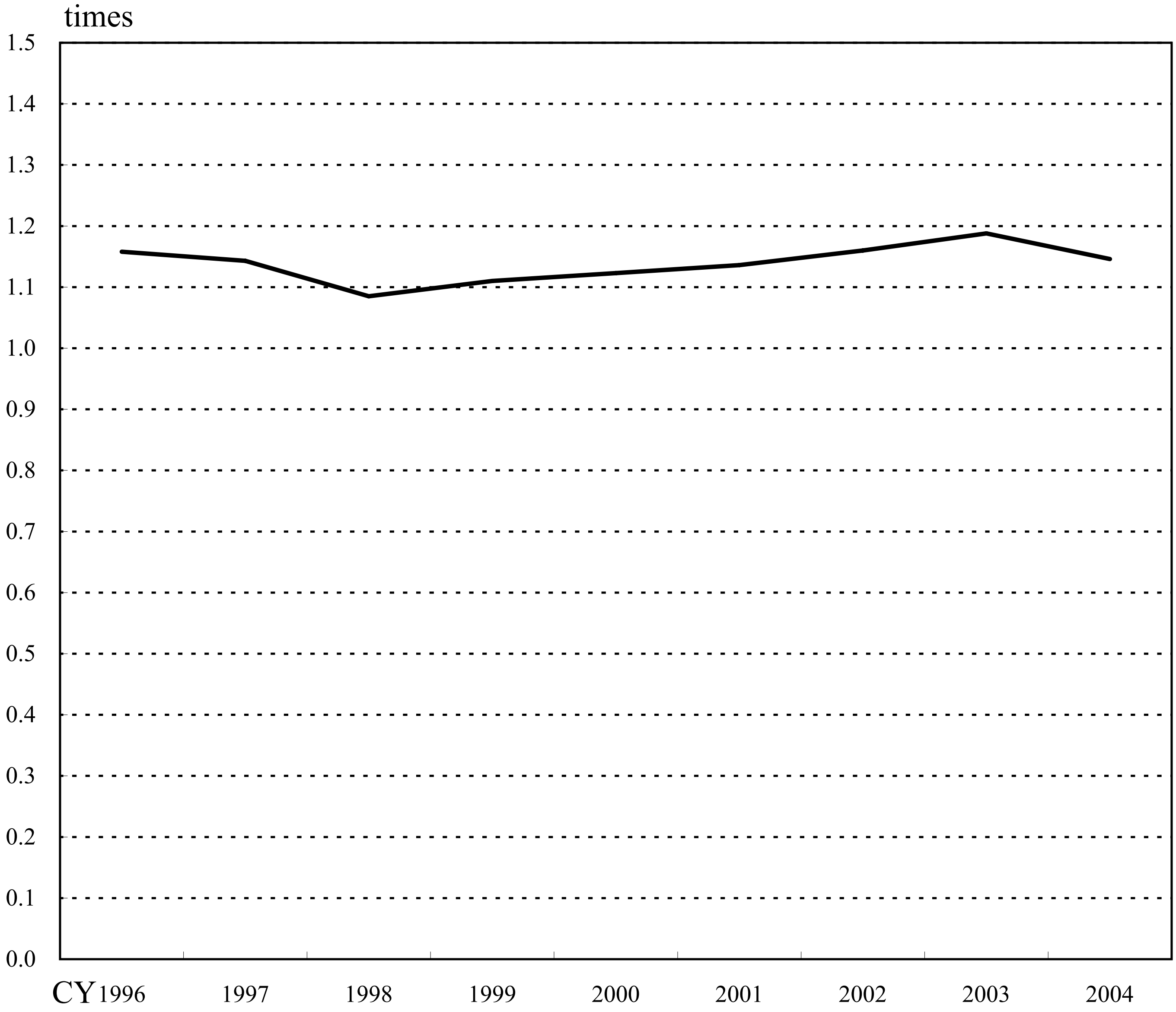


Figure 6: Pricing Strategy ("g sector")

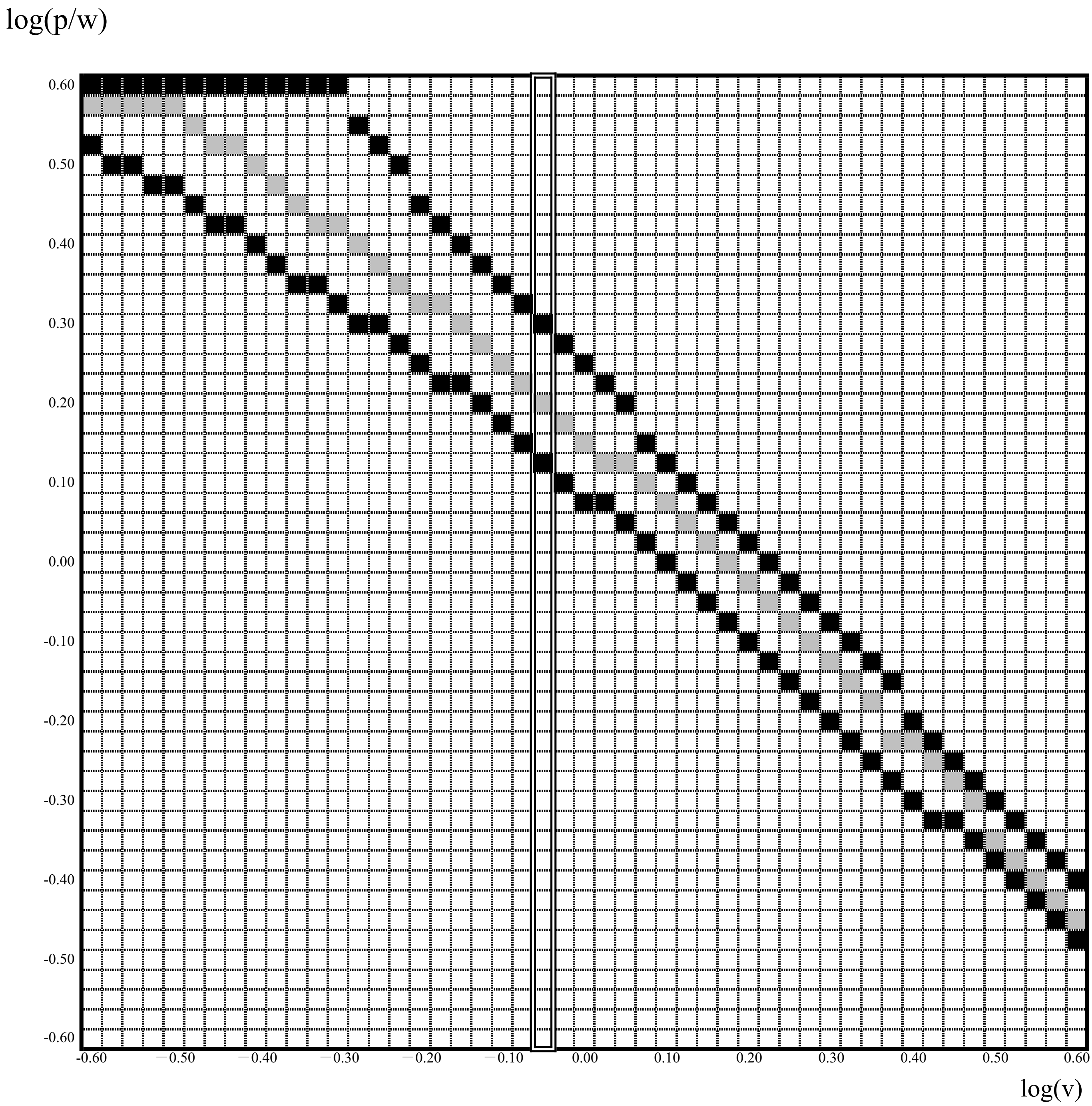


Figure 7: Pricing Strategy ("s sector")

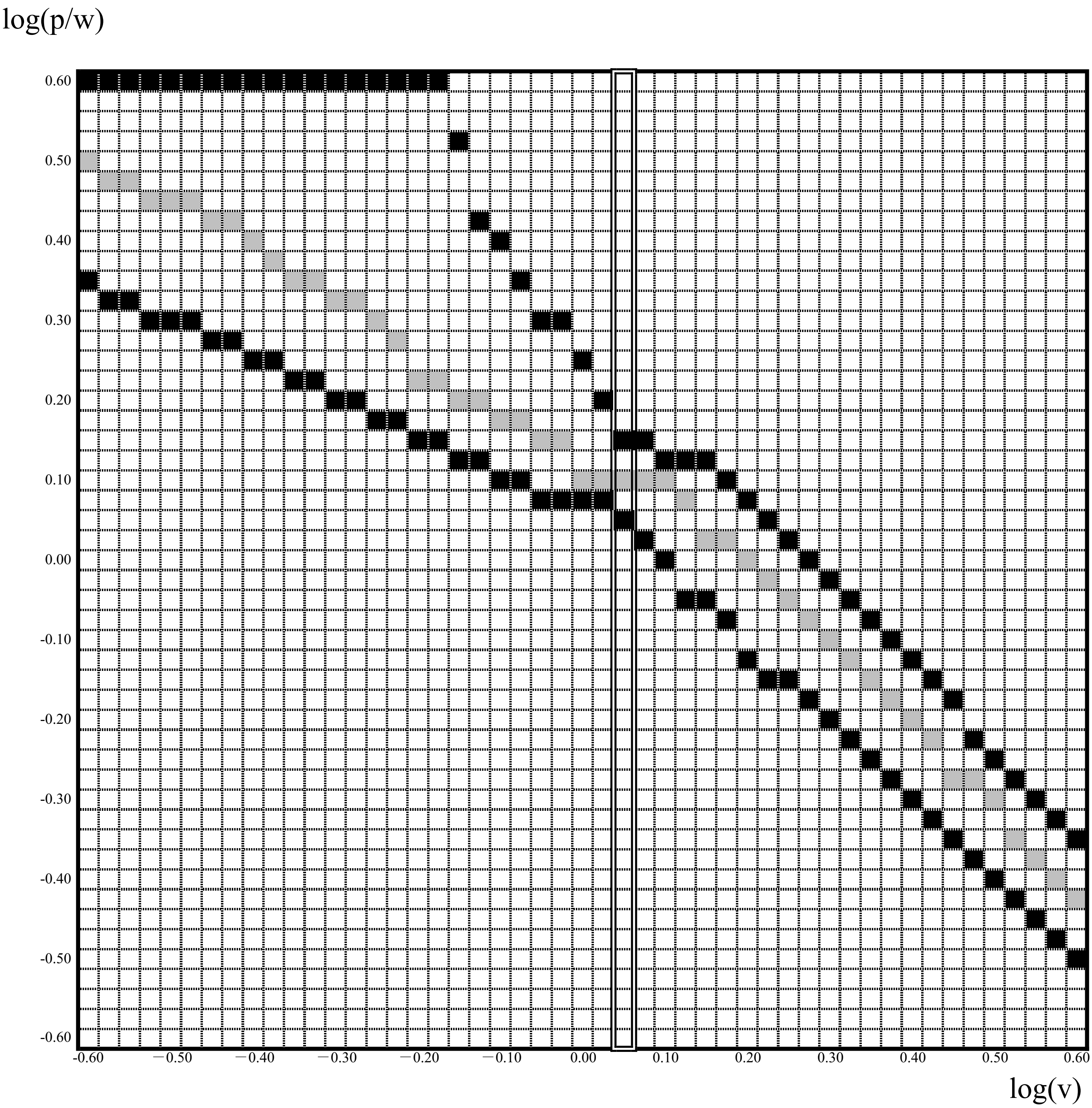
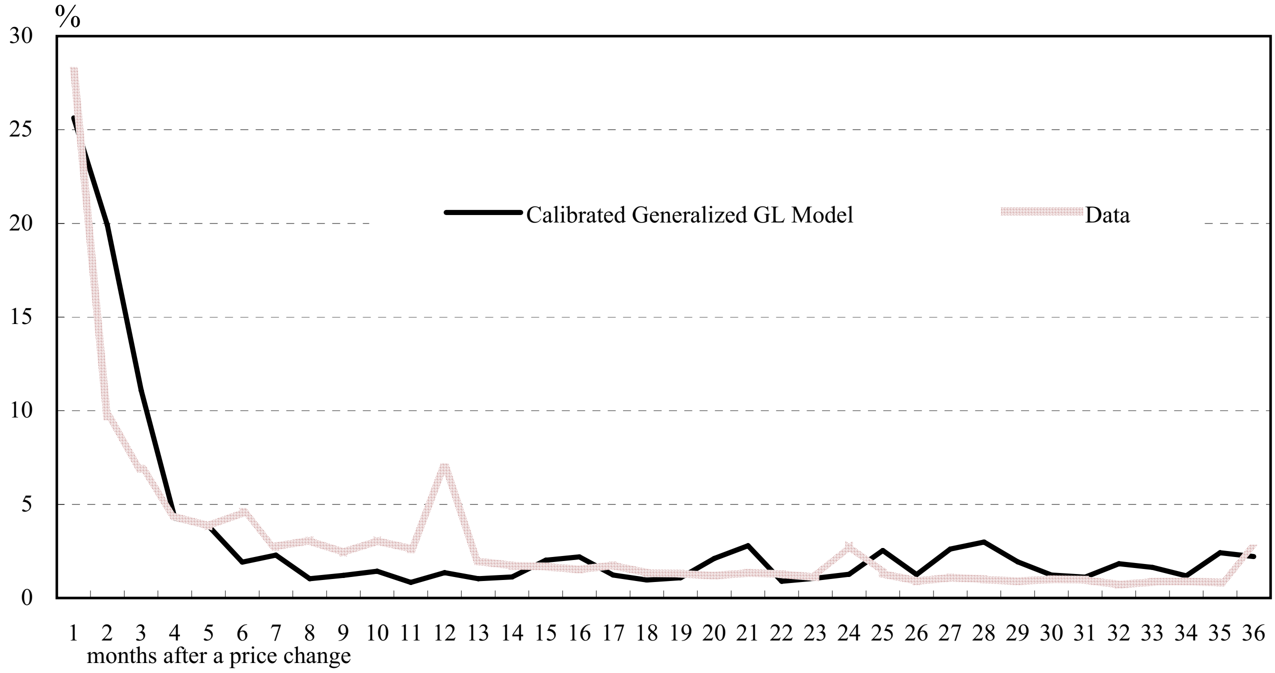


Figure 8: Hazard Rates and Survival Rates of Calibrated Generalized GL Model

(1) Hazard Rates




(2) Survival Rates

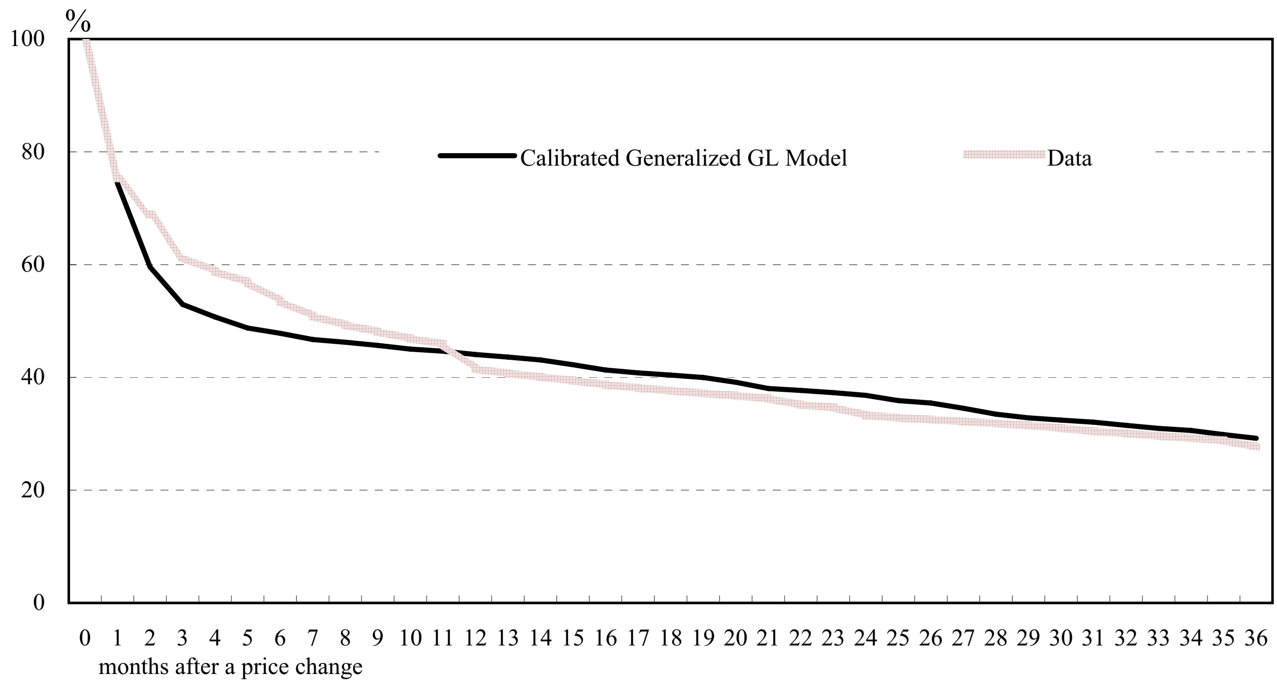
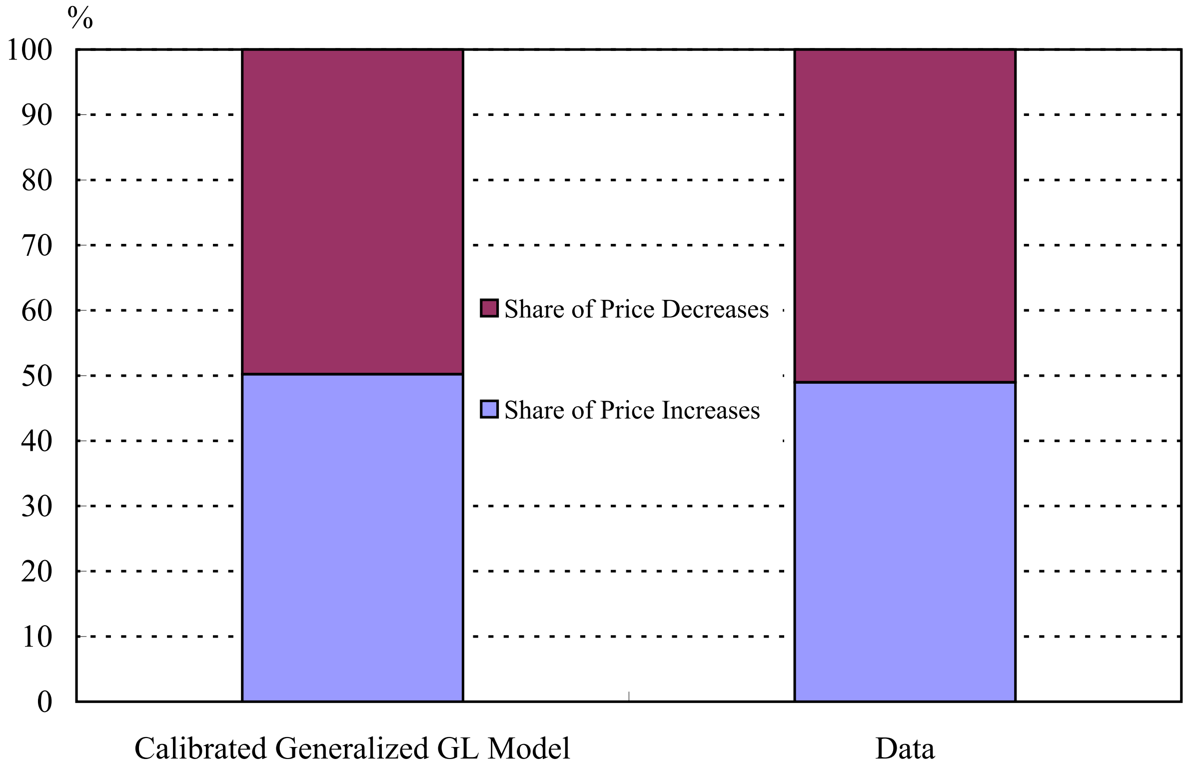


Figure 9: Share of Price Increases (Decreases) and Average Size of Price Changes

(1) Share of Price Increases (Decreases) in Price Changes




(2) Average Size of Price Changes

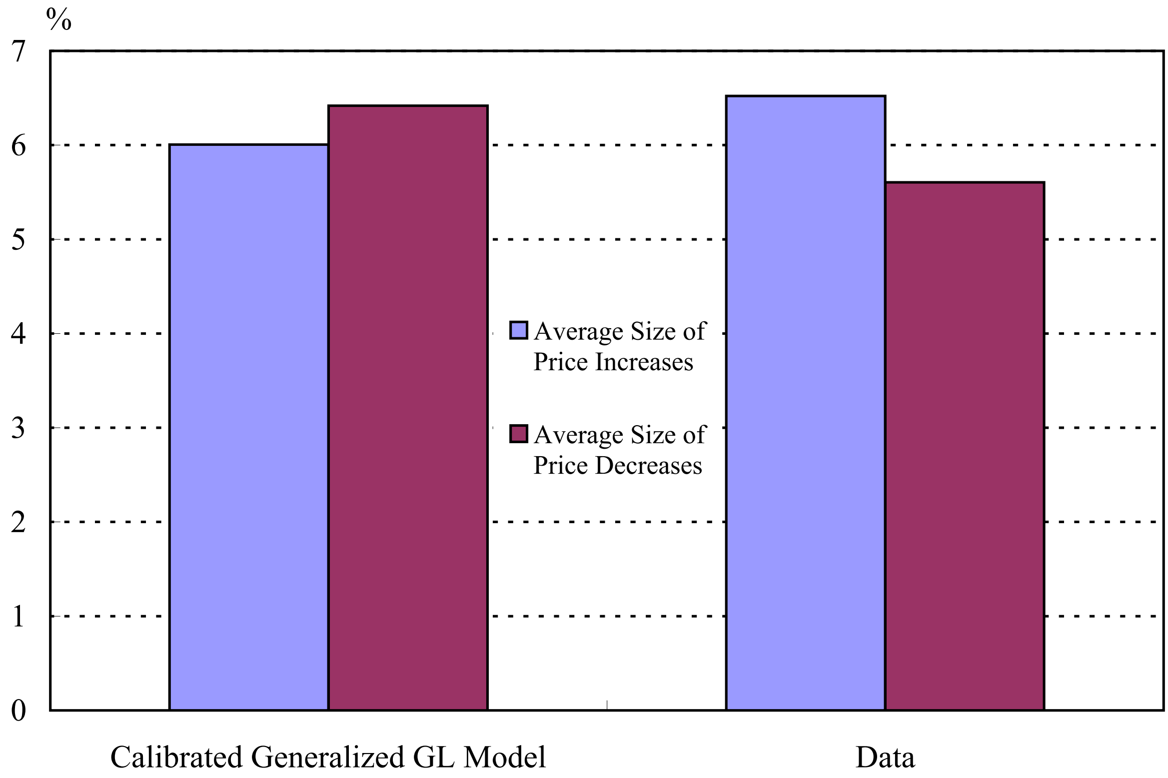
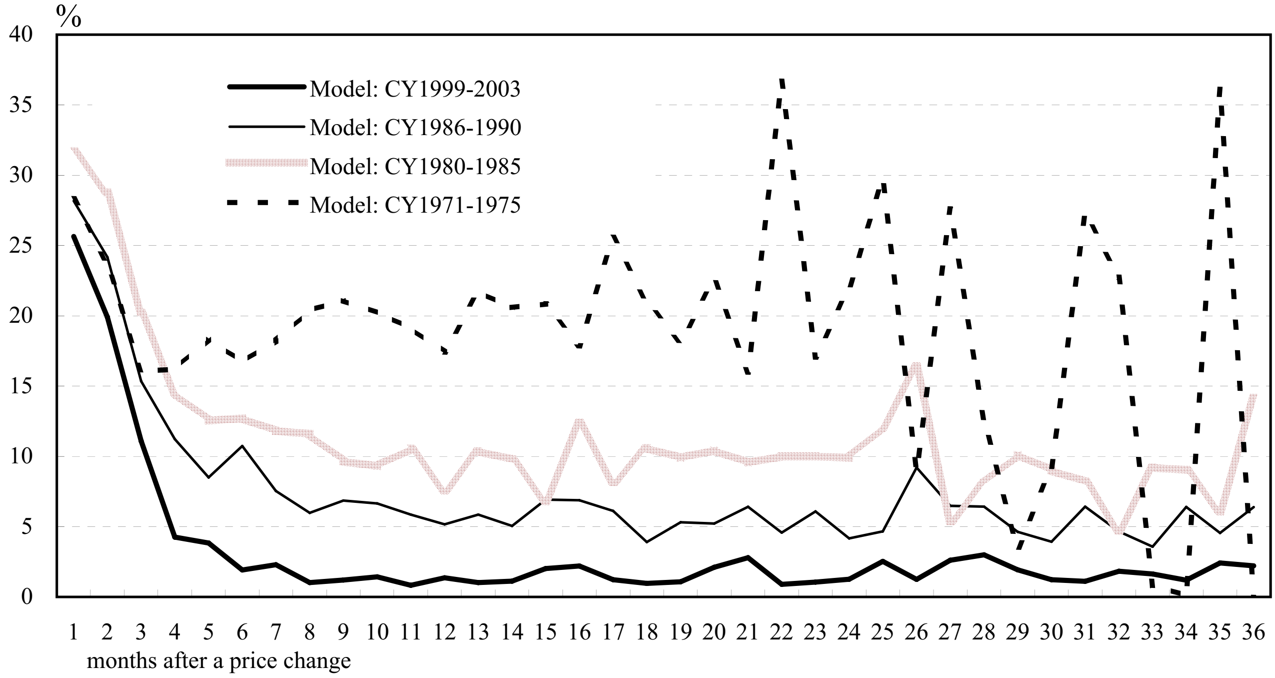


Figure 10: Hazard Rates and Survival Rates under Different Monetary Shock Processes

(1) Hazard Rates




(2) Survival Rates

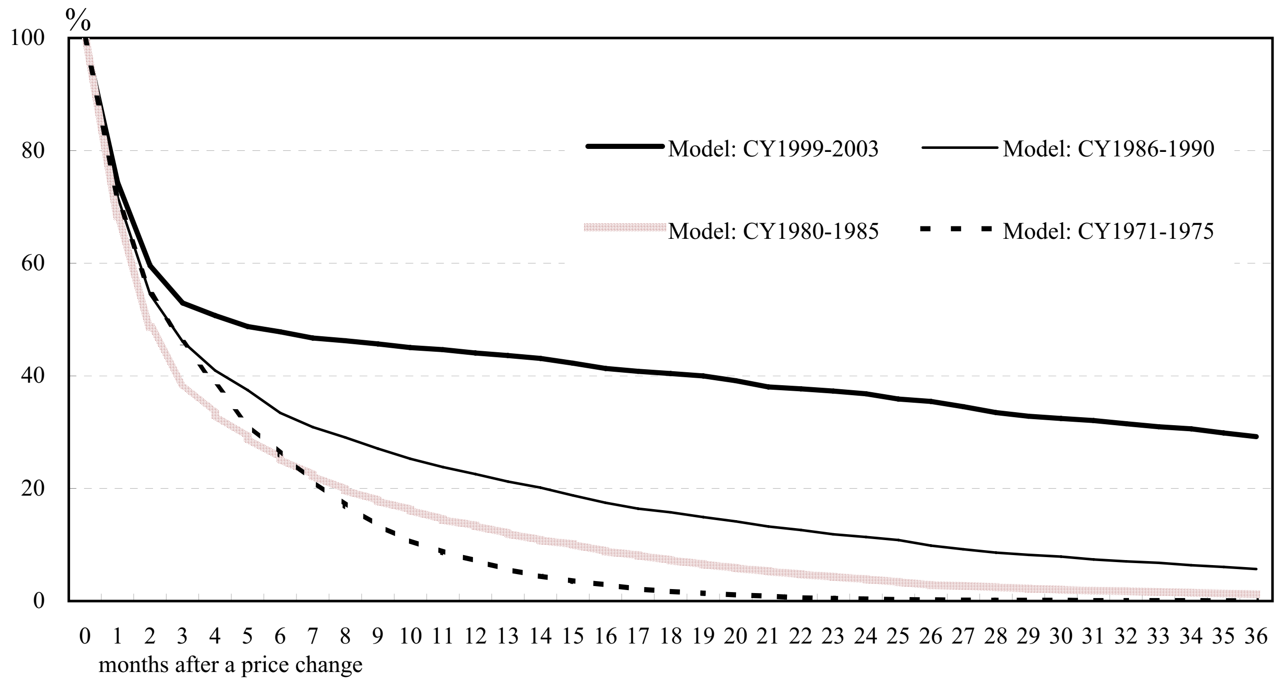
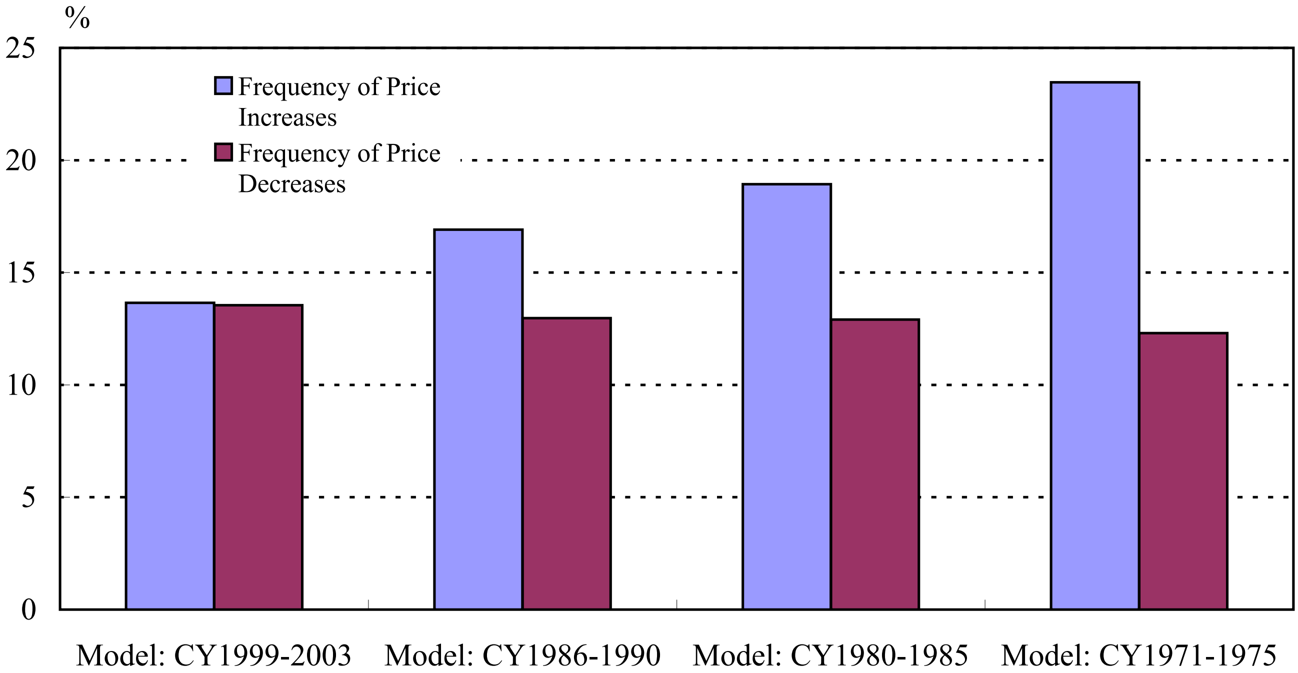
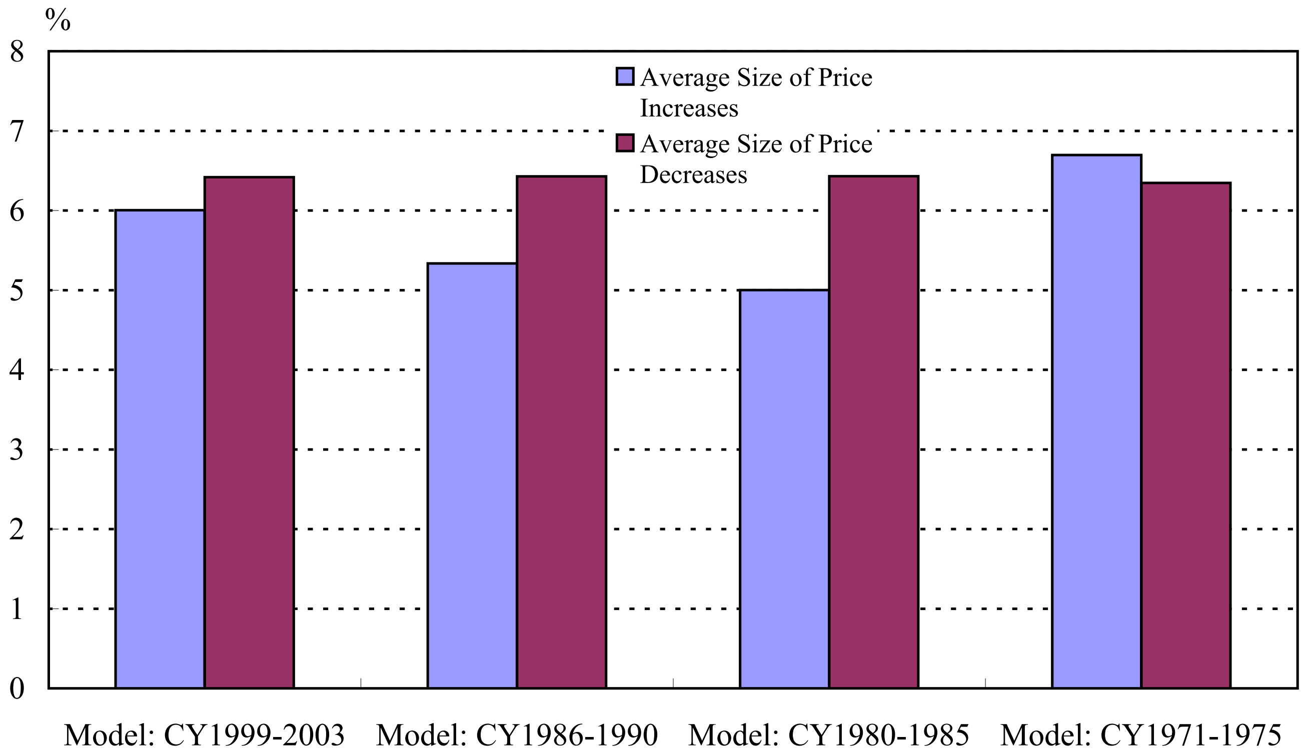


Figure 11: Frequency of Price Increases (Decreases) and Average Size of Price Changes under Different Monetary Shock Processes

(1) Frequency of Price Increases (Decreases)

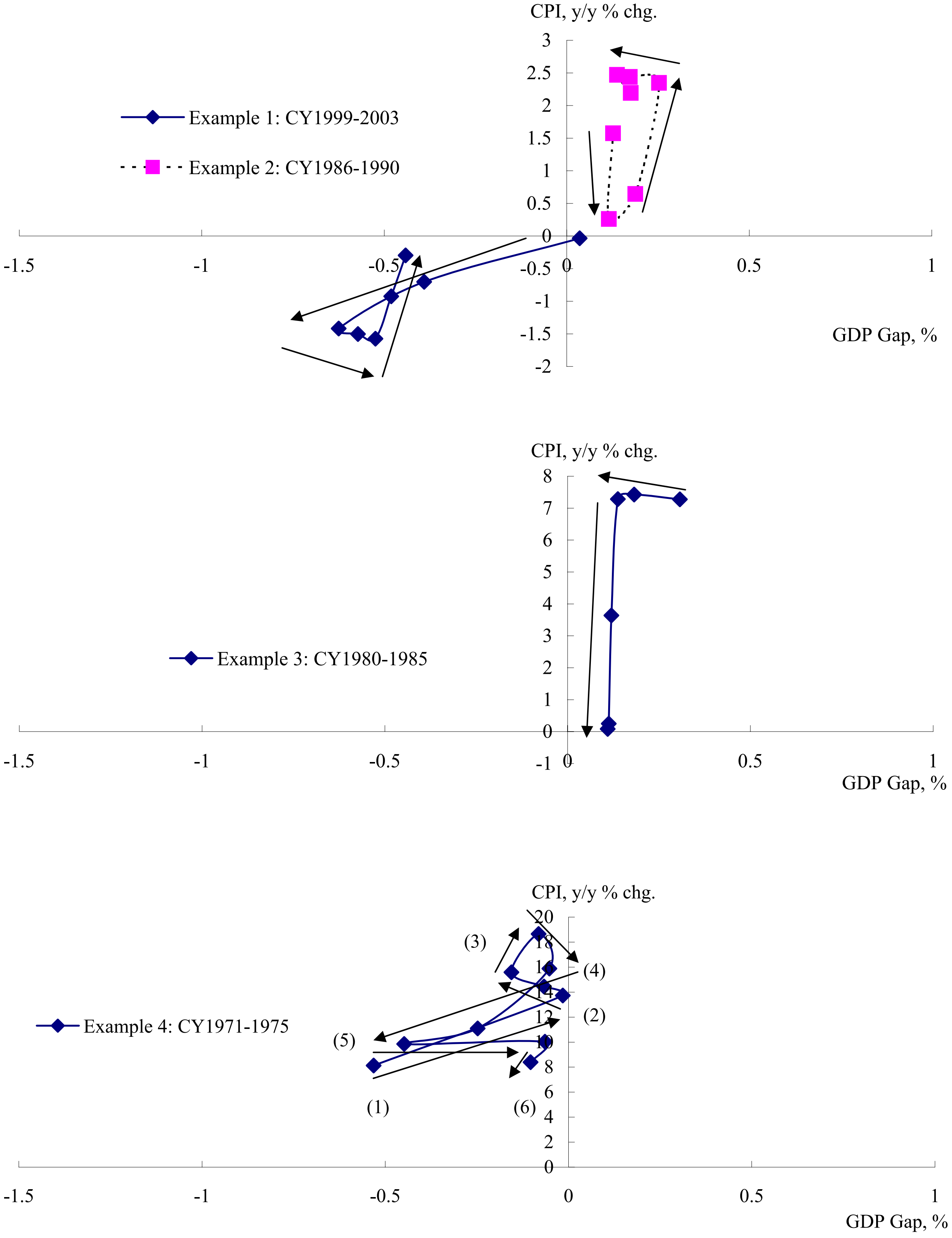



(2) Average Size of Price Increases (Decreases)




Note: The frequency of price changes is calculated based on the invariant distribution. For example, if the time interval is 1/10 month and the frequency of price changes per 1/10 month implied by the invariant distribution is 0.01, then my estimate for the frequency of price changes per month is 0.1. The frequency of price changes in (1) is too high. since there can be sequence of prices which change twice or more within one month. To estimate the measure of double (or more) counting, I perform some simulations. As a result, I found that about 2.5 percent is double counting in the case of CY1999-2003: Before excluding double counting, the frequency of price changes is 27.2 as seen in (1). After excluding double counting, the frequency becomes 24.8.

Figure 12: Phillips Curve




Notes: 1. Each point represents the quarterly value. I obtain each quarterly value by taking the means of the relevant variables over the quarter.

2. GDP Gap is defined as a percentage deviation of the real GDP from the benchmark GDP. Benchmark GDP is defined as GDP consistent with a stable price environment, i.e. ( $\mu$ , $\sigma_m$ ) = (0, 0)

Figure 13: Nominal Price Rigidity and Mean Reversion of Productivity Shocks

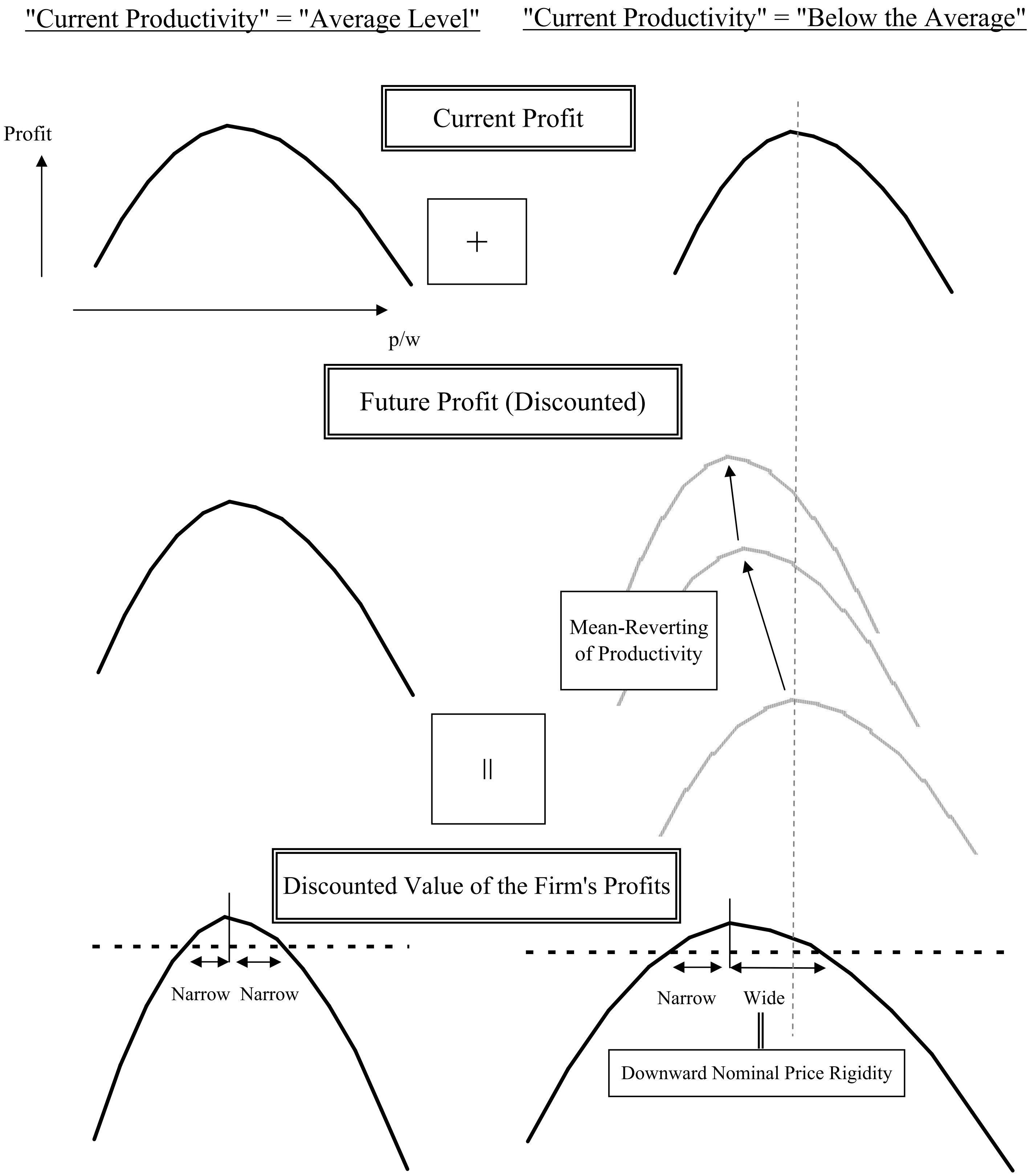
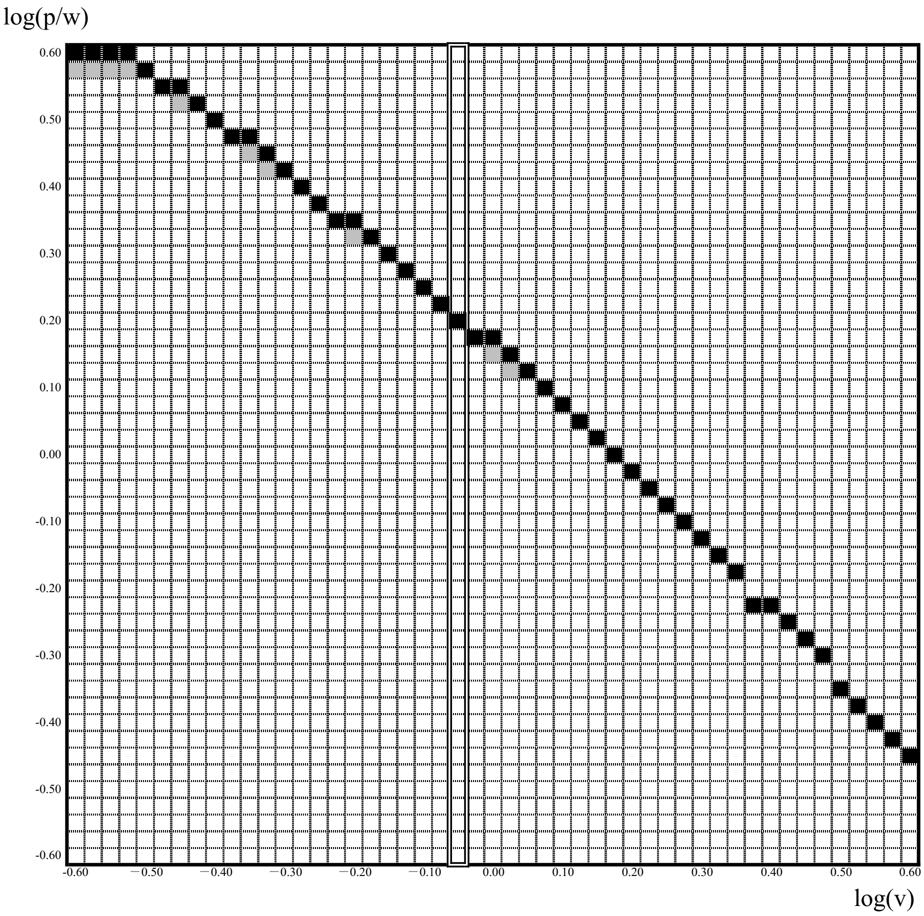


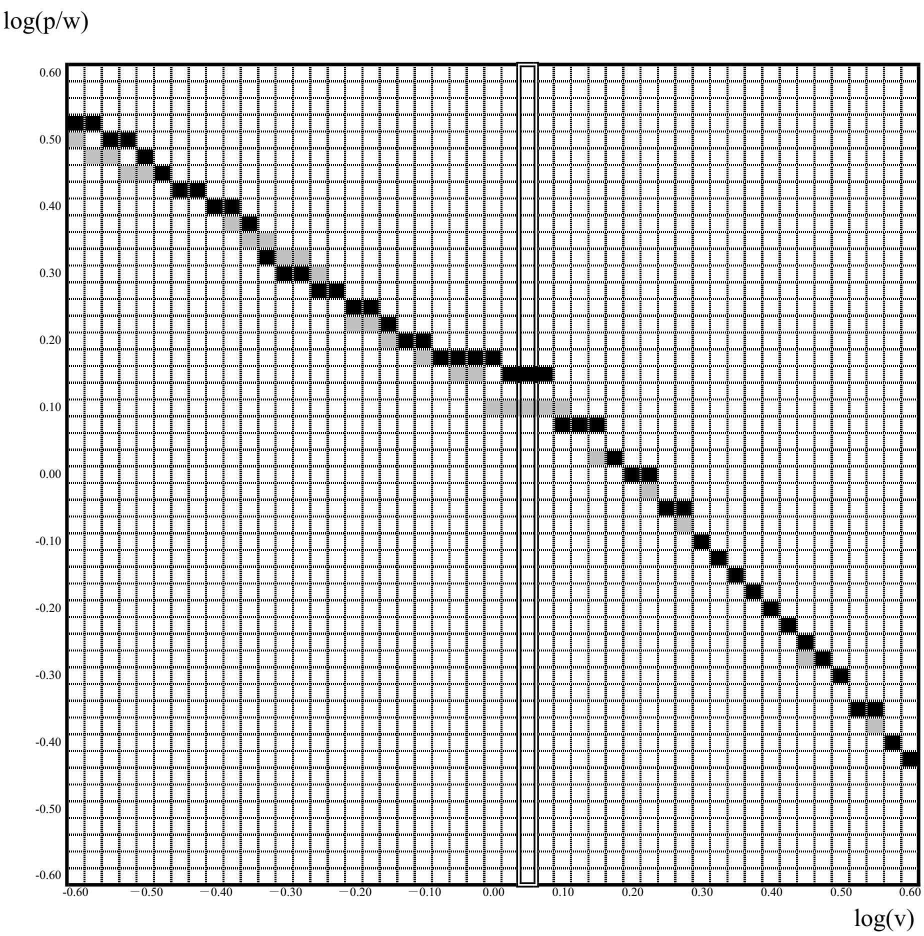
Figure 14: $\hat{\xi}_j$ under Different Monetary Shock Processes

(CY1999-2003: Gray →CY1971-1975: Black)

"g sector"




"s sector"



# Multi-Sector Menu Cost Model, Decreasing Hazard, and Phillips Curve

榎本　英高

2007年

# 問題意識

- 既存のモデル（時間依存型・状態依存型）はいずれも企業の価格設定行動を反映した価格のミクロデータが示す特性を統一的に説明するのに失敗

  （Nakamura and Steinsson(2006)）

  — とりわけ日米欧で共通してみられる減少ハザードを最適価格設定の枠組みで説明できていないことが現在、学界等で注目されている。

- （これに伴い、）既存のモデルの金融政策に関するインプリケーションの信頼性にも疑問符

# 時間依存型の問題点（1）

- Calvo型等の時間依存型はインフレ率が変化すれば価格改定頻度も変化するというファクトと矛盾。
  — 西岡・池田（2006）はやや複雑な時間依存モデルで減少ハザードを再現しているが、時間依存である故にインフレ率の変化に対応したハザードや価格改定頻度の変化をトレースできない。
  — 西岡・池田は、Klenow and Kryvtsov (2005)が開発した手法を用いて日本のデータは旧来型の状態依存型モデルから生み出される価格系列とは異なる特徴を持つことを示している。しかし、Klenow and Kryvtsovの方法では、本稿で紹介する生産性ショックを考慮した状態依存型については否定も肯定もできないことがGolosov and Lucasによって明らかにされている。
- 時間依存型モデルが導き出す金融政策へのインプリケーションは信頼できるのだろうか？

# 時間依存型の問題点（2）

*「政策変数（ここでは貨幣の平均的な伸び率）を変化させた場合には、経済主体の行動パターン（ここでは企業の価格変更戦略）自体が変化するので、モデルのパラメーターを固定したシミュレーションには意味がない」 Lucas批判*

*「メニューコストモデルでは企業にとっての最適価格から最も離れている企業から価格改定を行っていくが、Calvo型では最適価格から然程離れていない企業も価格改定を行うため、同じ価格改定頻度の下では貨幣が中立になるまでの時間が長くなる」*
*by Golosov and Lucas (2006)* ［以下、GLと略す］

*「ケインジアンは民間に不当に不利な仮定を置くことで政府に介入余地があるという結論を導き出そうとする・・・民間が金縛りにあうというCalvo型の仮定はアドホックであるし、民間“だけ”金縛りというのは不当な仮定である・・・」*

# 従来の状態依存モデルの問題点

- 従来の状態依存型価格設定モデル（メニューコストモデル）では、（１）ゼロインフレ周りでも価格改定が頻繁になされていることや、（２）インフレのもとで価格を引き下げる企業が存在することを説明できない。

— ゼロインフレのもとでは、価格を改定する必要性が生じない。

— インフレの下では、価格を据え置き続けると製品の実質価格が低下していき利潤が大きく減少するため、必要に応じて価格の引き上げが行われる。しかし、価格を引き下げる必要性が生じることはない。

# Golosov-Lucasモデルの概要

- GLはメニューコストモデルに企業特殊的な生産性ショックを導入することで、従来の状態依存型価格設定モデルで説明できなかった価格改定指標を再現。
  - 例えば、正の生産性ショックがあると、その企業の限界費用が低下し、価格を引き下げる誘因となる。
  - 企業特殊的な生産性ショックがあれば、ゼロインフレ周りでも価格改定は頻繁になされうるし、インフレのもとで価格が引き下げられることもありうる。
- しかし、依然として次に挙げる問題が残っている。

# Golosov-Lucasモデルの問題点

- 減少ハザードを説明できない

― 減少ハザードを説明するためには、(a)短い期間で価格改定される品目が多い一方で、(b)長い期間、価格が据え置かれる品目も多く存在することを同時に説明する必要があるが、1セクターであるGLでは説明困難

- 金融政策が短期でもほとんど中立的

― 米国データにカリブレートされたGLモデルでは1年以内にほとんど全ての品目が価格改定される。しかし、現実のデータはそうではない。

― 減少ハザード(価格を長い期間据え置く企業の存在)を説明できれば金融政策の非中立性を生み出すことが可能では？

# 本稿の分析のアイデア

- 才田等（2006）は価格改定頻度が高い品目群と低い品目群が存在すると指摘
- こうした特性を取り入れられるようにGLモデルを一般化すれば、企業の価格改定行動の特性をよりうまく説明できるのではないか？
- 本稿のモデル（一般化されたGLモデル）の金融政策へのインプリケーションは？

# 多部門への拡張の意義はあったか？

- ….your paper confirmed some of my conjectures and also I learned some things from it about which I have not thought before….

[by ゴロゾフ (GL modelの共著者)]

# 結論

1. 本稿のモデルは企業の価格改定の特性をうまく説明できる。
   – 減少ハザード、価格の引き上げ・引き下げ比率、平均価格改定幅を同時に再現可能。また、それらのインフレ率に対する反応もデータとモデルは整合的。
2. 金融政策の効果に関しても興味深い含意
   – インフレ率の低下に伴うフィリップス曲線のフラット化、金融政策の効果の非対称性等

# モデルの概略

- 無数の家計と無数の企業が存在
- 個別企業は「財セクター」か「サービスセクター」に所属（当初の所属で永久に固定）
- セクター毎の違いはメニューコスト、生産性のボラティリティ、平均生産性水準の３点

— 両セクターでこれら３点の特徴が全て同じと仮定すればGLと同じ（GLは本稿モデルの特殊例）

— カリブレーションの結果、財はサービス対比でメニューコストが低く、生産性のボラティリティが高く、平均生産性がやや低く設定される（カリブレーションについては後で詳述）。

# モデルの詳細（ショックの過程）

- 貨幣ショックと企業特殊的な生産性ショックが存在
- 貨幣ショック

$$d\log(m) = \mu dt + \sigma_m dZ_m$$

$Z_m$ はドリフト項ゼロ、分散 1 の標準ブラウン運動

- 企業特殊的な生産性ショック

$$d\log(v_a) = -\eta(\log(v_a) - \log(1 + e_a))dt + \sigma_{v_a} dZ_{v_a} \qquad a = g \text{ or } s$$

$Z_{v_a}$ はドリフト項ゼロ、分散 1 の標準ブラウン運動

“g”は「財セクター」、“s”は「サービスセクター」を表す。
個別企業のショックはそれぞれ独立。
貨幣ショックと個別企業のショックもそれぞれ独立。

# モデルの詳細（家計）

- 効用関数

$$E\left[\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\gamma}c_t^{1-\gamma}-\alpha l_t+\log\left(\frac{\hat{m}_t}{P_t}\right)\right)dt\right]$$

— $\gamma$ :相対的リスク回避度

— $c_t$: Spence-Dixit-stiglitz consumption aggregate

$$c_t=\left[\int C_t(p)^{1-\frac{1}{\epsilon}}\phi_t(dp,dv)\right]^{\frac{\epsilon}{\epsilon-1}}$$

— $\phi_t$: 名目価格と生産性水準の結合分布

— $\varepsilon$ :代替の弾力性

— $P_t$: 一般物価水準

$$P_t=[\int p^{1-\epsilon}\phi_t(dp,dv)]^{1/(1-\epsilon)}$$

# モデルの詳細(家計)

- 予算制約

$$E\left[\int_0^{\infty} Q_t\left(\int pC_t(p)\phi_t(dp,dv)+R_t\hat{m}_t-w_tl_t-\Pi_t\right)dt\right]\leq m_0$$

— $R_t$: 名目金利
— $\Pi_t$: 配当所得及び貨幣の一括移転所得の合計
— $Q_t$: 割引因子

- 消費:労働供給と消費の最適化条件から

$$C_t(p)=c_t^{1-\epsilon\gamma}\left(\frac{\alpha p}{w_t}\right)^{-\epsilon}$$

# モデルの詳細

- 名目金利と名目賃金
  - ― ショックの実現値にかかわらず、名目金利が$R_t=R=\rho+\mu$で一定となる均衡が存在（少なくとも数値計算で示すことが可能）。
  - ― 名目金利が$R_t=R=\rho+\mu$となる均衡に注目すると、家計の労働と貨幣保有に関する最適化の一階の条件から、この均衡では、

$$w_t = \alpha R m_t$$

が成り立つ。このことは、名目賃金が名目貨幣と同じ動きをすることを意味している。

# モデルの詳細（企業）

- 生産関数

$$C_t(p) = v_{at} l_t^f$$

- 利潤の割引現在価値　$\varphi_a$

$$\varphi_a(p, v_a, w, \phi_t) = \max_T \mathrm{E}_t \Big[ \int_t^{t+T} Q_s C_s(p)(p - w_s / v_{as}) ds$$
$$+ Q_T \cdot \max_q [\varphi_a(q, v_{a,t+T}, w_{t+T}, \phi_{t+T}) - k_a w_{t+T}] \Big] \quad (★)$$

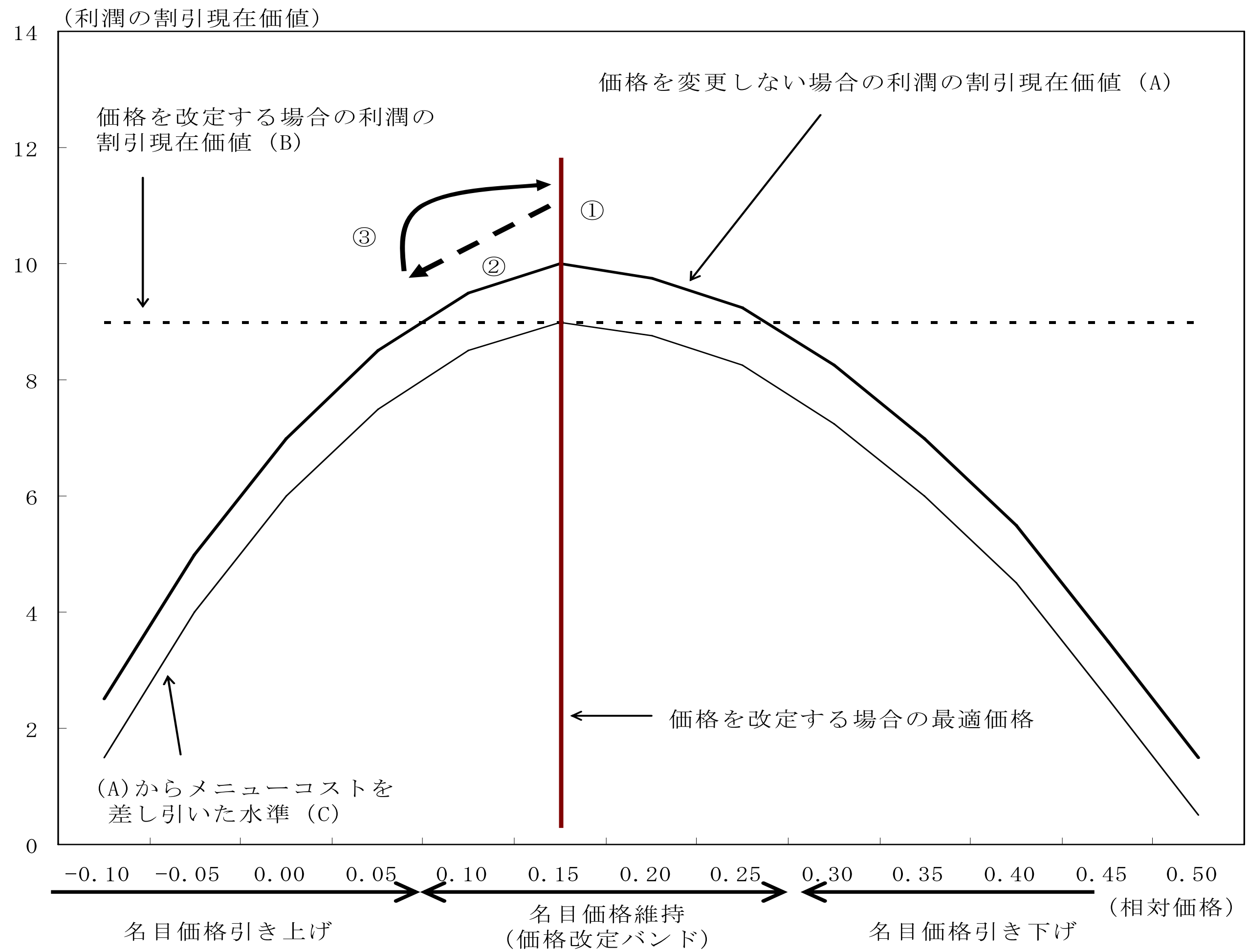
— $k_a$ :「aセクター」に属する企業が価格改定時に支払うメニューコスト（労働）
— Tは価格改定のタイミング、qは価格改定時に設定される新しい価格水準
— T,qともにショックに関する条件付の形で利潤を最大化するように選択
（詳細は次ページ）

# 最適価格設定（生産性ショックなし･賃金上昇の場合）

① 前回の価格改定の結果、最適相対価格からスタート。

② 名目賃金上昇率が正ならば、価格を改定するまで相対価格（本稿ではp/w）は徐々に低下。

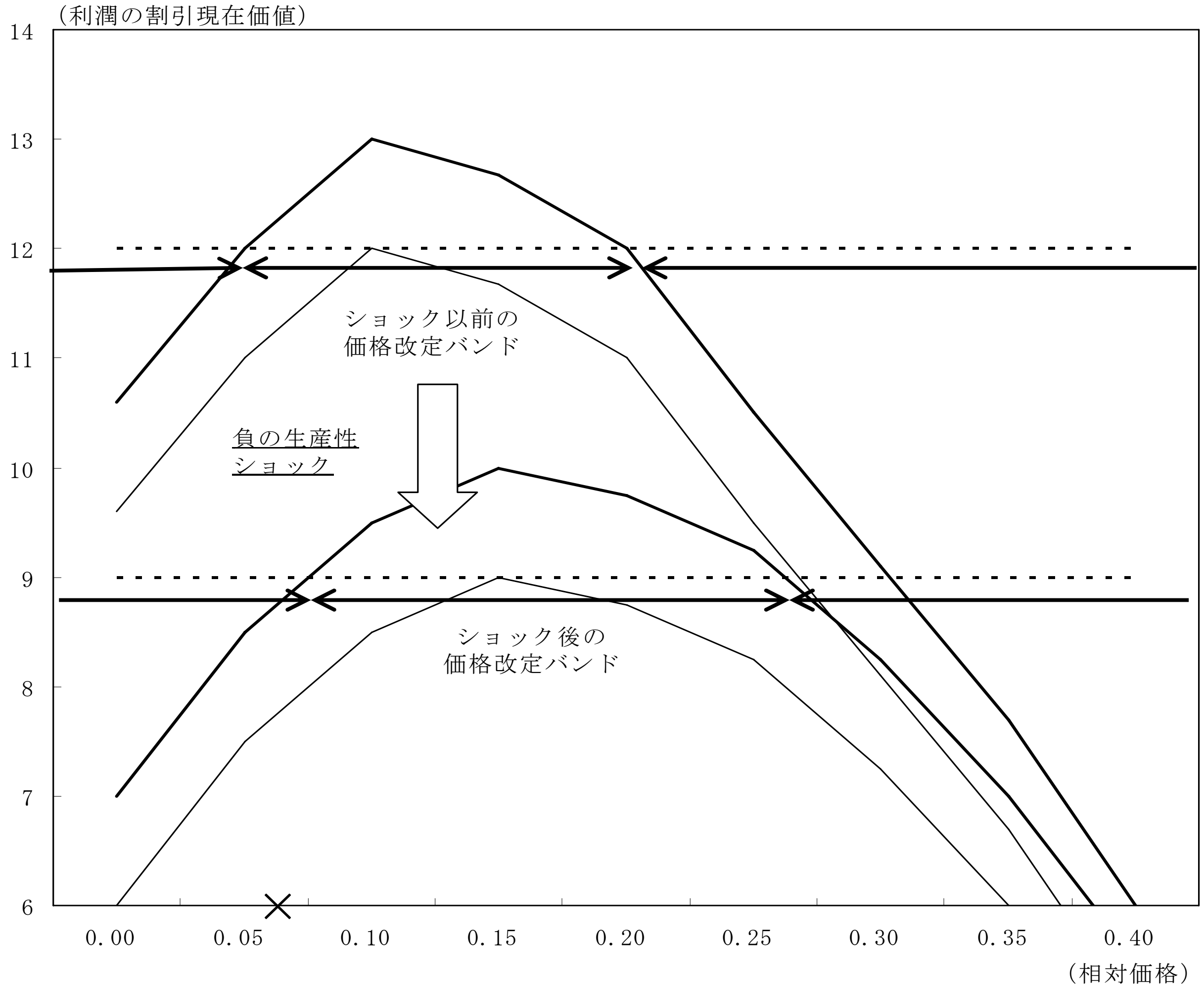
③ 価格改定のメリットが生じた時点（曲線Ａが点線Ｂを下回った時点）で名目価格を相対価格が最適値となるように調整（価格の引き上げ）。



# 最適価格設定（負の生産性ショック）

- 生産性が低い（＝限界費用が高い）ほど、最適価格及び価格改定バンドは高くなる。
- 生産性が低いほど、価格改定バンドの幅は広くなる。

― 今回のモデルでは、メニューコストは販売量等に拘わらず一定であると仮定。
そのため、生産性が低い場合のメニューコストの負担は相対的に重くなる。



# 均衡概念

- GLと同様に、均衡概念としては「独占的競争を行っている無数の企業の価格変更戦略のナッシュ均衡」を採用

― 均衡の定義により、この均衡においては、

(A)各企業の価格変更戦略は現在及び将来の価格と生産性水準の結合分布$\phi_t$を所与として(★)の解として求められるものであり、

(B)結合分布$\phi_t$の推移は、適当な初期分布$\phi_0$の下で、(A)で求めた個別企業の価格変更戦略と整合的である必要

・ どうやって均衡を求める？

― とても難しい問題！

# 技術的な問題(1)

- 利潤の割引現在価値は$\phi_t$を含んでいるため分析が困難
  — (実質価格と生産性水準の定常結合分布が存在すると想定し、それに注目することで)、論文の3.7節の方法で利潤最大化問題を以下のように分析の容易なリカーシブな形に書き換えることが可能。

$$\psi_a(x, v_a) = \max_T \mathrm{E}\Big[ \int_0^T e^{-\rho t} \overline{c}^{1-\epsilon\gamma} \quad (\alpha x_t)^{-\epsilon}(x_t - \frac{1}{v_{at}})dt + e^{-\rho T} \cdot \max_{x'}[\psi_a(x', v_{aT}) - k_a]\Big]$$

  — ここではx=p/w、$\varphi_a(p, v_a, w) = w\psi_a(x, v_a)$

# 技術的な問題（2）

- データとの比較や数値計算のためにモデルを離散近似する必要
  - 本稿のモデルは連続時間モデルなので、このままではデータとの比較が困難。
  - $\psi_a$を求めるためには数値計算（バリュー・ファンクション・イタレーション）を行う必要があるが、そのためには離散近似が必要。
- Kushner and Dupuis(2001)に従い離散近似
  - 離散近似の詳細は論文の3.8節を参照。
- 離散近似をしたうえで数値計算により$\psi_a$を求める
  - 数値計算（バリュー・ファンクション・イタレーション）の詳細は論文のAppendix Aを参照。

# カリブレーション（1）

- （1）生産性ショックの平均回帰性、（2）メニューコスト、（3）生産性ショックの分散については1999～2003年の日本の「価格のミクロデータの統計量」とモデルのフィットがよくなるように調整。
  - 用いた統計量は、サバイバル比率、価格の引き上げ・引き下げ比率及び平均価格改定幅（詳細・理由は次ページ）
- その他のパラメーターについてはデータ（価格のミクロデータ以外のデータ）や先行研究から得られる値を使用。

# カリブレーション（２）

- なぜ、カリブレーション時に「サバイバル比率」と「価格の引き上げ・引き下げ比率」と「平均価格改定幅」を重視するのか？

— サバイバル比率は品目毎の価格改定頻度の異質性の情報を含んでいる為、「財」、「サービス」のセクター毎にメニューコストと生産性の分散を定める本稿のモデルのパラメーター設定には不可欠。

— 本稿のモデルでは、価格改定は名目賃金の変化か生産性ショックにより引き起こされる。名目賃金の変化を所与とすると、生産性ショックが大きいほど、価格の引き上げ・引き下げ比率が５分５分に近づく（生産性ショックは左右対称だから）傾向があるため、この比率の持つ情報は重要（非対称性をもたらすのは貨幣（名目賃金）ショック）。

— メニューコスト引き下げは価格改定頻度の上昇、平均価格改定幅の低下をもたらすが、生産性ショックの分散上昇は基本的には価格改定頻度の上昇、平均価格改定幅の上昇をもたらす。よって、これらのパラメーターの値を特定するにあたって、（価格改定頻度の情報を含んでいる）サバイバル比率と平均価格改定幅の情報は重要。

（注）Xヶ月目のサバイバル比率とは、前回の価格改定からXヶ月目まで価格を据え置く品目の割合のこと。（後述する）ハザード確率分布は時間が経つと振れやすくなるので、カリブレーションではハザード確率分布と一対一の関係にあるサバイバル比率を参照した。

# カリブレーション（3）

- 主観的割引率、相対的リスク回避度、代替の弾力性についてはGLと同じ値を使用（決め手がない労働の不効用$\alpha$についてもGLと同じ6を使用＜37%の時間を労働＞）

（１）主観的割引率＜四半期＞ $\rho$ （Golosov and Lucasでは0.01）

| 論文 | 主観的割引率 | 備考 |
|---|---|---|
| 北村・藤木（1997） | 約0.01 | 公的資本を考慮した資本の限界生産性を用いる場合の推計結果 |
| Hayashi and Prescott(2002) | 約0.006 | — |

（２）相対的リスク回避度 $\gamma$ （Golosov and Lucasでは２）

| 論文 | 相対的リスク回避度 | 備考 |
|---|---|---|
| 北村・藤木（1997） | 0.6～2.5程度 | 公的資本を考慮した資本の限界生産性を用いる場合の推計結果 |
| 吉川（2001） | 1.36 | 90年代の推計値 |
| 森平・神谷（2001） | １程度 | — |

（３）財・サービスの代替の弾力性 $\varepsilon$ （Golosov and Lucasでは７）

| 論文 | 財・サービスの代替の弾力性 | 備考 |
|---|---|---|
| Nishimura, Ohkusa, and Ariga(1999) | 7 | 業種別に求めたマークアップの算術平均から計算 |
| 乾・権（2004） | 約６ | Rogerの方法で業種別に求めたマークアップの算術平均から計算 |

# カリブレーション（4）

- その他のパラメーターについては以下の通り

— 丸括弧内はGLで用いられた値

— 各セクターの企業の比率は、才田等（2006）のデータのウエイト等を勘案して決定。

— 貨幣ショックのパラメーターについてはGLと同様、（1）モデルでは貨幣は長期的には完全に中立、短期的にも概ね中立であること、（2）現実の貨幣のデータには信用不安による需要を映じた変動等、物価とは直接関係のない様々なノイズが含まれることを踏まえ、CPIの動きから逆算するアプローチを採った。

| | | | | |
|---|---|---|---|---|
| $\mu$ | 貨幣増加率（四半期） | −0.0017 | （0.0064） | 日本のCPI総合（1999年～2003年） |
| $\sigma_m$ | 貨幣増加率の標準偏差 | 0.0037 | （0.0062） | 日本のCPI総合（1999年～2003年） |
| $k_g$ | 財のメニューコスト | 0.00055 | （0.0025） | 現実の価格データとモデルとのフィット |
| $k_s$ | サービスのメニューコスト | 0.008 | （—） | 現実の価格データとモデルとのフィット |
| $\eta$ | 生産性ショックの平均回帰性 | 0.75 | （0.55） | 現実の価格データとモデルとのフィット |
| $\sigma^2_{Vg}$ | 財の生産性ショックの分散 | 0.011 | （0.011） | 現実の価格データとモデルとのフィット |
| $\sigma^2_{Vs}$ | サービスの生産性ショックの分散 | 0.0005 | （—） | 現実の価格データとモデルとのフィット |
| $e_g$ | 財の平均生産性水準 | −0.05 | （0） | 労働生産性のデータ |
| $e_s$ | サービスの平均生産性水準 | 0.05 | （—） | 労働生産性のデータ |
| | 「財セクター」に属する企業の比率 | 0.5 | （1） | |
| | 「サービスセクター」に属する企業の比率 | 0.5 | （—） | |

# カリブレーション（5）

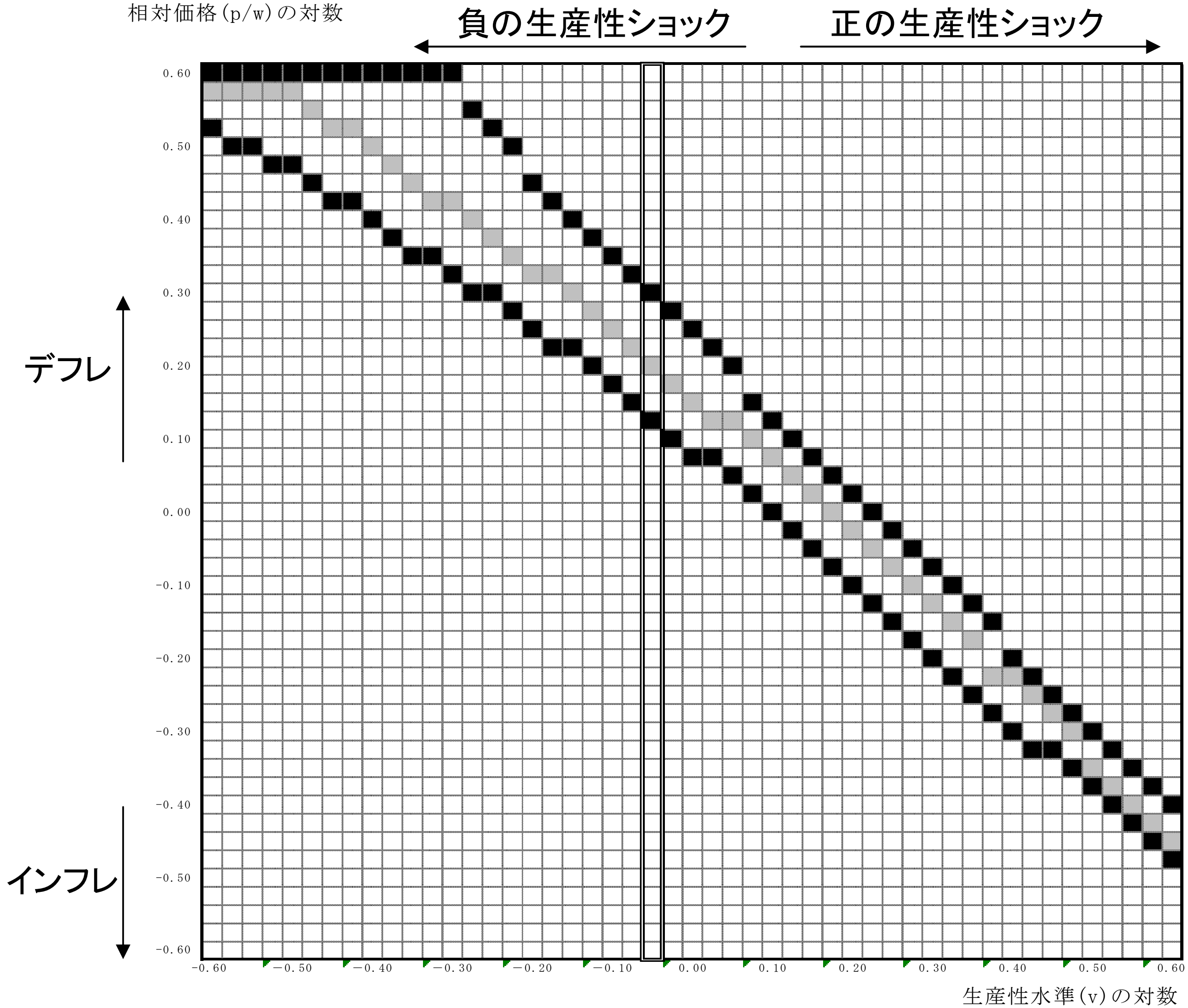
- SNAでみると1999～2003年までの労働生産性格差（サービス/財）は約1.1倍


「財セクター」：農林水産業、食料品、繊維、「サービスセクター」：サービス業、運輸通信業、電気・ガス・水道業
これらで才田等（2006）の小売物価データの8割弱をカバー。労働生産性は（生産/総労働時間）で計算。

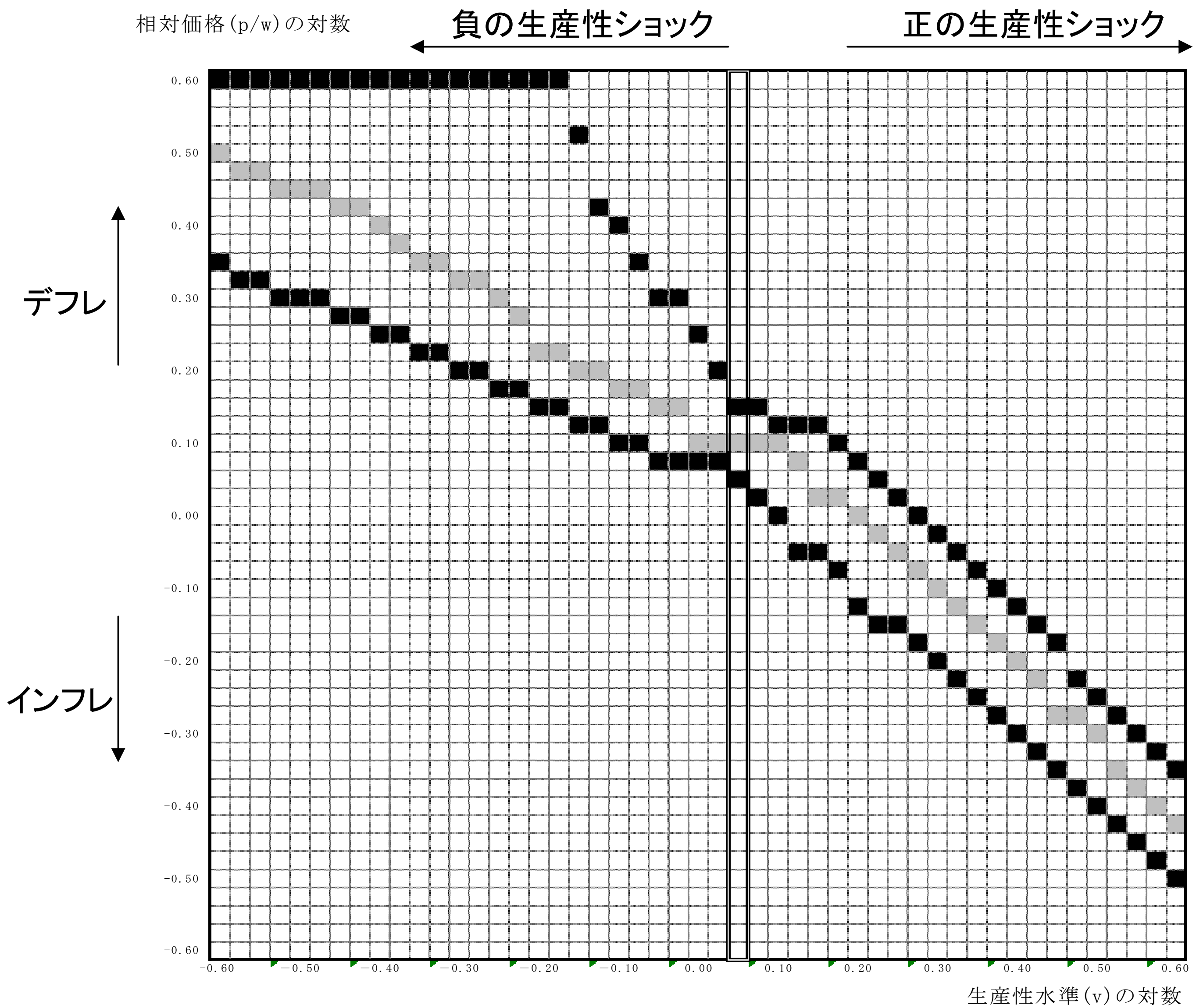
# 「財セクター」の価格改定バンド

- グレーの升目は各々の生産性水準にある企業が価格改定時に新たに設定しなおす相対価格の対数値を示す。

- 生産性水準を所与として黒の升目により囲まれている白とグレーの升目は企業が価格を維持する範囲を表している。黒の升目に移動した時点で直ちにグレーの升目に移動するように価格改定を行う。

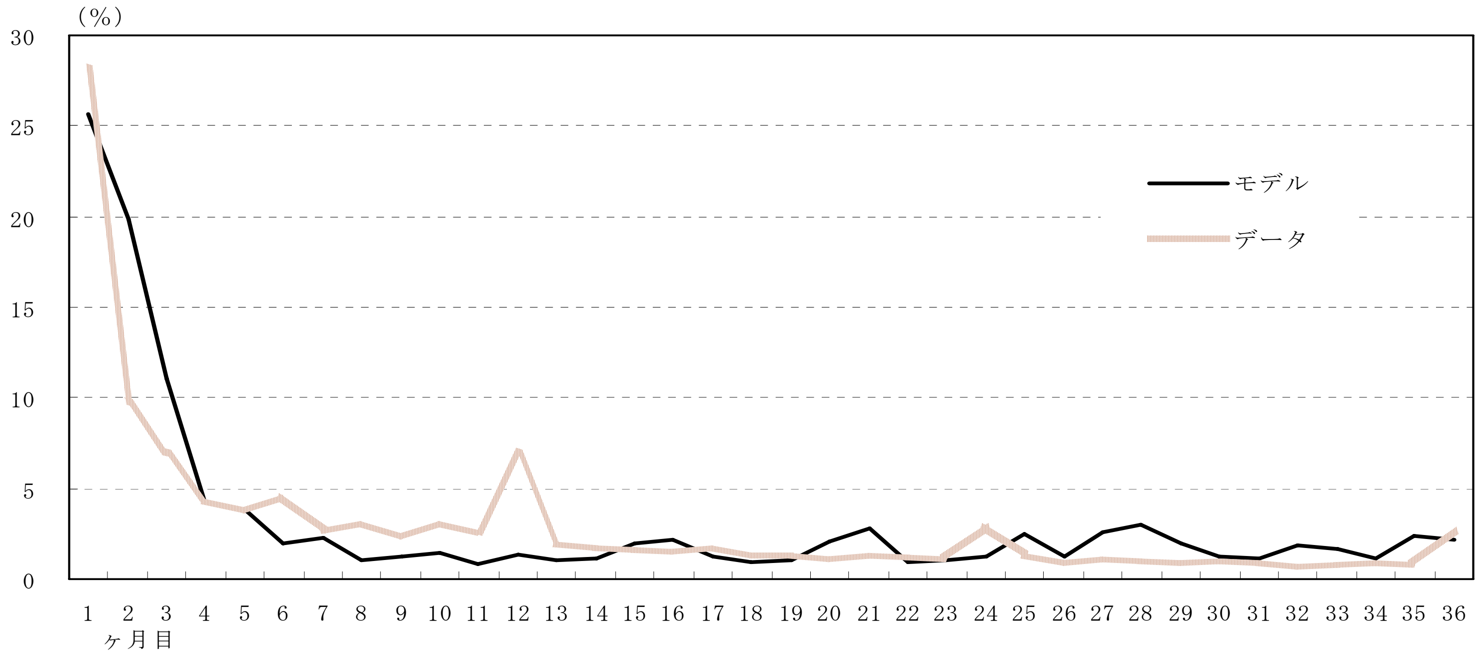


# 「サービスセクター」の価格改定バンド



# 結果（1）：減少ハザードは説明可能

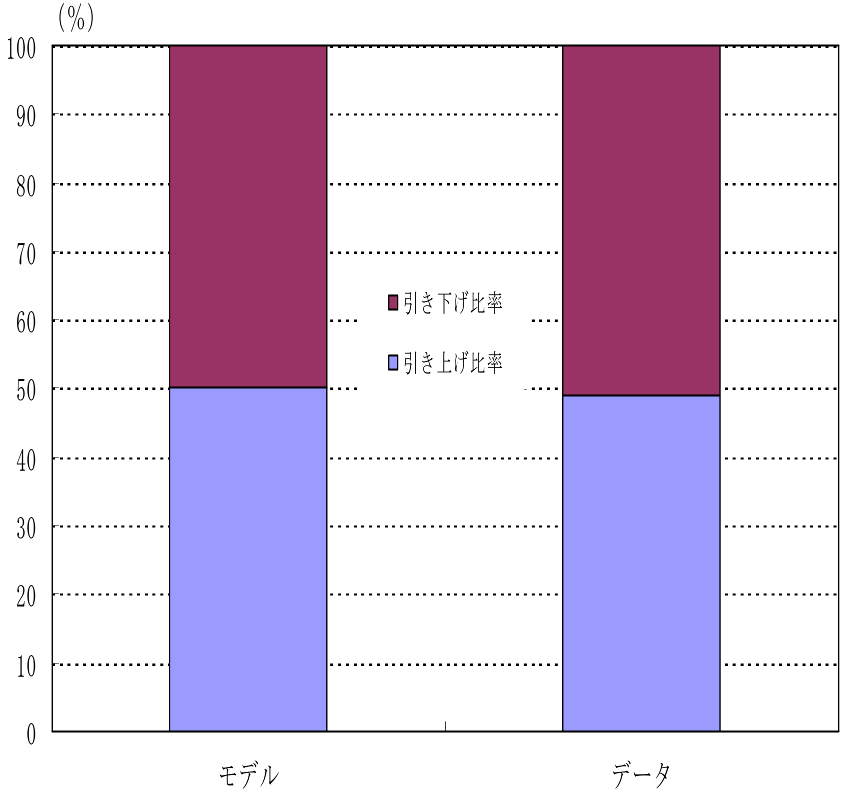
- メニューコストが小さく生産性ショックが大きい「財セクター」の価格が早い段階で改定されていく一方で、メニューコストが大きく、生産性ショックが小さい「サービスセクター」の価格は長く据え置かれる。そのため、両者を合成した結果として得られるハザードは減少型となる。
  - データは1999～2003年の小売物価統計（才田等（2006））

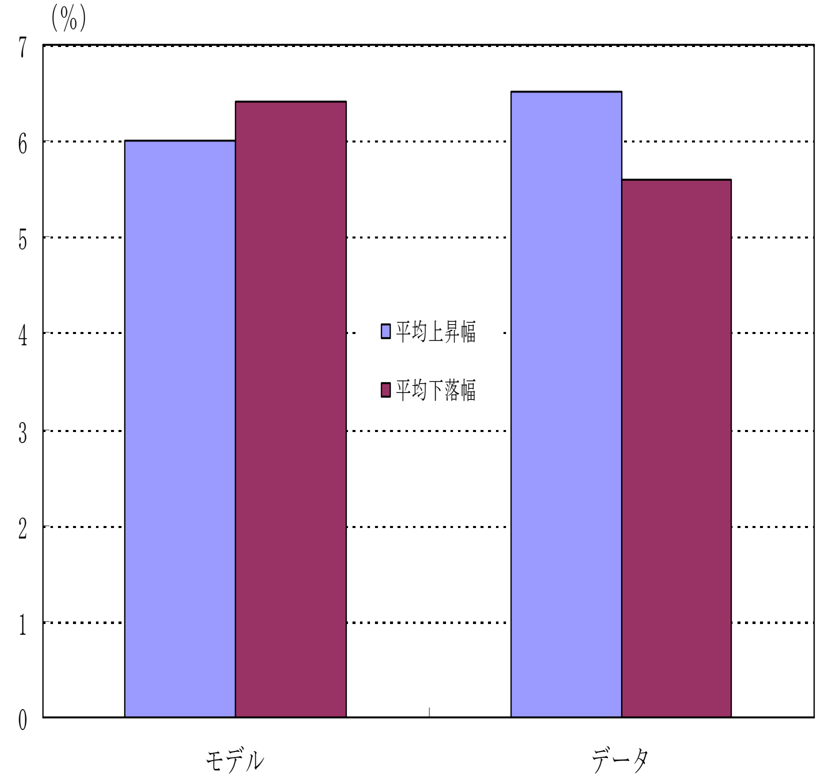



（Xヶ月目のハザード確率）
＝[（Xヶ月目に価格改定される割合）/（X-1ヶ月末までに価格が改定されていない割合）]×100

# 結果（2）

- 価格の引き上げ・引き下げ比率及び平均価格改定幅も概ね説明可能
  - ― デフレの下で価格引き上げが生じているのは負の生産性ショックによる。



# インフレ率と価格改定指標

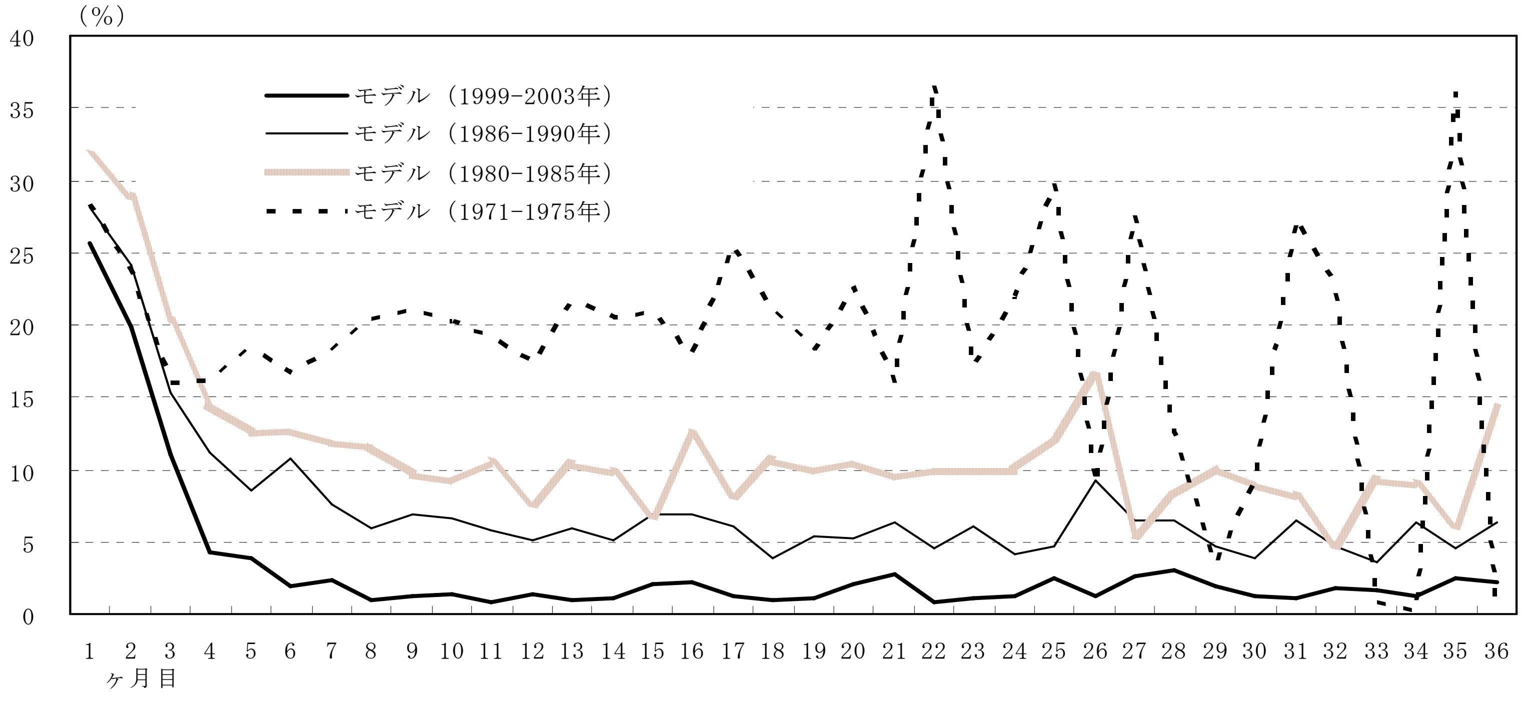
- 貨幣ショックの過程に関するパラメーターのみインフレ率が高かった時期の値に設定しなおすと、価格改定指標はどのように変化するか？

# 結果（3）：インフレ率と価格改定指標

- 貨幣の平均伸び率が高く賃金上昇率が高いほど、その分、価格改定バンドに頻繁に直面するようになるので、ハザード確率は高くなる傾向。

- 「現在の欧米程度のインフレ率ならハザード分布は減少型」というデータと整合的
  — 1986～1990年の平均インフレ率は1.5%強、1980～1985年は同3%強

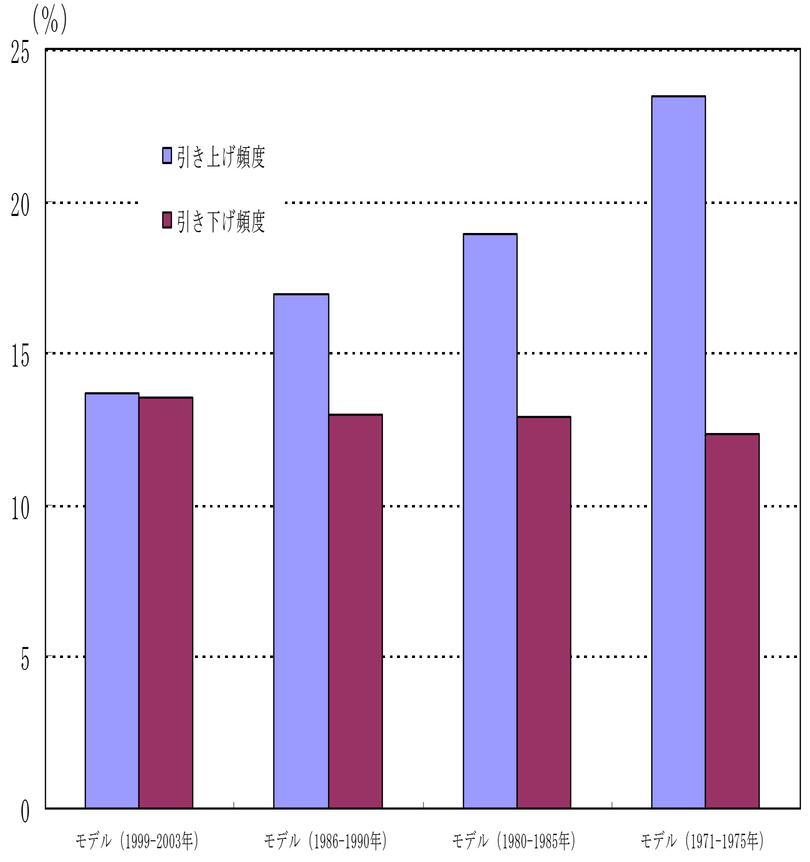
- 10%を大きく超えるインフレの下ではハザードの形状がかなり変化？
  — 1971～1975年の平均インフレ率は11%強

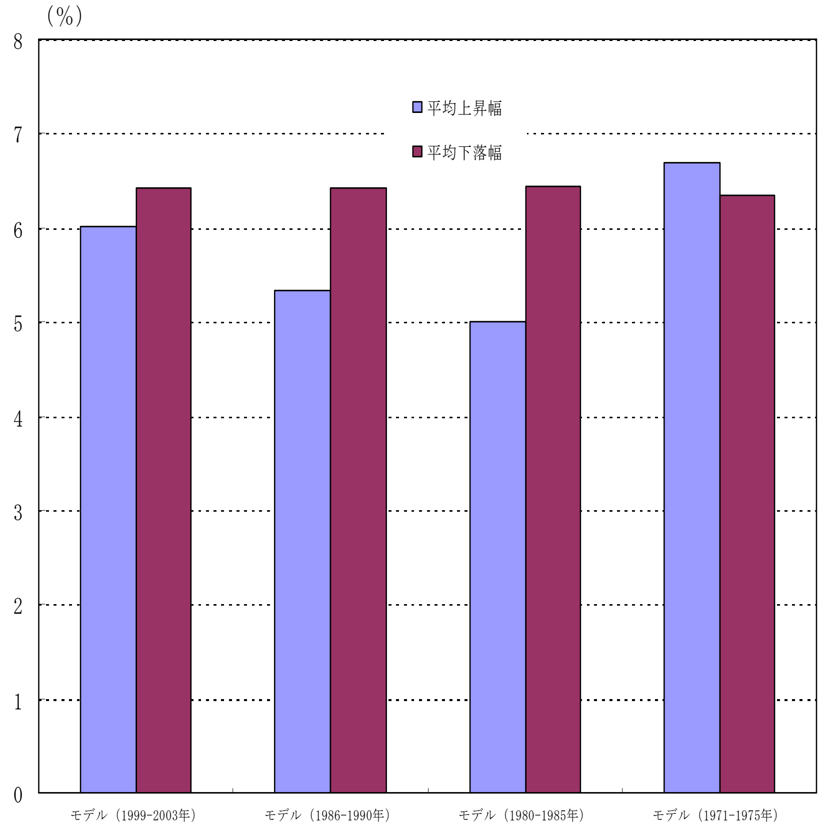


# 結果（4）：インフレ率と価格改定指標

- 「価格の引き上げ頻度はインフレ率の増加関数だが、引き下げ頻度や平均上昇幅、下落幅はインフレ率とあまり関係がない」というNakamura and Steinsson(2006)の米国のデータに関しての発見と整合的

— 価格の引き下げは生産性ショックが原因で行われている為、インフレ率が多少上昇しても引き下げ頻度は影響を受けにくい。



# 金融政策シミュレーション

- 物価とGDPの間にはどのような関係がみられるか（フィリップス曲線）？
- インフレ率が高いときと低いとき、あるいはデフレの時で、物価とGDPの関係は変化するか（フラット化するか）？

これら２点についてみるために、以下の実験を行った。

（１）Random　Numberを用いて分析対象期間（先程と同じ）のCPIと同じ平均伸び率と分散を持つ貨幣の系列を作成し、また生産性ショックも発生させ、それらに対する企業の最適な反応を求める。

（２）個別企業の価格改定行動を集計して、マクロのインフレ率を計算。また、個別企業の設定した価格を元に個別品目の需要量を計算し、それを集計することでGDPを算出。

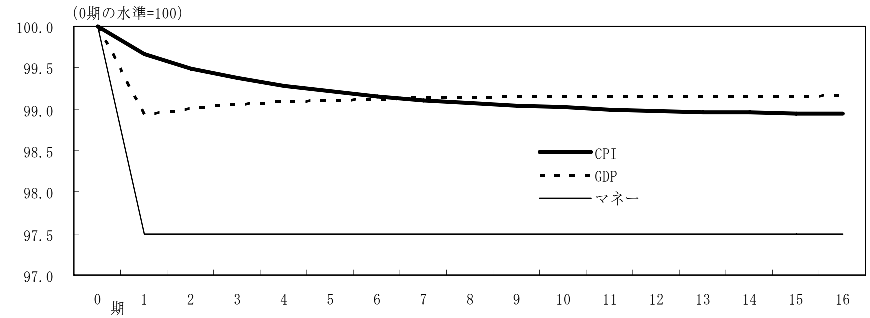
# 結果（５）：金融政策シミュレーション

- 金融引締め時は価格が粘着的になり、実質面への効果が大きめに出る一方、緩和時は価格が伸縮的なため実質面への影響が相対的に小さめになる傾向（27-28ページの価格改定バンドの図を参照）。

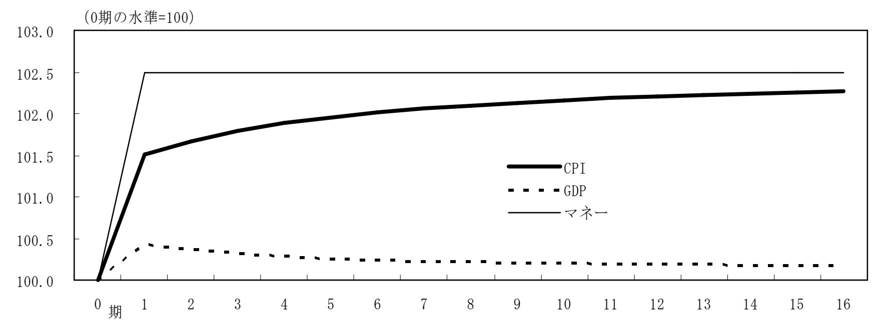
- 細かくみると、サービスの価格の下方硬直性が目立つ（次ページも参照）。そのため金融引締め時には、平均生産性が高いサービスを中心に生産が減少することになり、マクロの労働生産性が低下する。

- 貨幣ショックの効果は、ショックが発生するタイミングによって大きく異なりうることには留意が必要（（２－２）を参照）。

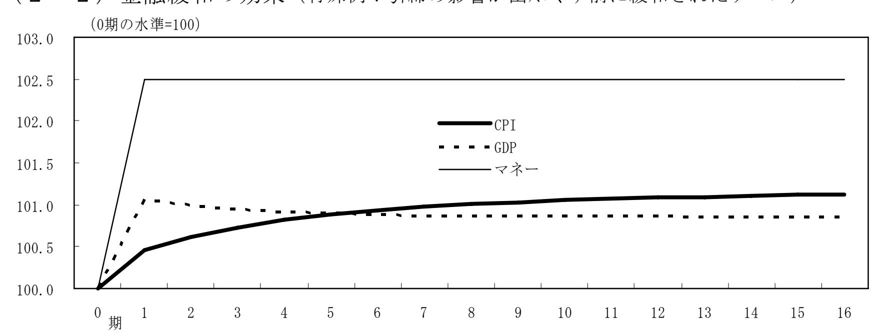
（１）金融引締めの効果（標準例）




（２－１）金融緩和の効果（標準例）



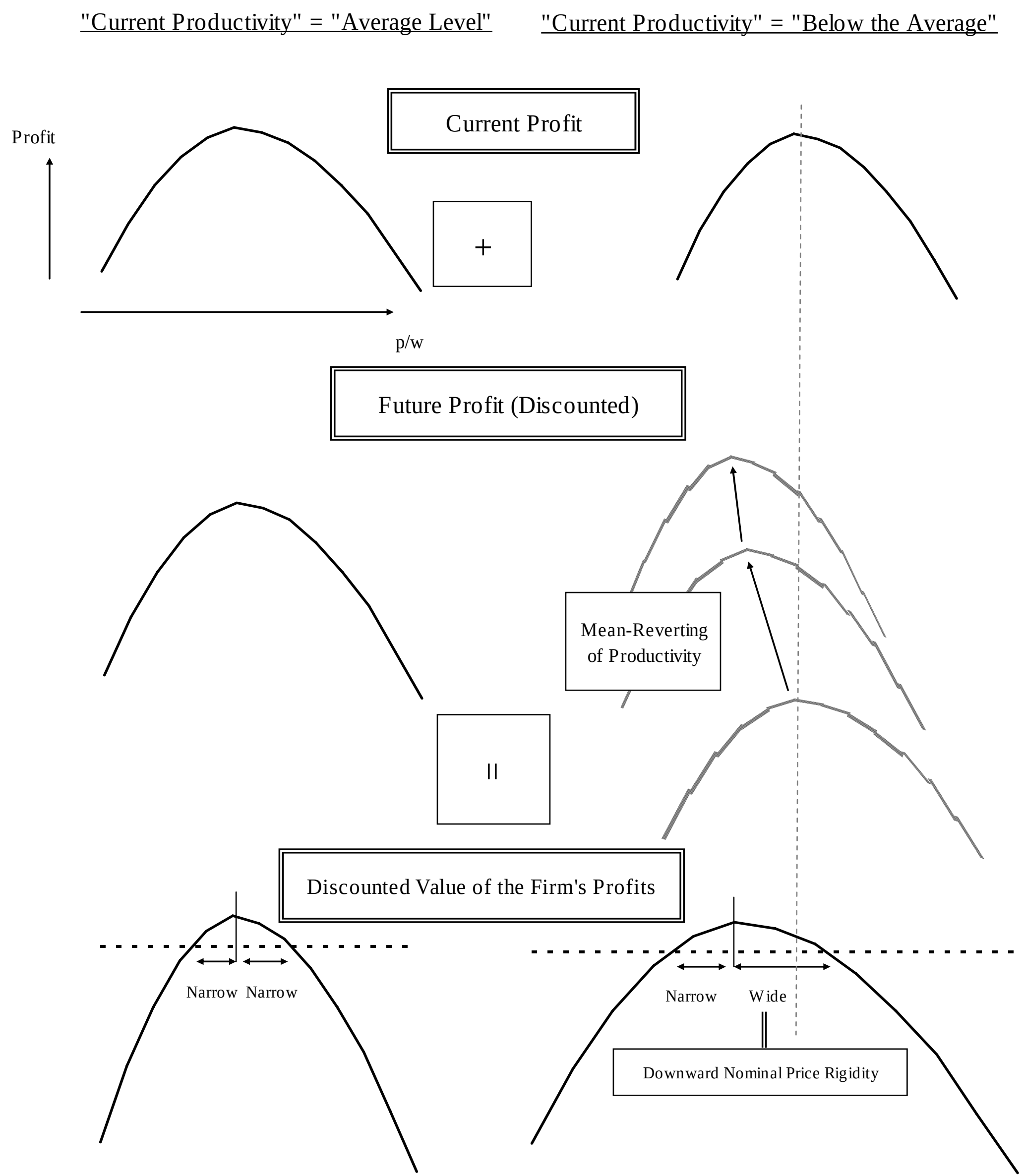

（２－２）金融緩和の効果（特殊例：引締め影響が出尽くす前に緩和されたケース）




（注１）１期に前期比2.5%のマネーショックを与えた場合の物価とGDPへの影響をシミュレーション。
（注２）12期で1ヶ月に相当。
（注３）標準例、特殊例ともに1980年～85年のCPI平均上昇率・分散の情報をもとに生成した貨幣ショックに対するモデルの反応例を示したもの。

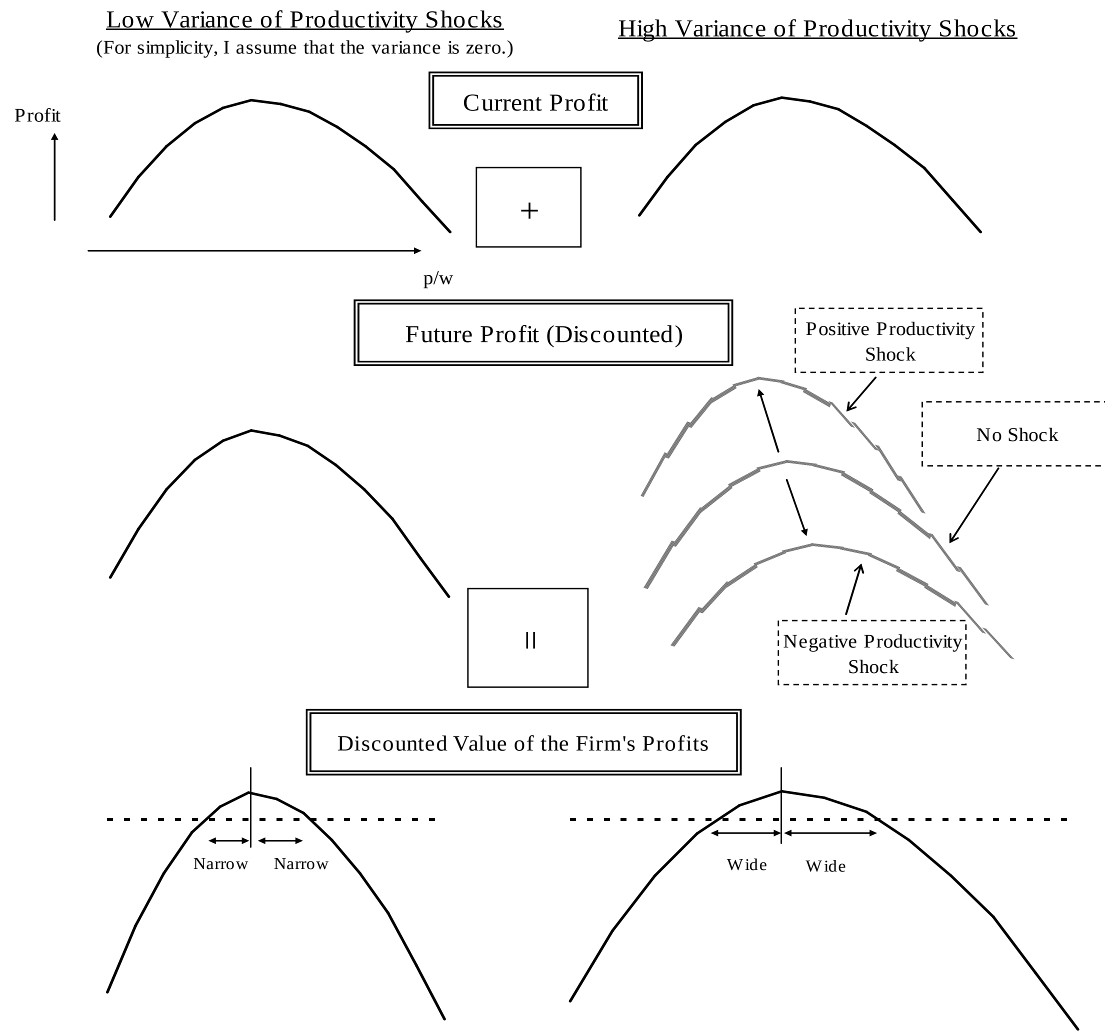
# 生産性の平均回帰性と価格改定バンド

- 生産性水準が平均生産性に近いほど価格改定バンドは狭くなる。
- 生産性が低い場合、価格が下方硬直的になる。
- 生産性が高い場合、価格は上方硬直的になる。しかし、生産性が高い場合、価格改定バンドが狭くなるという傾向がある（18ページの説明参照）ので、下方硬直性と比べると上方硬直性は目立たない。



# 生産性ショックの分散と価格改定バンド

- 生産性の分散が小さいほど、利益の割引現在価値の曲線がスティープ化し、結果として価格改定バンドは狭くなる。



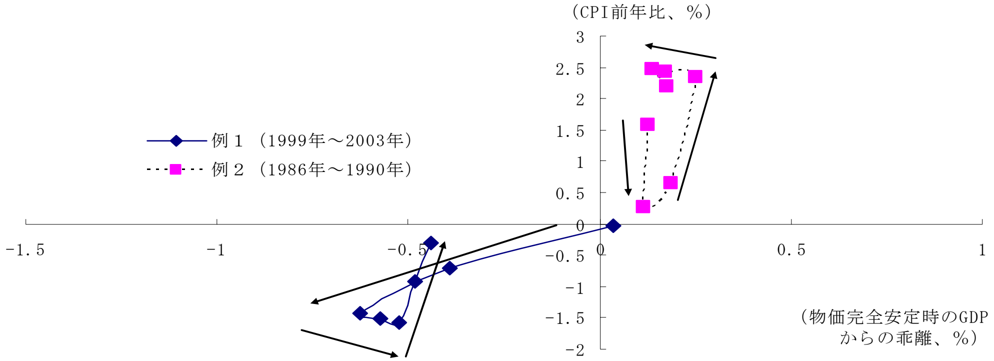
# 結果（6）：フィリップス曲線

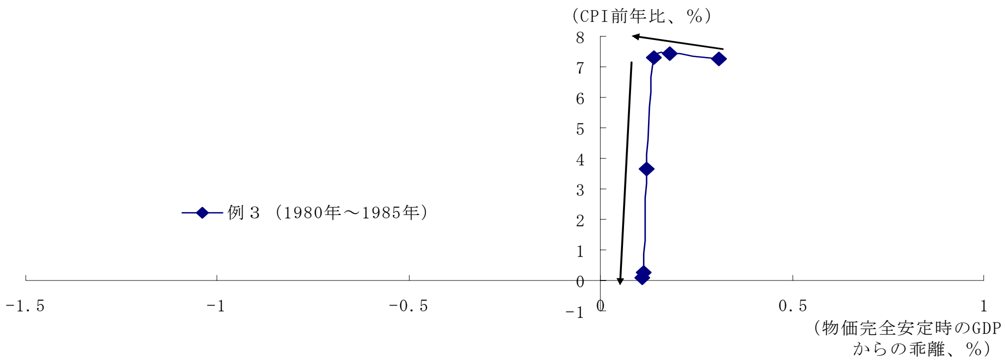
- 日米欧においてインフレ率とフィリップス曲線の傾きの大きさに正の相関がみられるとしたBenati(forthcoming)と概ね整合的

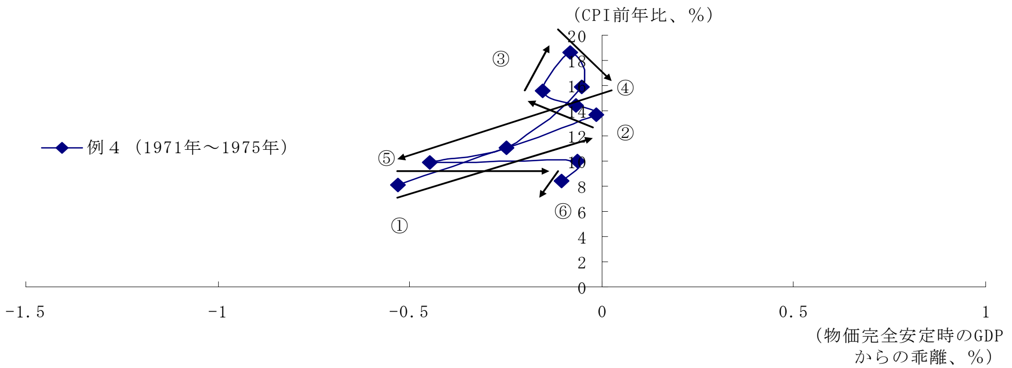
— 負の貨幣ショックの実物面への効果は正のショックよりも大きいので、デフレや超低インフレ下という負の貨幣ショックが生じやすい状況ではフィリップス曲線がフラットになる（結果（5））。

— 高インフレ下でも貨幣ショックのボラティリティが高いので負の貨幣ショックが稀に生じてGDPがやや大きく減少することがある。

— 高インフレ下でGDPが一貫して低レベルなのは、企業が将来インフレを考慮して価格改定時に設定する価格を低インフレ時と比べ割高に設定するため（40ページの図を参照）。

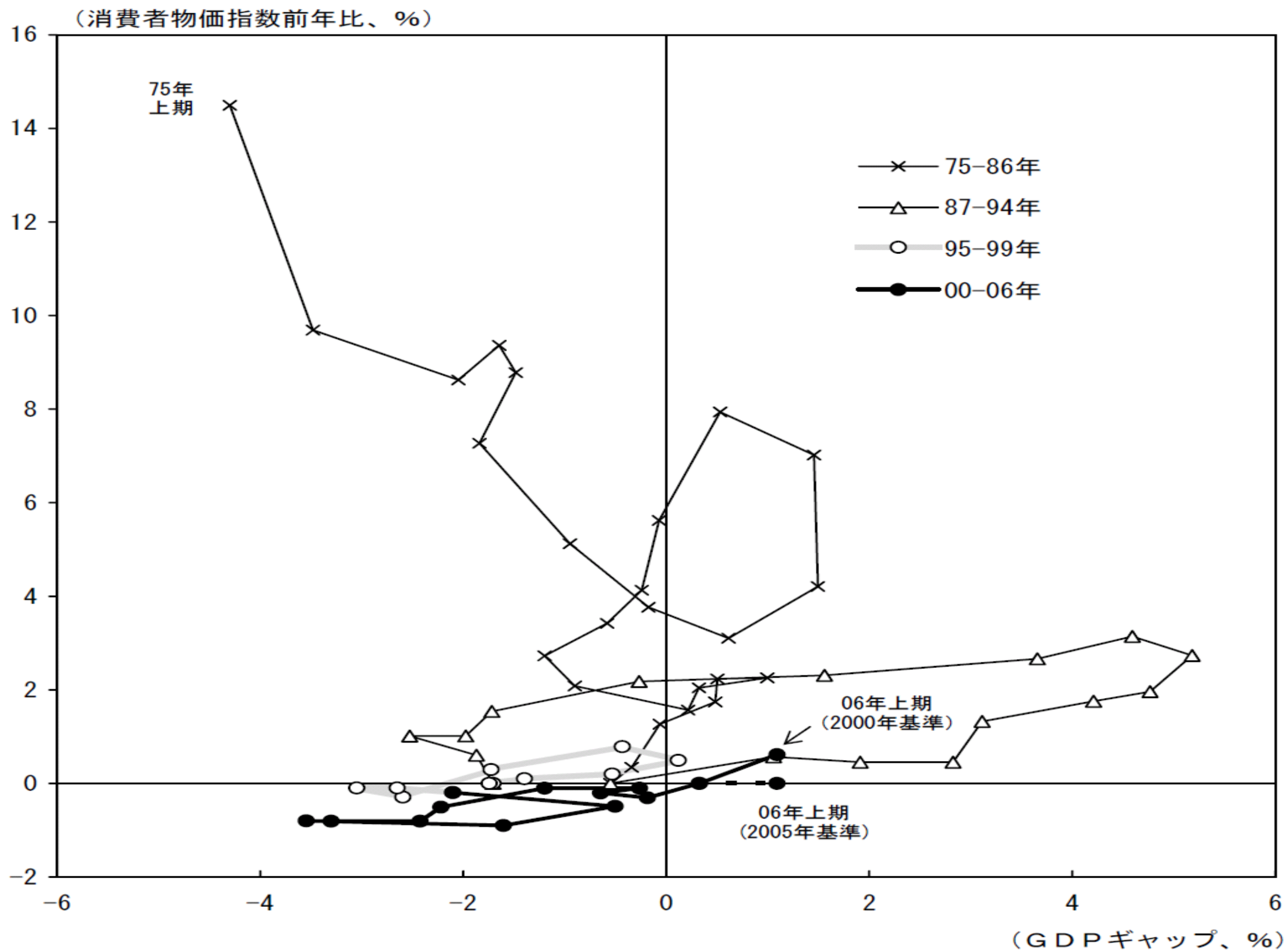



（注）各々のデータポイントは四半期

# （参考）日本のフィリップス曲線




（注）1. 消費者物価指数は総合除く生鮮食品。消費税調整済み。
2. 消費者物価指数の2000年までは1995年基準、2001～2005年は2000年基準のデータを用いて算出。

（資料） 内閣府「国民経済計算」、内閣府・財務省「法人企業景気予測調査」、総務省「消費者物価指数」「労働力調査」、厚生労働省「毎月勤労統計」「職業安定業務統計」、経済産業省「鉱工業指数統計」等

# インフレ率と最適価格

- インフレ率が上昇すると(グレー→黒)価格改定時に企業が設定する価格が割高になる傾向

— 企業は将来インフレを考慮して価格改定時に設定する価格を低インフレ時と比べ割高に設定する。
結果として、需要が減退する為、低インフレ時と比べGDPは低水準。

財の最適価格の変化（1999～2003年→1971～1975年）


サービスの最適価格の変化（1999～2003年→1971～1975年）


（注）グレーの升目は1999-2003年の、黒の升目は1971～1975年の貨幣ショックの下で各々の生産性水準にある企業が価格改定時に新たに設定し直す相対価格の対数値を表している（但し、グレーと黒が重なるところは黒で表示）。

# 結果（7）:トレンドインフレの変化とGDP

- 米国のインフレ率の低下を高インフレから低インフレへの移行、日本のインフレ率の低下を緩やかなデフレへの移行と考えれば桜等（2005）が整理したファクトとモデルの予想（フィリップス曲線の変化）は整合的。

［桜等（2005）によるファクト・ファインディング］

| | 実質成長率 | | インフレ率 | |
|---|---|---|---|---|
| | トレンド | トレンドからの乖離 | 平均 | 標準偏差 |
| 日本 | 低下 | 不変ないし若干拡大（不安定化） | 低下（デフレ） | 低下（安定化） |
| 米国 | 不変 | 縮小（安定化） | 低下 | 低下（安定化） |

# 今後の課題

- 更なるセクター分割

— 技術的には本稿と同じ方法で更にセクターを分割することが可能。

- GLモデルの更なる一般化（生産性の伸び率格差を扱えるモデルへの拡張など）

- より現実的な生産関数・効用関数のもとでのシミュレーションの実施（資本を導入すれば経済の変動は増幅されるか、等）