Working Less and Bargain Hunting More: Macro Implications of Sales during Japan's Lost Decade

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Abstract

We examine macroeconomic implications of sales. By focusing on the fact that bargain hunting is time consuming, we construct a DSGE model with sales and households' endogenous bargain hunting. The model reveals that trend declines in hours worked during Japan's lost decade account for actual rises in a sales frequency, rises in the fraction of bargain hunters, and a part of actual declines in inflation rates. The real effects of monetary policy weaken, because sales prices are frequently revised and endogenous bargain hunting enhances the strategic substitutability of sales.

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1 Introduction

This paper asks whether sales are important for the macroeconomy. This question comes from mainly two motivations. The first concerns Japan's lost decade during the 1990s and possibly 2000s. As we will show below, firms during that period continued to raise their sales frequency. At the same time, hours worked continued to decline due to the prolonged recession in combination with the statutory reduction in hours worked, *jitan*, and possibly a demographic change. We suspect that those developments in sales and hours worked are interconnected. The second concerns the real effect of monetary policy. A number of papers point out that temporary sales take place frequently, with their prices being revised flexibly.[1] That makes a contrast with staggering normal or regular prices. The existence of sales appears to reduce the stickiness of the aggregate price, thereby damping the real effects of monetary policy.

In this paper, we examine macroeconomic implications of sales by extending Guimaraes and Sheedy (2011, hereafter GS). GS develop a DSGE model with sales, in which households consist of price-insensitive loyal customers and price-sensitive bargain hunters. GS show that firms' best strategy is holding periodic sales. Because sales are strategic substitutes, firms strike an optimal balance between the two types of households. Due to such a property, GS show that the real effects of monetary policy remain strong, even though sales entail no explicit cost and their prices are perfectly flexible.

Our contribution lies in incorporating endogenous bargain hunting decisions by focusing on the fact that bargain hunting is time-consuming. While GS assume a constant fraction of loyal customers (bargain hunters), we relax this assumption by allowing for an endogenous, time-varying change in the fraction of loyal customers. We assume that households face tradeoff in bargain hunting. On the one hand, bargain hunting increases utility by optimally choosing their consumption level for each good. On the other hand, bargain hunting decreases utility. Bargain hunting involves disutility, like labor supply. More precisely, we assume that the fraction of loyal customers (bargain hunters) enters into a labor supply term in a utility function. As hours worked increase, households have less time in bargain hunting, increasing disutility from bargain hunting.

We reveal that macroeconomic implications are greatly modified when considering sales and endogenous bargain hunting. We report mainly two findings. First, Japan's

[1] See, for example, Bils and Klenow (2004), Nakamura and Steinsson (2008), Kehoe and Midrigan (2010), and Eichenbaum, Jaimovich, and Rebelo (2011).

trend declines in hours worked account for actual trend rises in sales frequency during Japan's lost decade, if the changes in hours worked are driven by technology or demand shocks. In addition, our model suggests a downward (upward) trend in the fraction of loyal customers (bargain hunters). Trend declines in hours worked contribute, in part, to actual declines in the inflation rate.

Second, the effect of an accommodative monetary policy shock on real economic activity is mitigated, when bargain hunting is endogenous. The shock increases hours worked, which, in turn, increases (decreases) the fraction of loyal customers (bargain hunters). Firms lower their sales frequency. Since sales-priced goods are sold more than normal-priced goods in terms of quantity, those changes in households' and firms' actions yield a downward pressure on aggregate demand for goods. The real effects of monetary policy diminish. This result is also explained by intensified strategic substitutability of sales. Suppose that all firms but firm A raise their sales frequency. As in GS, it loses an incentive for firm A to raise its sales frequency, because its decreases the marginal revenue from sales. In our model, additional channel emerges. When all firms but firm A raise their sales frequency, an aggregate price falls. That increases aggregate demand for goods, and in turn, aggregate demand for labor. Households supply more labor and lose time in bargain hunting. The fraction of loyal customers (bargain hunters) increases (decreases). By observing this, firm A lowers its sales frequency. Such intensified strategic substitutability of sales mitigates the real effect of monetary policy.

The following two papers suggest that hours worked and bargain hunting closely interact. First, Aguiar and Hurst (2007) use scanner data and time diaries to examine households' substitution between shopping and home production. They find that older households shop the most frequently and pay the lowest price. Second, Lach (2007) analyzes store-level price data following the unexpected arrival of a large number of immigrants from the former Soviet union to Israel. He finds that the immigrants have a higher price elasticity and a lower search cost for goods than the native population.[2]

Regarding sales models, Varian (1980) shows firms' randomizing pricing strategy in the presence of informed and uninformed consumers. Kehoe and Midrigan (2010) develop a DSGE model that incorporates not just menu cost associated with regular prices but also cost associated with deviations of sale prices from regular prices.[3]

[2]See also Pashigian and Bowen (1991), Sorensen (2000), Brown and Goolsbee (2002), McKenzie and Schargrodsky (2004), Pashigian, Peltzman, and Sun (2003).

[3]Although they are not the model of sales, Benabou (1988) and Watanabe (2008) construct a model

The structure of this paper is as follows. Section 2 provides evidence for endogenous bargain hunting by looking at Japan's micro price data. Section 3 develops a model. Section 4 presents the model's impulse responses. Section 5 discusses Japan's lost decade. Section 6 concludes this paper.

2 Evidence for Endogenous Bargain Hunting

In this section, we document various evidence to motivate and justify our modelling strategy regarding endogenous bargain hunting. First, from a goods-demand side, we look at Japan's household survey on time use. We show the existence of time use heterogeneity in working and shopping across differing cohorts as well as its changes in the last two decades. Second, from a goods-supply side, we look at Japan's Point-of-Sales (POS) data and present time-series paths of some economic variables associated with sales. We examine changes in the sales frequency. The fraction of loyal customers (bargain hunters) is hardly observable. So we infer its movement by calibration based on the GS model or the calculation of a price elasticity.

2.1 Survey on time use

We begin by looking at Survey on Time Use and Leisure Activities. The survey is conducted by the Statistical Bureau every five years. It asks around 200,000 people in 80,000 households about their daily patterns of time allocation. Questionnaire includes time use in working and shopping. In that respect, this survey helps us examine their relationship, which is the key to our model.

Tables XX and XX shows summary results of households' time use in shopping and working (including commuting time for work and school), respectively. The sample is that of over 15 year old. Numbers in the tables indicate minutes per week. Two results are worth highlighting. First, shopping time is longer for those who are not working than those who are working. We also find that female spends longer shopping time than male. Second, shopping time continued to increase from 1986 to 2006, in particular for male. At the same time, hours worked continued to decline, although they picked up slightly in 2006. Those results appear to provide a support for our assumption that bargain hunting depends negatively on hours worked.

incorporating consumer search and price setting.

Table XX: Time Use in Shopping (minutes)

	Both		Male		Female	
	Working	Not working	Working	Not working	Working	Not working
1986	15	29	6	9	27	37
1991	17	30	9	12	30	38
1996	19	32	11	15	30	39
2001	21	32	13	18	31	39
2006	21	33	14	20	31	39

Source: Statistical Bureau "Survey on Time Use and Leisure Activities"

Table XX: Time Use in Work (including commuting time, minutes)

	Both		Male		Female	
	Working	Not working	Working	Not working	Working	Not working
1986	383	28	493	42	371	21
1991	370	26	481	41	358	19
1996	358	22	469	35	345	17
2001	401	17	456	26	324	13
2006	412	16	470	25	335	12

Source: Statistical Bureau "Survey on Time Use and Leisure Activities"

Figure XXX demonstrates the life cycle patterns of time use as of 2006. We see a negative correlation between time in shopping and time in working, in particular, for male and people up to about 75 year old. For male, hours worked peak at around 45 year old, and at the age, time in shopping hits a bottom. After that age, hours worked decline and time in shopping rise. Over 80 year old, time in shopping begins to drop. This result is in line with Aguiar and Hurst (2007). When we point out that Japan's rapid aging population has influenced bargain hunting as well as working, readers may wonder if bargain hunting is totally exogenous caused by the demographic reason. We do not deny this possiblity, but using a model, we consider an endogenous relationship between bargain hunting and working.

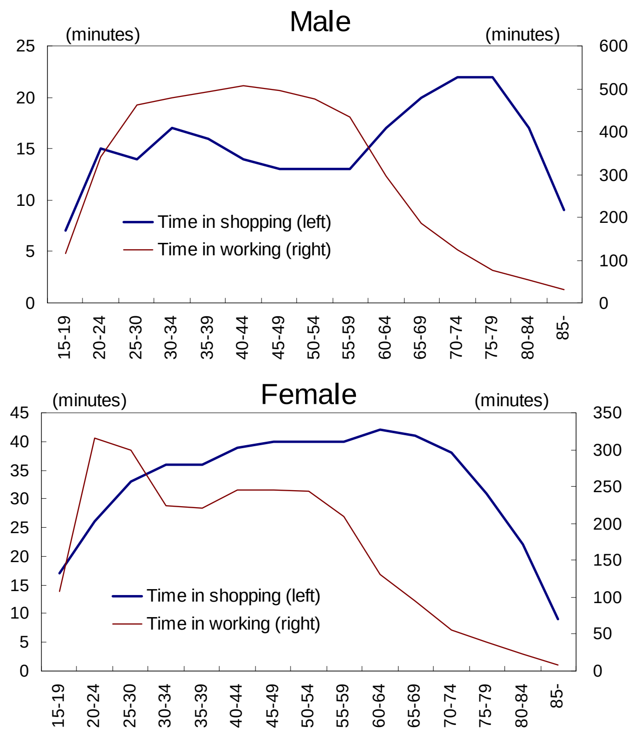


Figure XX: Time Use in Shopping and Working for Each Age and Sex (minutes)

Source: Statistical Bureau "Survey on Time Use and Leisure Activities"

2.2 POS Data

Next, from a goods-supply side, we examine indicators of sales using POS data.

The POS data are compiled by Nikkei Digital Media.[4] While existing literature often uses weekly or monthly data (e.g., Bils and Klenow [2004]), this POS data are daily. The sample period ranges from March 1, 1988 to XXX. The data are reported from various retail shops, including GMS and supermarkets throughout Japan. The number of products recorded exceeds XXX million, and the total number of observations is about XXX billion. The products covered in the data are restricted to ones with a product code, known as the JAN code. The POS contains processed foods and domestic articles, but not perishables, services, or expensive durable goods.

For each item and each shop, amount sold and proceeds are reported daily. Each

[4]See Abe and Tonogi (2010) for the pevious study using the POS data.

price is calculated as a unit price with proceeds being divided by amount sold. Proceeds exclude consumption tax. The unit price may be decimal due to the consumption tax, time sale during a day, and several other reasons.[5]

2.2.1 Sales frequency

Figure XXX demonstrate the aggregate, monthly time-series movements of three variables which are associated with sales and serve as key variables to the GS model.[6] They are the ratio of a sales price markup to a normal price markup μ, the ratio of quantities sold at sales price to those at regular price χ, and the frequency of sales s. Among them, this paper's focus is on the sales frequency s. Clearly, the sales frequency continues to rise during Japan's lost decade.

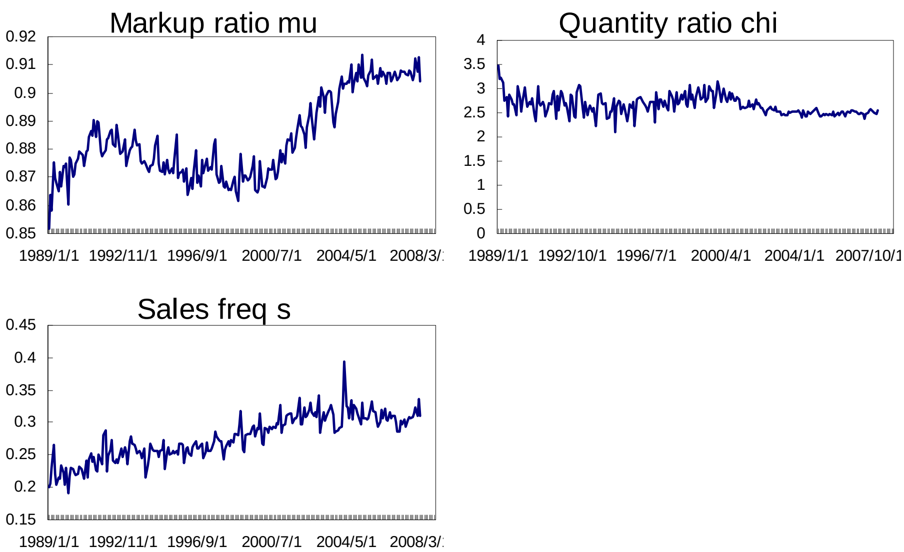


Figure XX: Key Sales Variables Obtained from POS Data

2.2.2 Fraction of loyal customers

We calibrate key deep parameters based on the GS model. In GS, aforementioned three variables in Figure XXX serve as targets to calibrate three key deep parameters. Cali-

[5]The revised tax law was put into effect on April 1, 2004, requiring shops to display their retail prices including consumption tax. This revision causes discontinuity in the POS data on that period, although the tax rate was unchanged and the POS data continued to compile proceeds excluding tax.

[6]For details, see Appendix XXX.

brated deep parameters are the elasticity of substitution between product types ϵ, the elasticity of substitution between brands for bargain hunters η, and the fraction of loyal customers λ.

We calibrate parameters monthly to investigate their changes. In doing so, we assume that the economy is at steady state at every period. Admittedly, this approach lacks justification. The following result is presented for the sake of illustration.

Figure XXX demonstrate the historical movements of the three calibrated parameters. We find that the fraction of loyal customers λ is not constant, although GS assume its constancy. The fraction of loyal customers tends to decrease over the sample period, partly owing to the steady increase in the sales frequency. As for other parameters, we find that the two elasticity parameters increase. That reflects a relative increase in sales prices to normal prices, while a relative quantity sold at sales prices to normal prices remains almost constant.

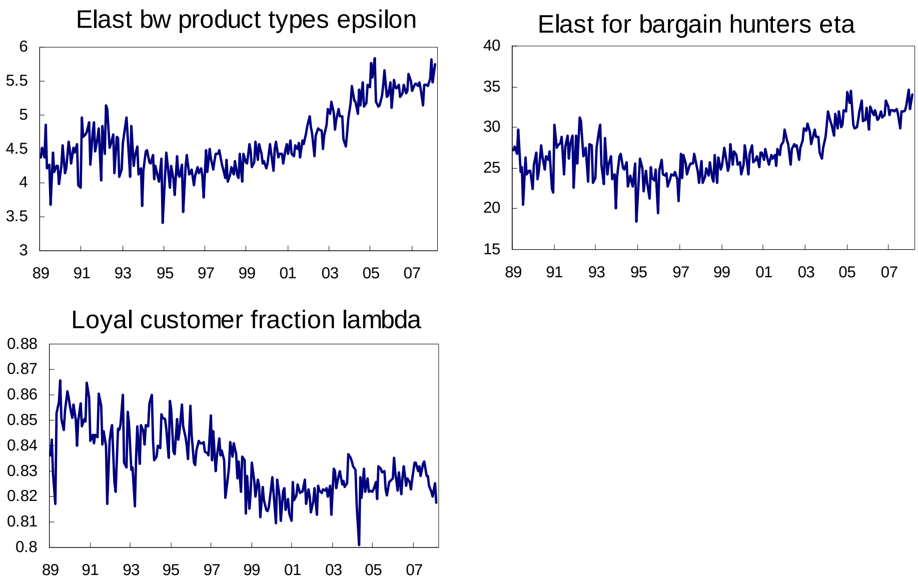


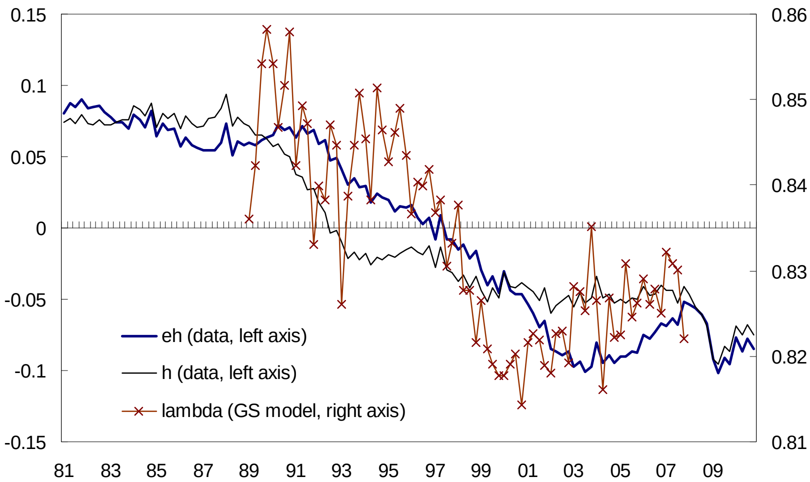
Figure XX: Key Sales Parameters Calibrated by the GS Model

When we look at the fraction of loyal customers more closely, we notice that it moves closely to labor market indicators. In Figure XXX, we plot the historical movements of labor market indicators on a left axis and the fraction of loyal customers on a right axis.

Two labor market indicators are (1) hours worked denoted by h and (2) hours worked times employment divided by the population over 15 denoted by eh.[7] The fraction of loyal customers is transformed into quarterly average.

The graph shows shrinking labor markets in the 1990s. Three forces are considered to be present. First, Japan was faced with the so-called lost decade after the burst of the asset price bubble in the early 1990s. That led to the prolonged recession. Second, the statutory *jitan* contributed to the fall in hours worked. *Jitan* was gradually introduced by the government thorough revisions of the Labor Standards Law: 1988:1Q to 1993:4Q and 1997:2Q to 1998:4Q, while the extent of jitan varied across industries and establishment sizes.[8] Third, demographic changes may have contributed to the declines in hours worked and employment, because Japan is one of the most rapidly growing aging countries.

Casual observations suggest a positive correlation between labor supply and the fraction of loyal customers. The trend declines in hours worked and employment are accompanied by the trend decrease in the fraction of loyal customers in the 1990s. In the early 2000s, the labor market recovered slightly. Coherently, the fraction of loyal customers picked up.



[7]Hours worked and employment are taken from Monthly Labour Survey in businesses with 30 or more employees. Hours worked represent those per capita. The data are seasonally adjusted and expressed as a logarithm deviation from their mean.

[8]Before the revision, legal work hours were 48 per week. Legal work hours were gradually reduced to 40. Hours worked exceeding this legal limit should be compensated by at least a 25-percent premium. See Kawaguchi, Naito, and Yokoyama (2008) for the analyses on *jitan* and Kuroda (2010) for the counter-argument asserting that hours worked hardly declined with demographic changes controlled.

Figure XX: Fraction of Loyal Customers and Labor Market Indicators
Source: Ministry of Health, Labour and Welfare "Monthly Labour Survey"

2.2.3 Price elasticity

The above discussion is logically inconsistent, however, because we used the GS model in which the fraction of loyal customers is assumed to be constant with the aim of examining the properties of the time-varying fraction of loyal customers. Without relying on a specific model like the GS model, we thus consider evidence for changes in bargain hunting. To this end, we look at how a price elasticity changes over time. Bargain hunters are considered to be more price sensitive than loyal customers. Therefore, if the fraction of loyal customers (bargain hunters) decreases (increases), the price elasticity should rise.

Figure XXX shows a scatter plot regarding quantity changes in response to price changes for the item of a cup noodle. The horizontal and vertical axes indicate daily price changes and quantity changes, respectively. When today's price or the quantity stays unchanged from the previous date, a dot is plotted at a position of one. The figure plots price changes for all periods and stores. A price elasticity is calculated as its slope. It is considered that supply shocks yield a negative slope, while demand shocks yield a positive slope. Therefore, we draw samples only from the second and fourth quadrants to calculate the elasticity. The figure is used for an illustration purpose. When we calculate the time-series path of elasticities, we use short sample periods and construct the aggregate elasticity as a weighted average in terms of stores and items.

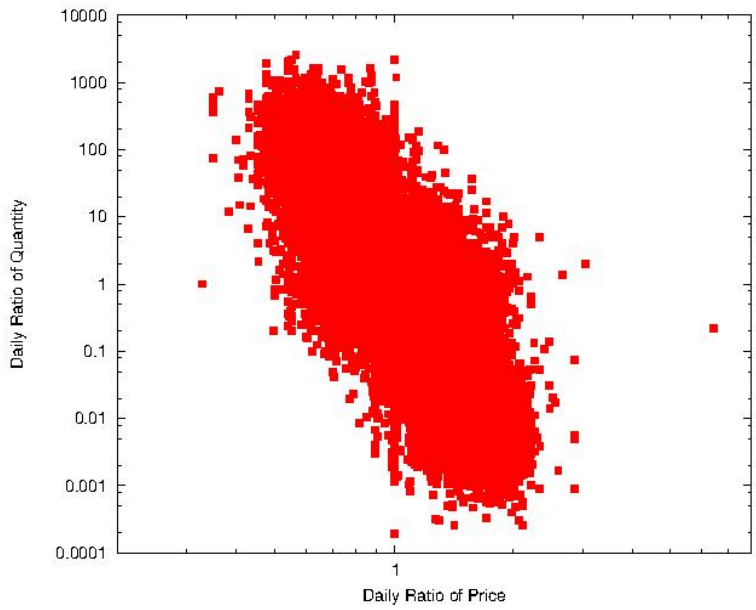


Figure XX: Price Elasticity for Cup Noodle Obtained from POS Data

Figure XXX plots the time-series movement of the price elasticity. That exhibits an upward trend in its absolute term, suggesting that households become more price sensitive. The price elasticity does not necessarily comove with the two labor market indicators, but their trends move in the same direction.

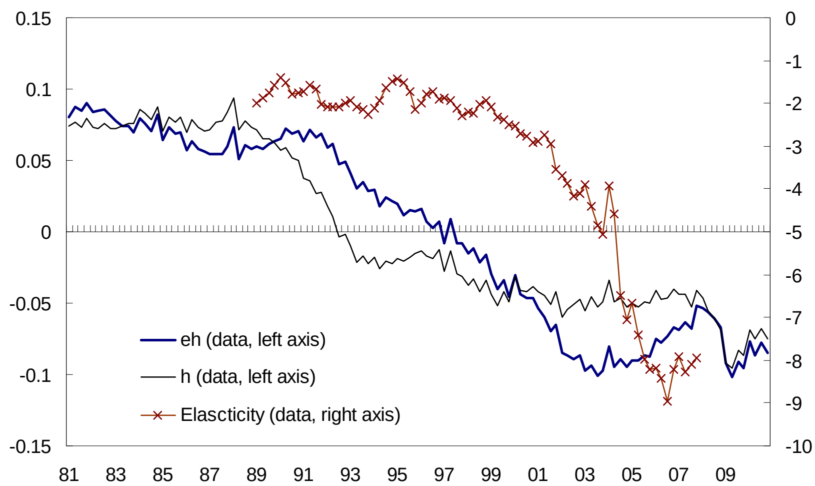


Figure XX: Price Elasticity and and Labor Market Indicators
Source: Ministry of Health, Labour and Welfare "Monthly Labour Survey"

To sum up, those observations support the idea that households' bargain hunting is endogenous, depending on their time spent in labor supply. As households are busy in work, they save time for bargain hunting, contributing to an increase (decrease) in the fraction of loyal customers (bargain hunters).

3 Model

Bearing the endogenous fraction of loyal customers in mind, we construct a sales model. Our model owes its great deal to GS.

3.1 Setup

Household We assume a cohort of households who has the following lifetime utility function:

$$u(t) = \sum_{j=0}^{\infty} \beta^j E_t \left[v(C_{t+j}) - Z^h_{t+j} \upsilon \left(H_{t+j} + \phi_L \frac{(1 - L_{t+j})^{\theta_L}}{(1-\lambda)^{\theta_L}} \right) \right], \tag{3.1}$$

where C_t is consumption, H_t is hours worked, and L_t is the share of loyal customers in the cohort ($0 < L < 1$). Z^h_t represents a stochastic shock to labor supply, with its logarithm deviation denoted by ε^h_t. The share of loyal customers L_t is endogenous, with its mean λ. Parameter β is the subjective discount factor ($0 < \beta < 1$), and ϕ_L and $\theta_L (> 0)$ represent the elasticity of utility from being loyal customers. The function $v(C_t)$ is strictly increasing and strictly concave in C_t, and $\upsilon(X_t)$ is strictly increasing and convex in X_t. The overall aggregator of consumption is given by

$$C = \left[\int \left(\int c(\tau, b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{3.2}$$

where $c(\tau, b)$ is the household's consumption of brand b of product type τ. GS give the example such that product types include beer and dessert and brand includes Corona beer and Ben & Jerry's ice cream. The above form of consumption differs from GS's definition [5]:

$$C = \left[\int c(\tau)^{\frac{\epsilon-1}{\epsilon}} d\tau + \int \left(\int c(\tau,b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}}.$$

[9] As in GS, we assume $\eta > \epsilon$, so that bargain hunters are more willing to substitute between different brands of a specific product type than households are to substitute between different product types.

Despite (3.2), a demand function for each good is assumed to be the same as GS's definition [7]:

$$c(\tau,b) = \begin{cases} \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* & \text{for } 1-L \text{ population} \\ \left(\frac{p(\tau,b)}{P}\right)^{-\epsilon} C^* & \text{for } L \text{ population,} \end{cases} \tag{3.3}$$

where $p(\tau, b)$ is the price of brand b of product type τ, $p_B(\tau)$ is an index of prices for all brands of product type τ, and P_t is the aggregate price level. C_t^* is aggregate consumption spending. The household is better off by selecting one's demand from the top of the demand function (3.3), because this is the optimal demand function derived from equation (3.2).

The household's budget constraint is given by

$$P_t C_t^* + E_t[Q_{t+1|t} A_{t+1}] = W_t H_t + D_t + A_t, \tag{3.4}$$

where W_t is the wage, D_t is dividends received from firms, Q_t is the asset pricing kernel, and A_t is the household's portfolio of Arrow-Debreu securities.

The endogenous L_t is the most important innovation made in this paper. In choosing the optimal L_t, the household confronts trade-off. On the one hand, an increase in L_t raises one's utility. As equation (3.1) shows, it increases time for leisure by decreasing the time for bargain hunting. On the other hand, the increase in L_t decreases the benefit from bargain hunting. The household decreases one's utility by selecting the suboptimal amount of demand as is specified by the bottom of the demand function (3.3). This second effect is more formally illustrated by the relationship between utility-related consumption C_t and spending-related consumption C_t^*. Appendix shows that C_t

[9]In what follows, a square bracket [ ] indicates a equation number in GS.

depends on not only C_t^* but also the following consumption wedge $\digamma_t$:[10]

$$C_t = \digamma_t \cdot \left(\frac{P_{B,t}}{P_t}\right)^{-\epsilon} C_t^*, \tag{3.5}$$

and that $\digamma_t < 1$ and $d\digamma_t/dL_t < 0$. As the household makes more bargain hunting, L_t decreases and F_t increases. Households enjoy higher utility from the same amount of consumption spending C_t^*. If the household makes bargain hunting for all goods, that is, $L_t = 0$, then we have $\digamma_t = 1$.

Additionally, Calvo-type wage stickiness is introduced as in GS. Households supply differentiated labor inputs to firms. Wages can be adjusted at a probability of $1 - \phi_w$.

Firms A good thing in our model is that firms' behavior is depicted in the same way as GS. Firms in our model face the same demand function given by equation (3.3) as those in GS. It is thus optimal for firms to randomize their price across shopping moments from a distribution with two prices. Firms set a normal high price $P_{N,t}$ with the frequency of $1 - s$ and a low sale price $P_{S,t}$ with the frequency of s. The only difference from GS is that firms optimize their pricing decisions by observing changes in the share of loyal customers L_t.

As GS argue, the strategic substitutability of sales plays a crucial role in firms' pricing. The more others have sales, the less an individual firm wants to have a sale. Suppose that other firms always have sales. If the individual firm stops a sale and sells its good at a normal price, its profit increases, because price-insenstive loyal customers tend to buy the good even at the normal price. As an opposite case, suppose that other firms have no sale. Because sales attract price-sensitive bargain hunters, the individual firm can increase its profit by having sales. Such strategic substitutability makes firms randomize their price.

Firms adjust their normal prices with Calvo-type price stickiness. In each period, firms have a probability of $1 - \phi_p$ to reset their normal prices. Sales prices can be adjusted freely.

Wholesalers produce goods using a labor input which consists of hours worked and the labor supply shock. Production technology is subject to a AR(1) shock ε_t^a.

[10] The utility-related consumption C also depends on the price ratio P_B/P, but that does not influence the household's decision of L because the household is a price taker. As in GS, the price index for bargain hunters is the same for all product types that is, $P_B = p_B(\tau)$.

Monetary authority A monetary authority sets a nominal interest rate i_t following the monetary policy rule of

$$i_t = \rho i_{t-1} + (1-\rho)\phi_\pi \pi_t^N + \varepsilon_t^i, \tag{3.6}$$

where ρ represents a policy inertia, ϕ_π represents the response to a normal price change π_t^N, and ε_t^i represents a shock to monetary policy.

Resource constraint A resource constraint is given by

$$Y_t = C_t^* + Z_t^g, \tag{3.7}$$

where Z_t^g is a government expenditure shock, with its logarithm deviation denoted by ε_t^g.

Exogenous shocks We consider four types of shocks. They are shocks to monetary policy, technology, government expenditure, and labor supply:

$$\varepsilon_t^i = \eta_t^i \tag{3.8}$$

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{3.9}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g \tag{3.10}$$

$$\varepsilon_t^h = \rho_h \varepsilon_{t-1}^h + \eta_t^h. \tag{3.11}$$

As for the monetary policy shock, we do not assume an inertia, because the monetary policy rule is persistent by construction.

3.2 Key equations

We provide key equations to the model in a log-linearized form.

Sales pricing It is optimal for a firm j to adjust its sale price $p_{S,j,t}$ by one-for-one with a change in its nominal marginal cost $x_t + p_t$,

$$p_{S,j,t} = x_t + p_t, \tag{3.12}$$

where a real marginal cost is denoted by x_t. This is the same as GS.

As the share of loyal customers l_t increases, firms decrease the sales frequency s_t:

$$ss_t = -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t \underline{- \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) l_t}. \tag{3.13}$$

In the equation above, a term with a underline represents a new term compared from GS. Like GS, an increase in the real marginal cost x_t decreases the sales frequency. Because the sales price responds by one-for-one to the marginal cost, the sales price increases more than the normal price. That decreases relative demand for sales, thereby decreasing the sales frequency.

Fraction of loyal customers The fraction of loyal customers l_t is described by

$$\begin{aligned}
0 = &\left(\theta_c^{-1} - 1 + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
&- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a - (\theta_h^{-1}-1)\varepsilon_t^h - (\theta_c^{-1}-1)\varepsilon_t^g \\
&- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L\frac{\lambda}{(1-\lambda)H}\right) l_t \\
&+ (\theta_c^{-1}-1)\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right)\right\} \\
&+ \frac{P_{SN}}{1-P_{SN}} p_{SN,t} + \frac{\eta-1}{\eta} f_t.
\end{aligned} \tag{3.14}$$

Two things are worth noting. First, the fraction of loyal customers l_t increases with hours worked h_t, which depends positively on aggregate demand y_t.[11] As hours worked lengthen, the disutility from bargain hunting increases.

Second, the fraction of loyal customers l_t increases with the consumption wedge f_t. An increase in the consumption wedge means an increase in utility from a given amount of consumption spending. As the wedge increases, the benefit from bargain hunting diminishes, raising the fraction of loyal customers. The consumption wedge increases with $p_{SN,t}$, which increases with the ratio of sales prices to normal prices: $\mu = P_{S,t}/P_{N,t}$ and decreases with the sales frequency s_t. In other words, as sales prices increase to converge to normal prices or sales become less frequent, prices become homogenous and

[11] See also equation (C.102)

the consumption wedge increases.

Phillips curve with sales The Phillips curve with sales is given by

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} \\ &+ \frac{1}{1-\psi}\left\{\kappa x_t + \psi(\Delta x_t - \beta E_t \Delta x_{t+1})\underline{+\kappa A l_t + A(\Delta l_t - \beta E_t \Delta l_{t+1})}\right\}.\end{aligned} \tag{3.15}$$

Compared with the standard New-Keynesian Phillips curve, the equation has two new terms. First, as in GS, changes in the real marginal cost, Δx_t, influence the inflation rate π_t. This is because the overall price changes through flexible sales prices as well as persistent normal prices. Second, unlike GS, the share of loyal customers l_t influences the inflation rate. As the share of loyal customers increases, the overall price increases. That results from the shift of demand for normal goods on a household side and a decrease in sales frequency on a firm side.

The real marginal cost x_t is described by

$$x_t = \frac{1}{1+\gamma\delta} w_t + \frac{\gamma}{1+\gamma\delta}\left(y_t \underline{- B l_t}\right). \tag{3.16}$$

As in the standard New-Keyensian model, the real marginal cost increases with both the real wage w_t and aggregate demand y_t. Furthermore, it decreases with the fraction of loyal customers l_t. Its mechanism runs as follows. When the share of loyal customers increases, demand for goods sold at the normal price increases and demand for goods sold at the sales price decreases. Such a shift of demand is amplified by a decrease in firms' sales frequency in response to the increase in the share of loyal customers. Since sales goods are generally sold more than normal goods in terms of quantity, total demand for the goods falls. That diminishes the supply of the goods, and in turn, the real marginal cost.

Moreover, the increase in the fraction of loyal customers functions to decrease the real wage for both labor demand and supply reasons, decreasing the real marginal cost further. The wage Phillips curve is given by

$$
\begin{aligned}
\pi_{W,t} &= \beta\pi_{W,t+1} \\
&\quad + \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w}\frac{1}{1+\varsigma\theta_h^{-1}}\Bigg[\\
&\quad \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) y_t - \left(1 + \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) w_t \\
&\quad - \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a - (\theta_h^{-1} - 1)\varepsilon_t^h - \theta_c^{-1}\varepsilon_t^g \\
&\quad \underline{- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B + \theta_h^{-1}\theta_L\phi_L\frac{\lambda}{(1-\lambda)H}\right) l_t} \\
&\quad +(\theta_c^{-1} - 1)\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\}\Bigg]. \qquad (3.17)
\end{aligned}
$$

On the labor demand side, for the same reason above, total demand for the goods falls, which decreases labor demand, and in turn, the real wage. On the labor supply side, the fraction of loyal customers increases (decreases) to offset an increase (decrease) in hours worked. Thus, for a given level of hours worked, the degree of real wage increases (decreases) declines.

4 Impulse Response Functions

We simulate impulse response functions (IRFs) of economic variables to four types of shock. The first shock is an accommodative shock to the monetary policy rule. The second shock is a positive shock to wholesalers' production technology. The third shock is a government spending shock as a demand shock. The fourth shock is a labor supply shock.

4.1 Calibration

Calibration is mostly based on GS. Unlike GS, we introduce an interest rate monetary policy rule and use $\rho = 0.8$ and $\phi_\pi = 1.5$. Also, we use values associated with sales so that they are consistent with Japan's POS data. More explanation XXX As for parameters associated with the fraction of loyal customers ϕ_L and θ_L, we target a steady state level of the fraction of loyal customers to calibrate ϕ_L given θ_L.

Table XX: Parameters

Parameters		
β	Discount factor	0.9975
θ_c	Elasticity of consumption	0.333
θ_h	Elasticity of labor supply	0.7
α	Elasticity of output to hours	0.667
γ	Elasticity of marginal cost	0.5
ς	Elasticity bw diff labor	20
ϕ_p	Calvo price stickiness	0.889
ϕ_w	Calvo wage stickiness	0.889
ρ	Mon pol rule inertia	0.8
ϕ_π	Mon pol rule on inflation	1.5

Parameters		
ρ_a	Persistence of tech shock	0.85
ρ_g	Persistence of gov shock	0.85
ρ_h	Persistence of labor shock	0.85

Target variables		
μ	Price ratio of sales to normal	0.883
χ	Quantity ratio of sales to normal	2.657
s	Sales frequency	0.276
Parameters		
ϵ	Elasticity bw product types	4.586
η	Elasticity bw brands	26.820
λ	Fraction of loyal customers	0.833

Target variable		
λ	Fraction of loyal customers	0.901
Parameter		
ϕ_L	Utility weight on loyal customers	1.6e-4 $\theta_L = 100$ 5.3e-3 $\theta_L = 3$

4.2 Comparison between the standard New-Keynesian model and the GS model

To understand GS's results, we begin with presenting IRFs in the GS model, in comparison with those in the standard New-Keynesian model. By the GS model, we mean the model discussed above without endogenous developments in the fraction of loyal customers. The standard New-Keynesian model corresponds to the GS model without sales.

Figure XXX presents the IRFs of three economic variables: aggregate demand, inflation rates excluding sales (normal price changes), and nominal interest rates. The horizontal axis indicates time up to eight quarters after a shock. Top and bottom panels demonstrate IRFs to the accommodative monetary policy shock and the positive technology shock, respectively. Dashed and solid lines indicate IRFs in the GS model and in the standard New-Keynesian model, respectively.

The top left panel shows that, as GS argue, the real effect of monetary policy in the model with sales remains large, which is close to that in the model without sales. Similarly, the bottom left panel shows that the real effect of the technology shock hardly changes by the incorporation of sales. Whether a price index includes sales or not, inflation rates become more volatile in the model with sales than in the model without sales. In the model with sales, sales prices keep a constant markup on marginal cost by flexible adjustments. That makes the aggregate price index move more flexibly than the model without sales. Since normal prices are reset with the consideration of the aggregate price index, they become more volatile in the model with sales than in the model without sales. Nevertheless, as GS argue, real effects are similar between the two models, owing to sales being strategic substitutes.

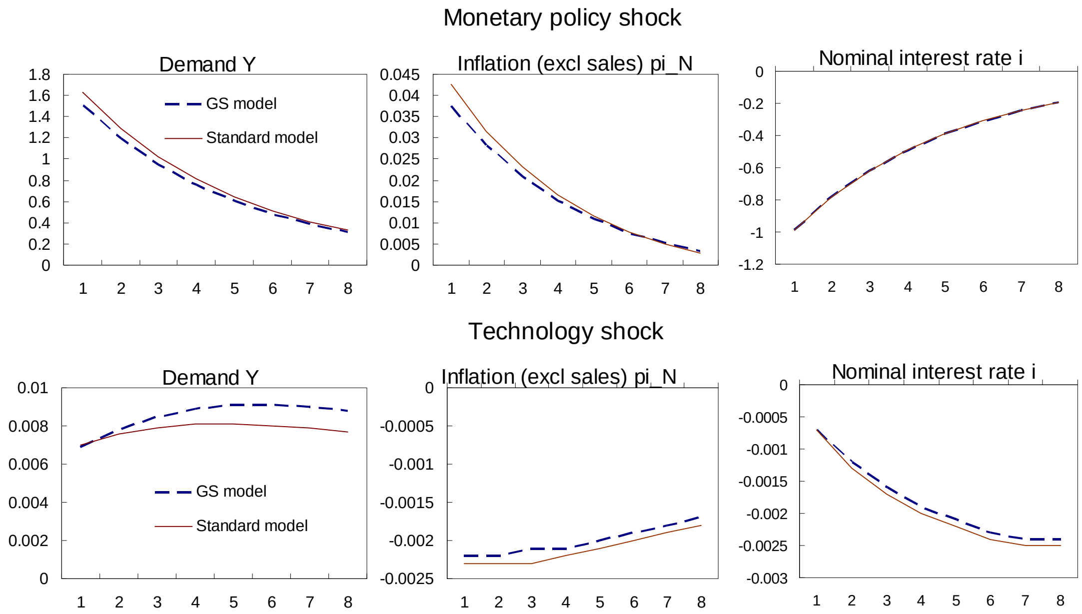


Figure XX: IRFs in the GS Model

4.3 Effects of endogenous bargain hunting

Now, we simulate IRFs in the model with endogenous developments in the fraction of loyal customers, in comparison with those in the GS model. We plot the IRFs of nine economic variables. In the figures below, dotted lines indicate IRFs in the GS model. Thick and thin solid lines both indicate IRFs in the model with endogenous developments in the fraction of loyal customers, with differing elasticity parameter values θ_L =3 and 100. A lower θ_L implies a higher elasticity of the fraction of loyal customers.

4.3.1 Monetary policy shock

Our simulation results reveal that sales can alter macroeconomic implications greatly. Figure XXX presents IRFs to the accommodative monetary policy shock. The effect of monetary policy on demand diminishes, in particular, when θ_L is as low as 3. The mechanism runs as follows. In response to the monetary policy shock associated with lowering the nominal interest rate, aggregate demand increases. That raises hours worked. Since households spend more time in works, their disutility from bargain hunting increases. With θ_L as low as 3, the fraction of loyal customers (bargain hunters) increases (decreases) in a relatively elastic manner. In viewing this, firms lower their sales frequency. Since sales-priced goods are sold more than normal-priced goods in terms of quantity, the decrease in the sales frequency mitigates the increase in aggregate demand.

The attenuated real effect of monetary policy is also explained by intensified strategic substitutability of sales. Suppose that all firms but firm A raise their sales frequency. As in GS, it loses an incentive for firm A to raise its sales frequency, because its decreases the marginal revenue from sales. In our model, additional channel emerges. When all firms but firm A raise their sales frequency, an aggregate price falls. That increases aggregate demand for goods, and in turn, aggregate demand for labor. Households supply more labor and lose time in bargain hunting. The fraction of loyal customers (bargain hunters) increases (decreases). By observing this, firm A lowers its sales frequency. Such intensified strategic substitutability of sales mitigates the real effect of monetary policy.

Inflation rates excluding sales (normal price changes) also move very differently in this model, compared with the GS model. In response to the accommodative monetary policy shock, inflation rates excluding sales increase far less in the model than in the GS model. As we explained in the previous section, the increase in the fraction of loyal customers functions to decrease the real wage and the real marginal cost. Although the increase in hours worked yields an upward pressure on the real marginal cost, the effect of the increase in the fraction of loyal customers functions dominates, when θ_L as low as 3. In contrast, the model yields greater increases in inflation rates including sales than the GS model. This is because the aggregate price index increases with both the fraction of loyal customers and the sales frequency.

When θ_L is as high as 100, IRFs resemble to those in the GS model. The fraction of loyal customers moves rigidly. That makes the model similar to the GS model in which the fraction of loyal customers as in GS is kept constant.

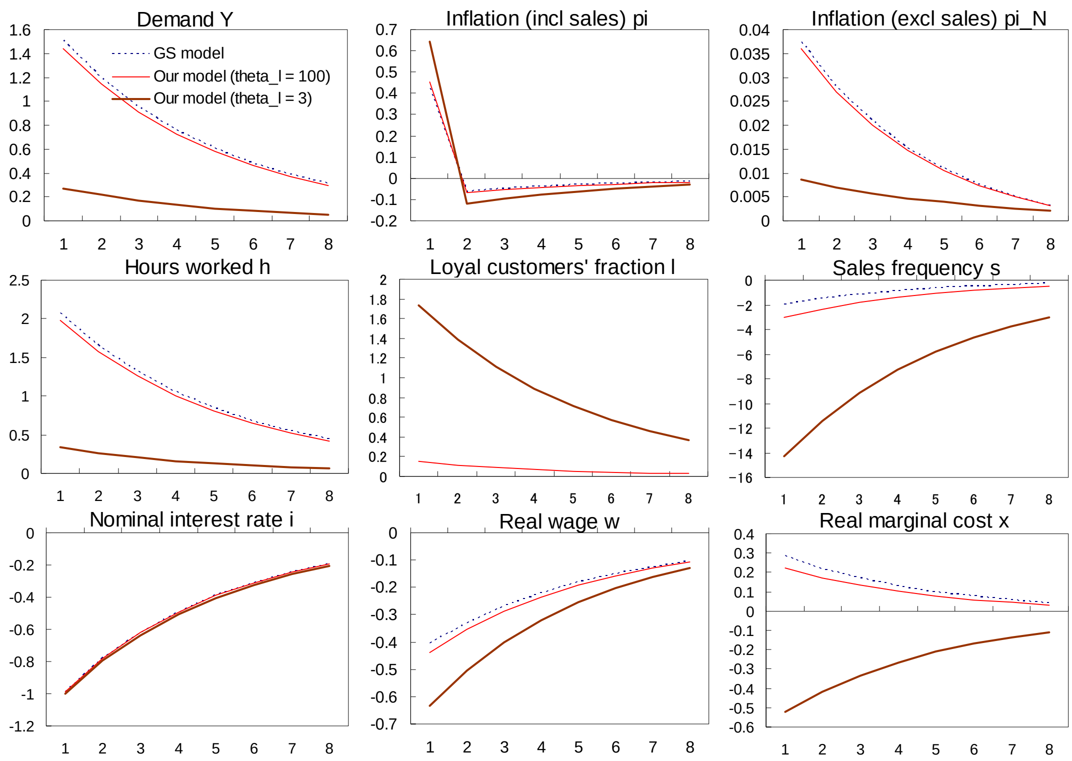


Figure XX: IRFs to an Accommodative Monetary Policy Shock

4.3.2 Technology shock

When the positive technology shock hits the economy, our model yields greater effects on aggregate demand and inflation than the GS model. In this type of sticky price model, the positive technology shock tends to decrease hours worked. That decreases (increases) the fraction of loyal customers (bargain hunters). Firms react to the shock by increasing their sales frequency. Because sales-priced goods are sold by a large amount, the increase in aggregate demand is magnified. The aggregate price falls, owing to the decrease in the fraction of loyal customers and the increase in the sales frequency. In contrast, the normal price increases. That results from increases in real wage and the real marginal cost, due to the decrease in the fraction of loyal customers. Although the graph plots only up to eight quarters after the shock, inflation excluding sales stays negative in the medium term in all of the models.

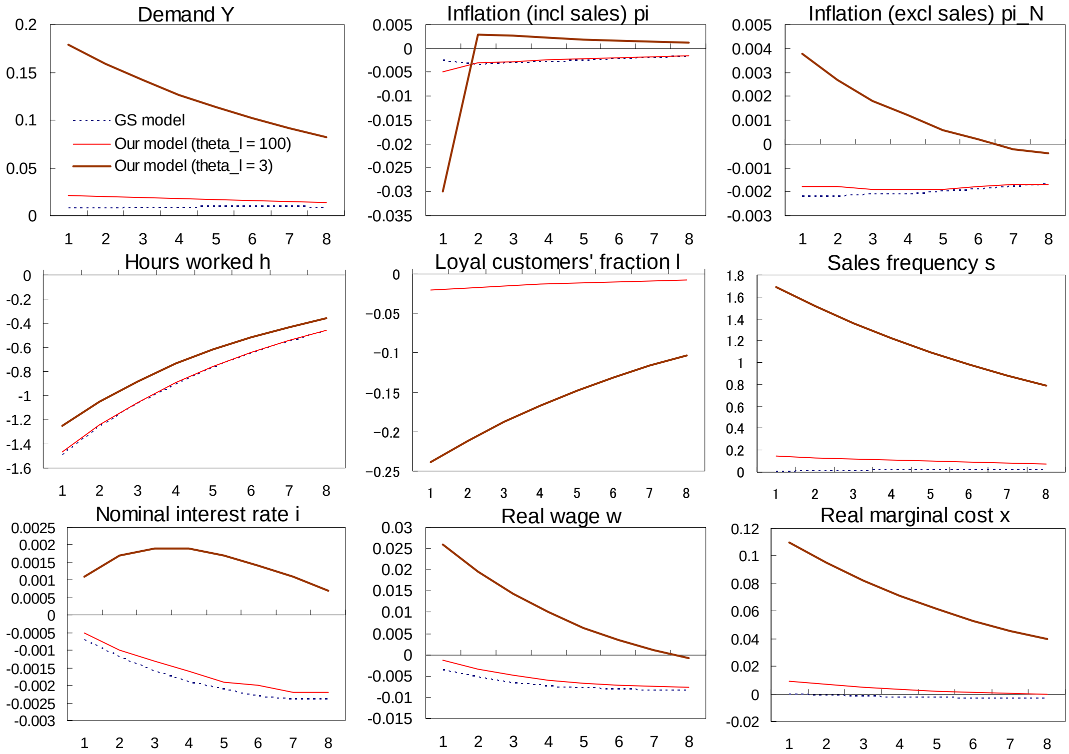


Figure XX: IRFs to a Positive Technology Shock

4.3.3 Government expenditure shock

Economic responses to the positive government expenditure shock resemble to those to the accommodative monetary policy shock.

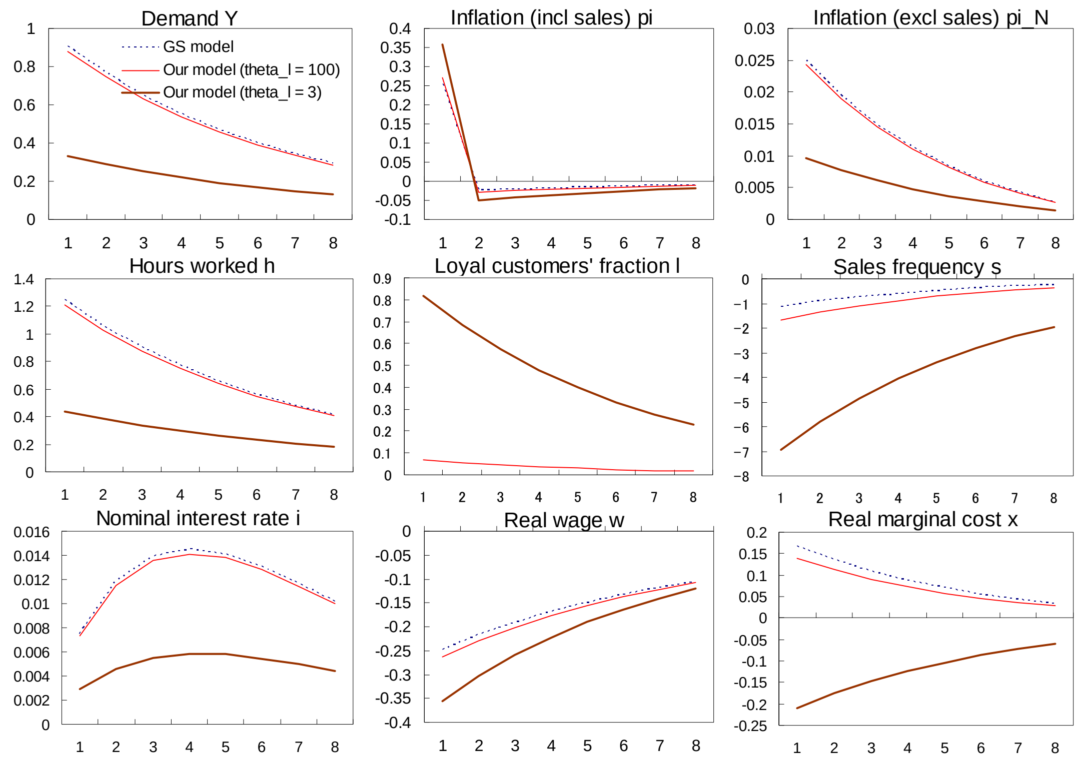


Figure XX: IRFs to a Positive Government Expenditure Shock

4.3.4 Labor supply shock

Finally, we simulate IRFs to a shock to labor supply. This shock is formulated, being motivated by Hayashi and Prescott (2002). In analyzing Japan's lost decade and incorporating the effects of *jitan*, they introduce the following utility function:

$$\log C_t - \alpha \frac{H_t}{40} E_t,$$

where H_t and E_t represent workweek length (hours) and the fraction of household members who work. Both H_t and E_t contribute to production. For 1990 to 1992, they take H_t as exogenous. In our model, as we showed in equation (3.1), we replace the exogenous $H_t/40$ for the labor supply shock Z_t^h and the endogenous E_t for labor supply H_t

with $\upsilon\left(H_t+\phi_L\frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}}\right)$. Both Z_t^h and H_t contribute to production. Note that, in our simulation, the elasticity of labor supply is 0.7, less than one. In Hayashi and Prescott (2002), it equals one.

Figure XXX demonstrates that, when a shock increases labor supply, labor input and the fraction of loyal customers move in the opposite direction, unlike when the above other types of shocks hit the economy. This is because the positive shock ε_t^h decreases h_t, although total labor input $(h_t+\varepsilon_t^h)$ increases. The decrease in h_t functions to lower the fraction of loyal customers, while the positive labor supply shock itself functions to raise the disutility of bargain hunting and thereby the fraction of loyal customers. With the elasticity of labor supply below one, the former effect dominates the latter; the fraction of loyal customers decreases. Although we do not show here, the fraction of loyal customers increases (unchanges), when the elasticity of labor supply exceeds (equals) one.

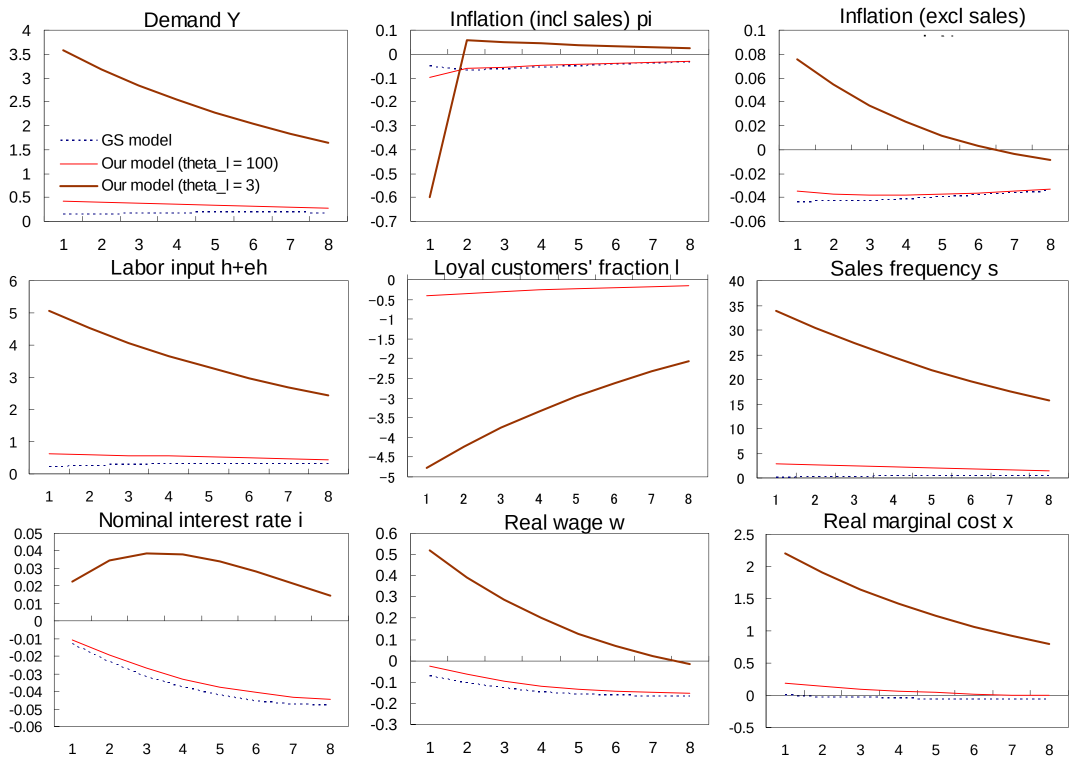


Figure XX: IRFs to a Labor Supply Shock

5 What Happened during Japan's Lost Decade

In Figure XXX, we have pointed out that the sales frequency s continues to rise during Japan's lost decade. Using our model, we argue that its reason is attributed to the decline in hours worked. We then discuss macroeconomic implications.

5.1 Approach

In doing simulation, we start with boldly assuming that only the technology shock drives the economy. We obtain the time-series path of the technology shock that accounts for actual hours worked in Japan (Figure XXX). Although we do not fully claim the validity of this assumption, we point out the following reasons. First, it is a natural step to regard the technology shock as a chief driving force of the economy, alongside the literature of RBC. In addition, Hayashi and Prescott (2002) argue that the slowdown of the TFP contributes to Japan's lost decade. Second, when *jitan* shortened the workweek length, labor hoarding may have decreased. Resulting enhanced labor efficiency is regarded as a positive technology shock.

We fix sales parameters calibrated for Japan's POS data and estimate the persistence of the technology shock only. Our sample ranges from 1981Q1 to 2008Q4. After obtaining the time-series path of the technology shock, we calculate the time-series paths of the sales frequency, the fraction of loyal customers, the inflation rate, and the sales markup. We use two models: the GS model and our model with endogenous developments in the fraction of loyal customers characterized by $\theta_L = 3$, which is chosen to fit data. For simplicity, we neglect the zero lower bound on the nominal interest rate, which constrained the effectiveness of monetary policy during Japan's lost decade.

5.2 Simulation results

Figure XXX illustrates that our model explains the movement of the sales frequency very well. It plots the model-based and actual sales frequency. In terms of the direction of its trend and the size of its changes, the model-based sales frequency moves very closely to actual one. Both series show steady increases in the sales frequency in the 1990s and 2000s. In the 1980s, when the actual data are missing, our model suggests a stable sales frequency. In comparison, the GS model predicts much attenuated changes in the sales frequency, which in the graph is almost flat.

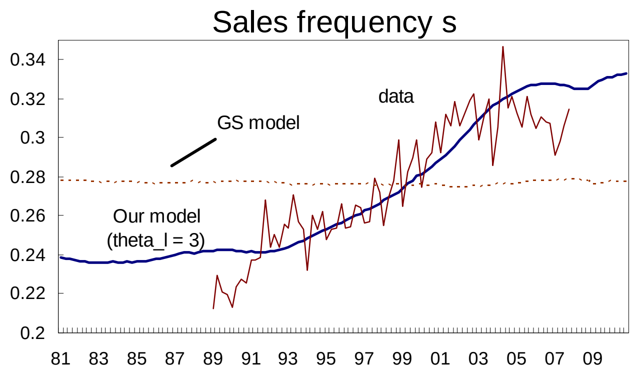


Figure XX: Model and Actual Sales Frequency

Our model demonstrates unobservable changes in the fraction of loyal customers. Figure XXX shows this. Our model predicts that the fraction of loyal customers stays almost constant in the 1980s. In Japan's so-called lost decade, the 1990s and 2000s, it exhibits a downward trend. Put differently, the fraction of bargain hunters increases during that period. Obviously, in the GS model, it remains constant. To check whether the fraction of loyal customers actually decreased, we plot the time series of the price elasticity calculated from the POS data and the time use in shopping obtained from the survey data. Their scales are adjusted to compare three series in one graph. The price elasticity increased in its absolute term. That indirectly supports a decrease in loyal customers, because the price elasticity of loyal customers is considered to be lower than that of bargain hunters.

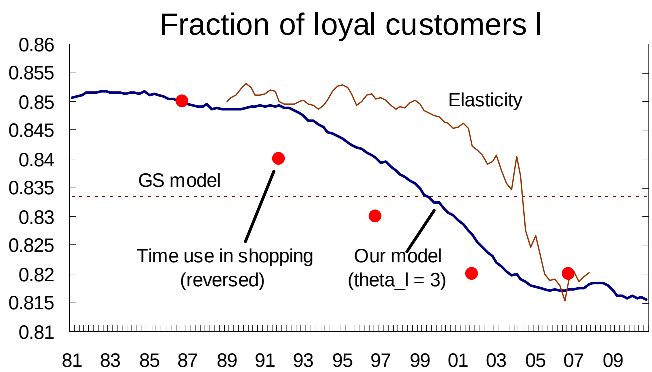


Figure XX: Model and Proxy Fraction of Loyal Customers

Next, we turn to inflation. Figure XXX shows simulation results with actual price changed measured by CPI and POS in an annual basis in percent. Using the technology shock obtained by the above method, we simulate the time-series path of inflation rate. The model-based inflation rate excludes sales, so that it corresponds to the official CPI. Our model predicts much attenuated fluctuations in inflation rates, compared with data. Our model has almost no advantage over the GS model. In terms of the trend, our model as well as the GS model generates the decline in the 1990s and 2000s, which is consistent with the data.

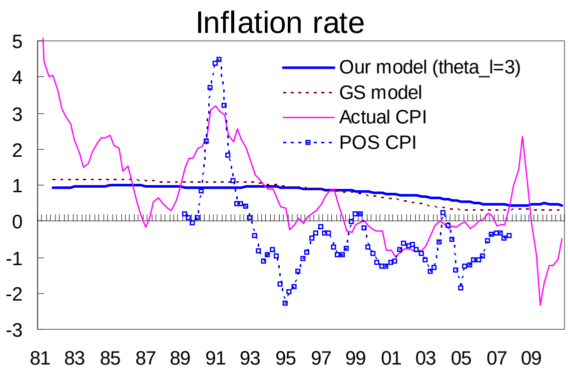


Figure XX: Model and Actual Inflation Rate

The difference between the aggregate price index and the normal price is demonstrated in Figure XXX. The aggregate price index includes sales prices. The normal price index is the one which corresponds to CPI. The aggregate price index was mostly negative during the lost decade, which implies that CPI underestimates the deflation. The GS model yields an attenuated difference between the two price indexes. This is because the sales frequency hardly changes in the GS model. Those model-generated series do not match the actual series obtained from the POS, although they are not directly comparable because weights between sales and normal goods are different.

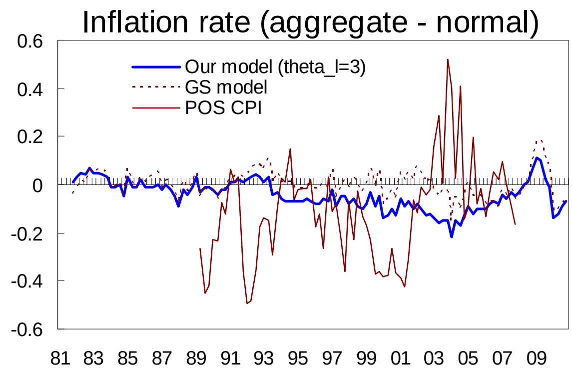


Figure XX: Model and Actual Inflation Rate Difference between Aggregated and Normal Price Indexes

Figure XXX shows the time-series path of the markup ratio of sales prices to normal prices. The performance of our model is as poor as the GS model.

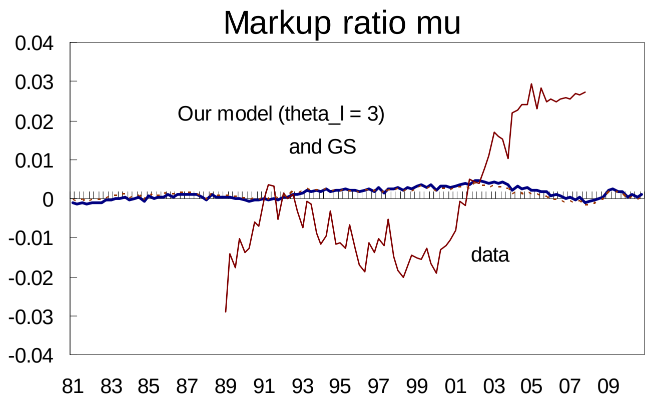


Figure XX: Model and Actual Markup Ratio

In sum, those simulation results suggest that our model improves the GS model in explaining the extensive margin of sales (sales frequency) but not the intensive margin (sales markup).

5.3 Other explanations

Although we implemented simulation by assuming that the temporary technology shock drives the economy, this assumption is not necessarily guaranteed. Other types of shocks may be suitable to account for the actual decline in hours worked. Instead of transitory shocks, structural changes may have shifted hours worked in their steady state.

We check robustness of our results in three ways. First, we investigate cases where other shocks than the technology shock drive the decline in hours worked. Second, we investigate cases where hours worked changes in their steady state. Third, we investigate cases where an innovation in bargain hunting technology influences bargain hunting in steady state.

5.3.1 Other stochastic shocks

We consider two other types of shocks: a government expenditure shock and a labor supply shock. A government expenditure shock, in part, captures the idea that the statutory decline in hours worked is subsidized by fiscal policy, influencing governmental expenditure. The government expenditure shock is also categorized as a demand shock, opposed to the technology shock analyzed above. If Japan's lost decade is understood as a situation where demand was insufficient, negative demand shocks become a candidate for the driving force of the Japanese economy. For example, Sugo and Ueda (2008) estimate a sticky-price DSGE model and find that an investment adjustment cost shock was a main driving force. Bayoumi (2001) and Caballero, Hoshi, and Kashyap (2008) emphasize a financial reason including a zombie lending as a cause of Japan's lost decade. Although the financial shock is not directly linked to the demand shock, the former is considered to influence demand for investment on the firm side. A labor supply shock is motivated by Hayashi and Prescott (2002), as we explained in the previous section.

Figure XXX shows that, if we assume that the government expenditure shock drives the actual changes in hours worked, our model performs as good as or even better than the previous case with the technology shock. First, as for the sales frequency, the model fits the best when θ_L is 10. It tracks the trend rise in the sales frequency as the previous case. Moreover, it explains its fall around 2005, as well. Second, the fraction of loyal customers is shown to decline over the 1990s and 2000s. Third, as for the inflation rate and the difference between the normal and aggregate price index, our model succeeds in yielding more volatile and closer movements to the actual one than the previous case.

On the other hand, if we assume the labor supply shock drives the actual changes in hours worked, our model predicts opposite movements. The sales frequency continues to fall, and the fraction of loyal customers and the inflation rate continue to rise. They are contrary to data and our aforementioned simulation results. Its reason is understood from Figure XXX. In the model, *jitan* is captured by a negative labor supply shock. To compensate the decrease in hours worked, labor supply h_t increases endogenously. With the labor supply elasticity below one, the increase in h_t is costly, preventing households' bargain hunting. The fraction of loyal customers thus increases and the sales frequency decreases. As we noted in the previous section, if we assume the unit elasticity of labor supply, the shock has no effect on total labor input, the sales frequency, and the fraction of loyal customers.

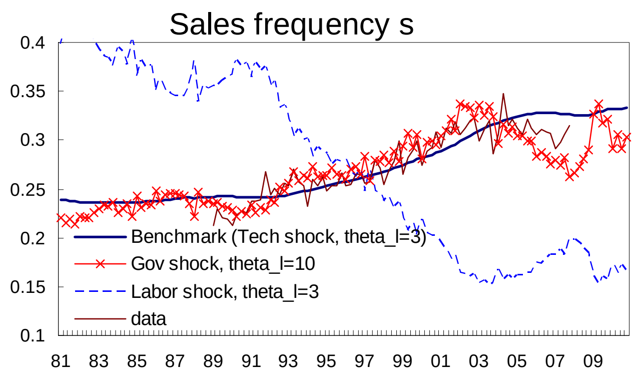


Figure XX: Model and Actual Sales Frequency 2

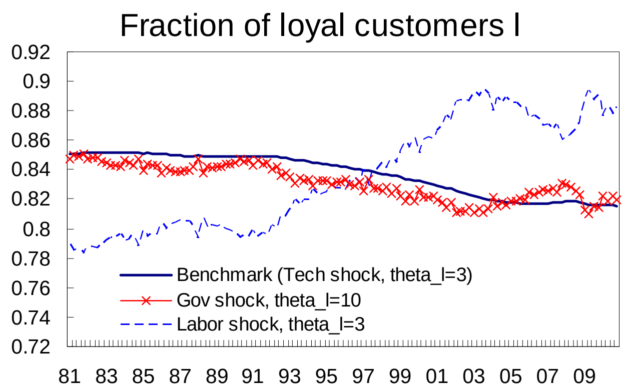


Figure XX: Model and Proxy Fraction of Loyal Customers 2

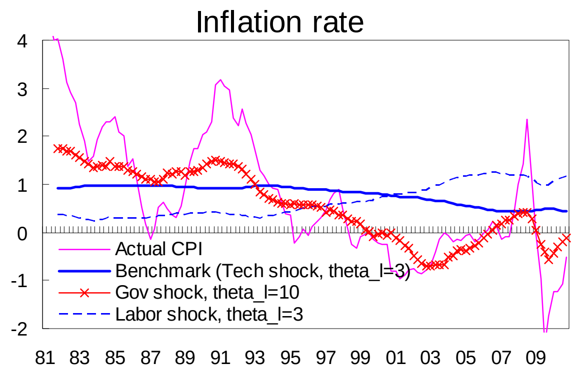


Figure XX: Model and Actual Inflation Rate

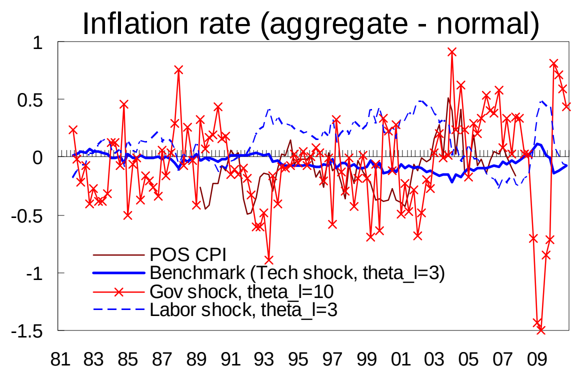


Figure XX: Model and Actual Inflation Rate Difference between Aggregated and Normal Price Indexes 2

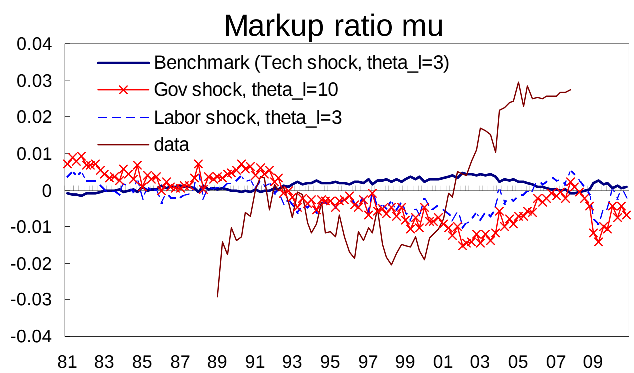


Figure XX: Model and Actual Markup Ratio 2

As a bottom line, our exercise suggests that both demand and supply shocks can account for the rise in the sales frequency, the fall in the fraction of loyal customers, and, in part, the fall in the inflation rate, by matching data for hours worked. The labor supply shock, however, yields completely opposite results.

5.3.2 Steady-state changes in hours worked

An alternative approach to accounting for changes in sales behavior is to assume that steady state has changed, instead of transitory shocks. To examine this possibility, we examine the effects of changes in steady-state hours worked on the sales frequency and the fraction of loyal customers. We fix parameters associated with sales, such as ϕ_L, θ_L, μ, and χ, assuming that steady-state hours worked change for other reasons. Other reasons include changes in technology, monetary policy, and the household' preference outside the arguments in function $v()$.

Figure XXX shows that decreases in steady-state hours worked raise the sales frequency and lowers the fraction of loyal customers. In the figure, the horizontal axis represents changes in steady-state hours worked in logarithm. The scale of vertical axis is identical with that in the top panel of Figure XXX. That suggests that about five to ten percent declines in hours worked account for the actual increase in Japan's sales frequency. The bottom panel shows that the declines in hours worked lead to declines in the fraction of loyal customers.

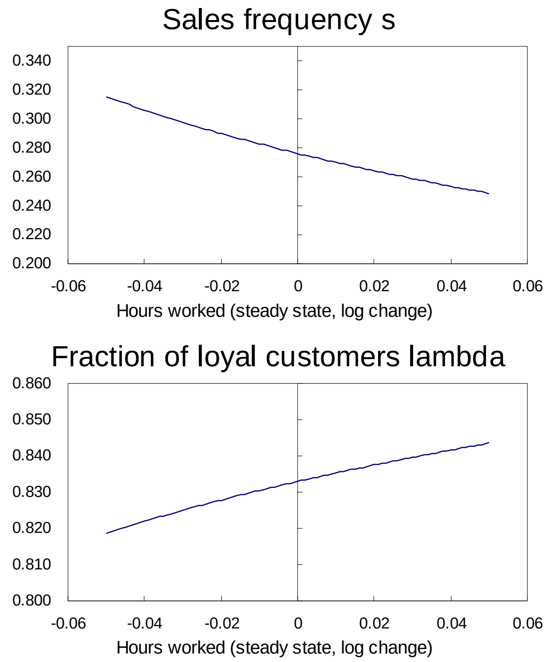


Figure XX: Effects of Steady-State Changes in Hours Worked

Hence, similar to transitory shocks analyzed in the previous section, steady-state declines in hours worked also account for the rise in the sales frequency and the fall in the fraction of loyal customers.

5.3.3 Innovation in bargain hunting technology

Another explanation may be provided for the rise in the sales frequency, by relating it to an innovation in bargain hunting technology. Brown and Goolsbee (2002) argue that the internet lowers search cost for customers. In our model, ϕ_L in equation (3.1) serves as a candidate to capture bargain hunting technology, in that ϕ_L is interpreted as the degree of disutility from bargain hunting. We calculate how the steady state values of the sales frequency and the fraction of loyal customers respond to changes in ϕ_L.[12]

Figure XXX suggests that an innovation in bargain hunting technology leads to a rise in the sales frequency and a fall in the fraction of loyal customers. A decrease in ϕ_L mitigates disutility from bargain hunting. If we interpret this as an innovation in bargain hunting technology, then the innovation encourages more bargain hunting (a fall in λ). That increases s.

[12] In equation (3.1), we fix λ in the denominator with its benchmark value.

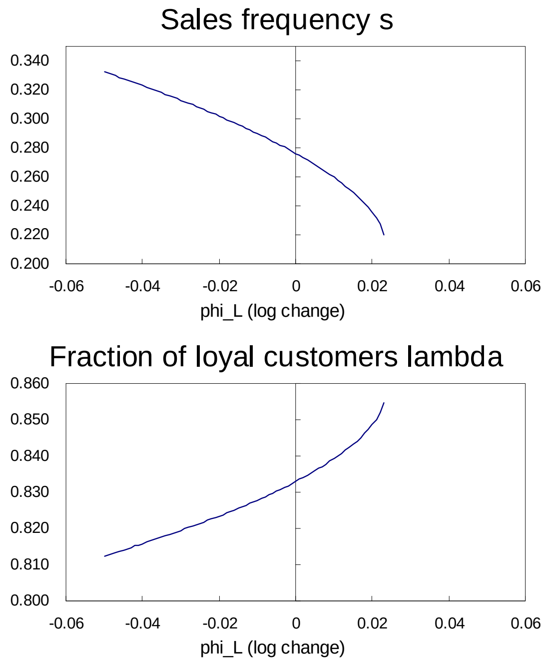


Figure XX: Effects of a Change in Disutility from Bargain Hunting

This simulation result implies that not just a reduction in hours worked but also an innovation in bargain hunting technology, possibly brought by the internet technology, contributes to the actual rise in sales frequency during Japan's lost decade.

6 Concluding Remarks

We have examined macroeconomic implications of sales. To this end, we have constructed a DSGE model with sales and households' endogenous bargain hunting. The model has revealed that trend declines in hours worked during Japan's lost decade account for actual rises in a sales frequency, rises in the fraction of bargain hunters, and a part of actual declines in inflation rates. Because sales prices are frequently revised and endogenous bargain hunting enhances the strategic substitutability of sales, the real effects of monetary policy weaken.

Albeit indecisive, our analyses have suggested that the adverse demand shock was a main driving force during Japan's lost decade. The shock succeeds in explaining not only rises in the sales frequency, but also declines in inflation rates and a difference between the price index excluding sales and the price index including sales. The shock is considered to reflect weak demand for fixed investment due to heavy debt burden on firms' and banks' sides.

Future research needs to securitize the sources of business cycles. Moreover, further qualitative and quantitative evidence for endogenous bargain hunting needs to be presented.

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Appendix

A POS Data

This appendix explains how we define sales and normal (regular) prices and how we compute elasticity and aggregated price indexes from the POS data.

We define daily time by t; a monthly time bin by t_2; a sub category by c; an item (JAN Code as a unique product identifier) by i; a store by s, a sales indicator r ($r = 0$ if sold at a normal price and $r = 1$ if sold at a sales price; a sales dummy by $I_t^{c,i,s}$; an official CPI weight in cateogy c by ω_t^c; sales quantity for item i in day t by $q_t^{c,i,s} = q_t^{c,i,s,0} + q_t^{c,i,s,1}$; price for item i sold at store s on day t by $p_t^{c,i,s}$; and expenditure for item i on day t by $e_t^{c,i} = \sum_s \sum_{r=0,1} q_t^{c,i,s,r} p_t^{c,i,s,r}$.

A.1 Normal (Regular) Price

Following Eichenbaum, Jaimovich, and Rebelo (2011), we define a normal price by the mode price in the window of about three months, that is, six weeks before and after each date. A good is judged as sales if its price differs from its normal price. That is, a normal price for item i sold on day t is defined by

$$\text{mode}_{t-42 \leq t \leq t+42}(p_t^{c,i,s}). \tag{A.1}$$

If multiple modes exist, we select the highest value as the mode price.

A.2 Elasticity

Price elasticity is defined by

$$\epsilon_t^{c,i,s} = \frac{\ln(q_t^{c,i,s}/q_{t-1}^{c,i,s})}{\ln(p_t^{c,i,s}/p_{t-1}^{c,i,s})} \tag{A.2}$$

In this paper, ϵ is estimated using least-square regression.

We calculate the Spearman's rank correlation C_s between $\ln(q_t^{c,i,s}/q_{t-1}^{c,i,s})$ and $\ln(p_t^{c,i,s}/p_{t-1}^{c,i,s})$. The null hypothesis is taken to be

$$H_0 : C_s = 0, \tag{A.3}$$

while the alternative hypothesis is taken to be

$$H_1 : C_s \neq 0. \tag{A.4}$$

The probability density of the test criterion $\frac{C_s}{\sqrt{((1-C_s^2)/(n-2))}}$ under the null hypothesis obeys the t-distribution. We estimate the elasticity ϵ if the null hypothesis is rejected at the 0.05 significance level.

A.3 Aggregation Procedures

A.3.1 Aggregation of the same item at different stores

The sales frequency is given by

$$s_t^{c,i} = \frac{\sum_s q_{t_2}^{c,i,s} I_t^{c,i,s}}{\sum_s q_{t_2}^{c,i,s}}, \tag{A.5}$$

where $I_t^{c,i,s}$ is defined by

$$I_t^{c,i,s} = \begin{cases} 0 & p_t^{c,i,s} = P_t^{c,i,s} \\ 1 & p_t^{c,i,s} \neq P_t^{c,i,s}. \end{cases}$$

The inflation rate is given by

$$\pi_t^{c,i} = \frac{\sum_s q_{t_2}^{c,i,s} \ln(p_t^{c,i,s}/p_{t-1}^{c,i,s})}{\sum_s q_{t_2}^{c,i,s}}. \tag{A.6}$$

The magnitude of price changes becomes

$$\mu_t^{c,i} = \frac{\pi_t^{c,i}}{s_t^{c,i}}. \tag{A.7}$$

Price elasticity is given by

$$\epsilon_t^{c,i} = \frac{\sum_s q_{t_2}^{c,i,s} \epsilon_t^{c,i,s}}{\sum_s q_{t_2}^{c,i,s}}. \tag{A.8}$$

The ratio of quantities sold at the sales price is monthly and given by

$$\chi_{t_2}^{c,i} = \frac{\sum_s Q_{t_2}^{c,i,s} (Q_{t_2}^{c,i,s,1}/\sum_r Q_{t_2}^{c,i,s,r})}{\sum_s Q_{t_2}^{c,i,s}}. \tag{A.9}$$

A.3.2 Aggregation of the same subcategory and different aggregated items

The sales frequency is given by

$$s_t^c = \frac{\sum_i e_{t_2}^{c,i} s_t^{c,i}}{\sum_i e_{t_2}^{c,i}}. \tag{A.10}$$

Price elasticity is given by

$$\epsilon_t^c = \frac{\sum_i e_{t_2}^{c,i} \epsilon_t^{c,i}}{\sum_i e_{t_2}^{c,i}}. \tag{A.11}$$

The ratio of quantities sold at the sales price is monthly and given by

$$\chi_{t_2}^c = \frac{\sum_i e_{t_2}^{c,i} \chi_{t_2}^{c,i}}{\sum_i e_{t_2}^{c,i}}. \tag{A.12}$$

A.3.3 Aggregation of different subcategories

The sales frequency is given by

$$s_t = \frac{\sum_c \omega_c s_t^c}{\sum_c \omega_c}. \tag{A.13}$$

Price elasticity is given by

$$\epsilon_t = \frac{\sum_c \omega_c \epsilon_t^c}{\sum_c \omega_c}. \tag{A.14}$$

The ratio of quantities sold at the sales price is monthly and given by

$$\chi_{t_2} = \frac{\sum_c \omega_c \chi_{t_2}^c}{\sum_c \omega_c}. \tag{A.15}$$

A.4 POS CPI

The POS CPI is defined by

$$CPI_t = C_0 \exp(\sum_{s=0}^{t} \pi_s). \tag{A.16}$$

B Summary of the Model

Equation [A.9a] in GS becomes equation (C.83):

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} \\ &+ \frac{1}{1-\psi} \left\{ \kappa x_t + \psi(\Delta x_t - \beta E_t \Delta x_{t+1}) + \kappa A l_t + A(\Delta l_t - \beta E_t \Delta l_{t+1}) \right\}.\end{aligned} \tag{B.1}$$

Equation [A.9b] in GS becomes equation (C.97):

$$x_t = \frac{1}{1+\gamma\delta} w_t + \frac{\gamma}{1+\gamma\delta} \left(y_t - B l_t \right). \tag{B.2}$$

Equation [A.9c] in GS becomes equation (C.107):

$$\begin{aligned} \pi_{W,t} = {} & \beta \pi_{W,t+1} \\ & + \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w} \frac{1}{1+\varsigma\theta_h^{-1}} [\\ & \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} \right) y_t - \left(1 + \frac{\delta}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} \right) w_t \\ & - \frac{\theta_h^{-1}}{\alpha} \varepsilon_t^a - (\theta_h^{-1} - 1) \varepsilon_t^h - \theta_c^{-1} \varepsilon_t^g \\ & - \left(\frac{1}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} B + \theta_h^{-1} \theta_L \phi_L \frac{\lambda}{(1-\lambda)H} \right) l_t \\ & + (\theta_c^{-1} - 1) \left\{ f_t - \epsilon \left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right) \right\} \Bigg]. \end{aligned} \tag{B.3}$$

Equation [A.9d] in GS holds:

$$\Delta w_t = \pi_{W,t} - \pi_t. \tag{B.4}$$

Equation [A.9e] in GS becomes equation (C.101):

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \theta_c (i_t - E_t \pi_{t+1}) + \varepsilon_t^g - \varepsilon_{t+1}^g \\ & + (1-\theta_c) \left\{ \Delta f_{t+1} - \epsilon \left(\Delta x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)} \Delta l_{t+1} \right) \right\}. \end{aligned} \tag{B.5}$$

A monetary policy rule is described as

$$i_t = \rho i_{t-1} + (1-\rho) \phi_\pi \pi_t^N + e_t^i. \tag{B.6}$$

The fraction of loyal customers is given by

$$
\begin{aligned}
0 = & \left(\theta_c^{-1} - 1 + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
& - \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a - (\theta_h^{-1} - 1)\varepsilon_t^h - (\theta_c^{-1} - 1)\varepsilon_t^g \\
& - \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L \frac{\lambda}{(1-\lambda)H}\right) l_t \\
& + (\theta_c^{-1} - 1)\left\{ f_t - \epsilon \left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t \right) \right\} \\
& + \frac{P_{SN}}{1-P_{SN}} p_{SN,t} + \frac{\eta - 1}{\eta} f_t. \qquad \text{(B.7)}
\end{aligned}
$$

The consumption wedge is given by equations (C.99) and (C.100):

$$
f_t = \frac{\eta}{\eta - 1}\lambda \frac{P_{SN} p_{SN,t} - (1-P_{SN}) l_t}{\lambda P_{SN} + (1-\lambda)}, \qquad \text{(B.8)}
$$

where

$$
\begin{aligned}
p_{SN,t} = & \frac{\left(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1\right)\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}(\mu^{1-\eta} - 1)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} ss_t \\
+ & \frac{\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}\mu^{1-\eta}(1-\eta)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} s\mu_t. \qquad \text{(B.9)}
\end{aligned}
$$

The sales price markup is given by equation (C.85):

$$
\mu_t = \frac{1}{1-\psi}\left(x_t + A l_t\right). \qquad \text{(B.10)}
$$

The frequency of sales is given by equation (C.91):

$$
ss_t = -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi} x_t - \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) l_t. \qquad \text{(B.11)}
$$

Production input is given by equation (C.105):

$$
\varepsilon_t^h + h_t = \frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - B l_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a. \qquad \text{(B.12)}
$$

As for the normal price index, equation (C.79) gives its Phillips curve:

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + \kappa(x_t + p_t - p_{N,t}). \tag{B.13}$$

C Model Details

Households Households maximize their utility

$$u(t) = \sum_{j=0}^{\infty} \beta^j E_t \left[v(C_{t+j}) - Z_{t+j}^h \upsilon \left(H_{t+j} + \phi_L \frac{(1-L_{t+j})^{\theta_L}}{(1-\lambda)^{\theta_L}} \right) \right], \tag{C.1}$$

where

$$C = \left[\int_\Lambda \left(\int_B c(\tau,b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \tag{C.2}$$

and Z_t^h represents a stochastic shock to labor supply, with its logarithm deviation denoted by ε_t^h. A demand function for each good is assumed to be the same as GS's definition [7]:

$$c(\tau,b) = \begin{cases} \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* & \text{for } 1-L \text{ population} \\ \left(\frac{p(\tau,b)}{P}\right)^{-\epsilon} C^* & \text{for } L \text{ population} \end{cases} \tag{C.3}$$

Substitution yields

$$\begin{aligned} C &= \left[\int_\Lambda \left(\int_B c(\tau,b)^{\frac{\eta-1}{\eta}} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\int_\Lambda \left(\begin{array}{c} L \int_B \left(\frac{p(\tau,b)}{P}\right)^{-\epsilon\frac{\eta-1}{\eta}} C^{*\frac{\eta-1}{\eta}} db \\ +(1-L) \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta\frac{\eta-1}{\eta}} \left(\frac{p_B(\tau)}{P}\right)^{-\epsilon\frac{\eta-1}{\eta}} C^{*\frac{\eta-1}{\eta}} db \end{array} \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} d\tau \right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\begin{array}{c} \int_\Lambda \left(L \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\epsilon\frac{\eta-1}{\eta}} db + (1-L) \int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{1-\eta} db \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} \\ \left(\frac{p_B(\tau)}{P}\right)^{1-\epsilon} d\tau \end{array} \right]^{\frac{\epsilon}{\epsilon-1}} C^* \end{aligned} \tag{C.4}$$

The price index for bargain hunters, $p_B(\tau)$, is given by

$$p_B(\tau) = \left(\int_B p(\tau,b)^{1-\eta} db \right)^{\frac{1}{1-\eta}}. \tag{C.5}$$

As in equation [20] in GS, given the a fraction s of all prices are at P_S and the remaining $1-s$ are at P_N, we obtain

$$P_B = p_B(\tau) = \left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{C.6}$$

Note that the above holds true under the flexible price model and the Rotemberg-type sticky price model, in which the normal price P_N is the same across goods τ. Under the Calvo-type sticky price model, P_N differs between goods τ, so the above equation does not hold precisely. Its log-linearization form guarantees the validity up to the first order. Also as is shown in equation [E.6] in GS, P_N needs to be defined as

$$P_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty}\phi_p^j R_{N,t-j}, \tag{C.7}$$

where $R_{N,t-j}$ is a new normal price set at $t-j$.

Terms inside equation (C.4) are given by

$$\begin{aligned}\int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{1-\eta} db &= \frac{sP_S^{1-\eta} + (1-s)P_N^{1-\eta}}{P_B^{1-\eta}} \\ &= 1, \end{aligned} \tag{C.8}$$

$$\begin{aligned}\int_B \left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\epsilon\frac{\eta-1}{\eta}} db &= \frac{sP_S^{-\epsilon\frac{\eta-1}{\eta}} + (1-s)P_N^{-\epsilon\frac{\eta-1}{\eta}}}{P_B^{-\epsilon\frac{\eta-1}{\eta}}} \\ &= \frac{sP_S^{\epsilon\frac{1-\eta}{\eta}} + (1-s)P_N^{\epsilon\frac{1-\eta}{\eta}}}{\left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{\epsilon}{\eta}}} \\ &= \frac{s\left(\frac{P_S}{P_N}\right)^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\left(\frac{P_S}{P_N}\right)^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}}, \\ &= \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)}{\left(s\mu^{1-\eta} + (1-s)\right)^{\frac{\epsilon}{\eta}}}, \end{aligned} \tag{C.9}$$

where the ratio of the sale-price markup to the nominal-price markup is defined as

$$\mu \equiv \frac{P_S}{P_N}. \tag{C.10}$$

Equation (C.4) thus becomes

$$
\begin{aligned}
C &= \left[\left(L \frac{s\mu^{\epsilon \frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}} + (1-L) \right)^{\frac{\eta(\epsilon-1)}{\epsilon(\eta-1)}} \left(\frac{P_B}{P} \right)^{1-\epsilon} \right]^{\frac{\epsilon}{\epsilon-1}} C^* \\
&= \left(L \frac{s\mu^{\epsilon \frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}} + (1-L) \right)^{\frac{\eta}{\eta-1}} \left(\frac{P_B}{P} \right)^{-\epsilon} C^* \\
&\equiv \digamma \cdot \left(\frac{P_B}{P} \right)^{-\epsilon} C^*.
\end{aligned} \tag{C.11}
$$

Here a consumption wedge $\digamma$ is defined as

$$\digamma = (LP_{SN} + (1-L))^{\frac{\eta}{\eta-1}}, \tag{C.12}$$

$$P_{SN} \equiv \frac{s\mu^{\epsilon \frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}}. \tag{C.13}$$

If $\mu < 1$, $\epsilon > 1$, $\eta > 1$, and $\epsilon/\eta < 1$, which is often the case, we have

$$P_{SN} < 1, \tag{C.14}$$

because $f(x) = x^{\epsilon/\eta}$ is a concave increasing function, and the denominator and numerator of P^{SN} are the weighted average of 1 and $\mu^{1-\eta}(> 1)$. Therefore, the consumption wedge satisfies $\digamma < 1$. That means that, because households do not optimally demand for goods, their utility from consumption decreases. The wedge increases as L decreases: the first differential $d\digamma/dL$ is given by

$$
\begin{aligned}
\frac{d\digamma}{dL} &= \frac{\eta}{\eta-1} (LP_{SN} + (1-L))^{\frac{\eta}{\eta-1}-1} (P_{SN} - 1) \\
&= -\frac{\eta}{\eta-1} (LP_{SN} + (1-L))^{\frac{1}{\eta-1}} (1 - P_{SN}) \\
&< 0
\end{aligned} \tag{C.15}
$$

If households make bargain hunting for all goods, that is, $L = 0$, then we have $\digamma = 1$. Households enjoy higher utility from consumption. However, it is accompanied with a decrease in utility by bargain hunting.

An aggregate price index P satisfies

$$
\begin{aligned}
PC^* &= \int_\Lambda \int_B p(\tau,b)c(\tau,b)dbd\tau \\
&= \int_\Lambda \Bigg\{ L\int_B p(\tau,b)\left(\frac{p(\tau,b)}{P}\right)^{-\epsilon} C^* db \\
&+(1-L)\int_B p(\tau,b)\left(\frac{p(\tau,b)}{p_B(\tau)}\right)^{-\eta}\left(\frac{p_B(\tau)}{P}\right)^{-\epsilon} C^* db \Bigg\} d\tau. \qquad \text{(C.16)}
\end{aligned}
$$

It yields

$$
\begin{aligned}
P &= \left[\left(\int_\Lambda \begin{array}{c} L\int_B p(\tau,b)^{1-\epsilon}db \\ +(1-L)\int_B p(\tau,b)^{1-\eta}db\,(p_B(\tau))^{\eta-\epsilon} \end{array}\right) d\tau\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\int_\Lambda \left(L\int_B p(\tau,b)^{1-\epsilon}db + (1-L)P_B^{1-\eta}P_B^{\eta-\epsilon}\right) d\tau\right]^{\frac{1}{1-\epsilon}} \\
&= \left[L\{s{P_S}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \qquad \text{(C.17)}
\end{aligned}
$$

Thus, in equation (C.11), we have

$$
\begin{aligned}
\left(\frac{P_B}{P}\right)^{-\epsilon} &= \frac{P_B^{-\epsilon}}{\left(L\{s{P_S}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}\right)^{-\frac{\epsilon}{1-\epsilon}}} \\
&= \left(\frac{L\{s{P_S}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\} + (1-L)P_B^{1-\epsilon}}{P_B^{1-\epsilon}}\right)^{\frac{\epsilon}{1-\epsilon}} \\
&= \left(L\frac{s{P_S}^{1-\epsilon} + (1-s)P_N^{1-\epsilon}}{\left(sP_S^{1-\eta} + (1-s)P_N^{1-\eta}\right)^{\frac{1-\epsilon}{1-\eta}}} + (1-L)\right)^{\frac{\epsilon}{1-\epsilon}} \\
&= \left(L\frac{s\mu^{1-\epsilon} + (1-s)}{\left(s\mu^{1-\eta} + (1-s)\right)^{\frac{\epsilon-1}{\eta-1}}} + (1-L)\right)^{-\frac{\epsilon}{\epsilon-1}}. \qquad \text{(C.18)}
\end{aligned}
$$

That is larger than one and decreases as L decreases. As the share of bargain hunting $(1-L)$ increases, the weight of bargain price index P_B increases, and the aggregate price index P decreases. Thus, the relative bargain price to the aggregate price increases, which decreases demand for bargain goods and increases demand for normal goods. Because bargain goods are sold more than normal goods, total demand decreases.

Households' budget constraint is

$$P_t C_t^* + E_t[Q_{t+1|t} A_{t+1}] = W_t H_t + D_t + A_t. \tag{C.19}$$

Each household optimizes ones behavior given P_B/P. The first-order conditions are written as follows: with respect to C,

$$\begin{aligned}\beta E_t \left[\frac{v_C(C_{t+1})}{v_C(C_t)} \frac{P_t}{P_{t+1}} \frac{\digamma_{t+1}}{\digamma_t} \left(\frac{P_{B,t+1}/P_{B,t}}{P_{t+1}/P_t} \right)^{-\epsilon} \right] &= E_t \left[Q_{t+1|t} \right] \\ &= \frac{1}{1+i_t}; \end{aligned} \tag{C.20}$$

with respect to H,

$$\frac{Z_t^h v_H \left(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}} \right)}{v_C(C_t)} = \frac{W_t}{P_t} \digamma_t \left(\frac{P_{B,t}}{P_t} \right)^{-\epsilon}; \tag{C.21}$$

and with respect to L,

$$\theta_L \phi_L \frac{(1-L_t)^{\theta_L - 1}}{(1-\lambda)^{\theta_L}} \frac{Z_t^h v_H \left(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}} \right)}{v_C(C_t)} = -\frac{C_t}{\digamma_t} \frac{d\digamma_t}{dL_t}. \tag{C.22}$$

The last equation is rearranged as

$$\begin{aligned} &\theta_L \phi_L \frac{(1-L_t)^{\theta_L - 1}}{(1-\lambda)^{\theta_L}} \frac{Z_t^h v_H \left(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}} \right)}{v_C(C_t)} \\ &= \frac{C_t}{\digamma_t} \frac{\eta}{\eta - 1} \left(L_t P_{SN,t} + (1-L_t) \right)^{\frac{1}{\eta-1}} \left(1 - P_{SN,t} \right) \\ &= \frac{\eta}{\eta-1} C_t \frac{\left(L_t P_{SN,t} + (1-L_t) \right)^{\frac{1}{\eta-1}} \left(1 - P_{SN,t} \right)}{\left(L_t P_{SN,t} + (1-L_t) \right)^{\frac{\eta}{\eta-1}}} \\ &= \frac{\eta}{\eta-1} C_t \frac{1 - P_{SN,t}}{L_t P_{SN,t} + (1-L_t)}. \end{aligned} \tag{C.23}$$

Resource constraint A resource constraint is given by

$$Y_t = C_t^* + Z_t^g \tag{C.24}$$

$$= \frac{C_t}{\digamma_t \cdot \left(\frac{P_B}{P} \right)^{-\epsilon}}, \tag{C.25}$$

where Z_t^g is a government expenditure shock, with its logarithm deviation denoted by ε_t^g. It is log-linearized as

$$c_t = y_t - \varepsilon_t^g + f_t - \epsilon \left(p_{B,t} - p_t \right). \tag{C.26}$$

Monetary policy A monetary policy rule is described as

$$i_t = \rho i_{t-1} + (1-\rho)\phi_\pi \pi_t^N + e_t^i, \tag{C.27}$$

where the inflation rate for normal prices is defined by $\pi_{N,t} \equiv p_{N,t} - p_{N,t-1}$ and e_t^i indicates a monetary policy shock.

Firms (Proof of Theorem 3 in GS) Firms' problem is almost the same as that in GS, because firms face the same demand function (C.3). The share of loyal customers L is endogenous, but each firm take the L given, so this fact does not change firms' optimization problem.

Regarding the demand function at the sale and normal prices, equation [22] in GS becomes

$$Q_S = (L + (1-L)v_S)(P_S/P)^{-\epsilon} Y \tag{C.28}$$

$$Q_N = (L + (1-L)v_N)(P_N/P)^{-\epsilon} Y, \tag{C.29}$$

where v is the purchase multiplier defined in equation [10] in GS:

$$v(p; P_B) = (p/P_B)^{-(\eta-\epsilon)}. \tag{C.30}$$

This is defined as the ratio of the amounts sold at the same price to a given measure of bargain hunters relative to the same measure of loyal customers. By log-linearizing the above demand functions, equations [E.1a] and [E.1b] in GS become

$$q_{S,j,t} = \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S} v_{S,j,t} - \epsilon(p_{S,j,t} - p_t) + y_t, \tag{C.31}$$

$$q_{N,j,t} = \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} l_t + \frac{(1-\lambda)v_N}{\lambda + (1-\lambda)v_N} v_{N,j,t} - \epsilon(r_{N,t-j} - p_t) + y_t, \tag{C.32}$$

where $r_{N,t-j}$ is a normal price set j periods ago. Regarding the purchase multiplier v, equations [E.2] in GS are the same:

$$\begin{aligned} v_{S,j,t} &= -(\eta-\epsilon)(p_{S,j,t}-p_{B,t}), \\ v_{N,j,t} &= -(\eta-\epsilon)(r_{N,t-j}-p_{B,t}). \end{aligned} \tag{C.33}$$

Above four equations yield equivalent equations to [E.3a] and [E.3b] in GS:

$$\begin{aligned} q_{S,j,t} &= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t - \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}(\eta-\epsilon)(p_{S,j,t}-p_{B,t}) - \epsilon(p_{S,j,t}-p_t) + y_t \\ &= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t \\ &\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S} p_{S,j,t} + (\eta-\epsilon)\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} p_{B,t} + \epsilon p_t + y_t, \end{aligned} \tag{C.34}$$

$$\begin{aligned} q_{N,j,t} &= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}(\eta-\epsilon)(r_{N,t-j}-p_{B,t}) - \epsilon(r_{N,t-j}-p_t) + y_t \\ &= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t \\ &\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} r_{N,t-j} + (\eta-\epsilon)\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} p_{B,t} + \epsilon p_t + y_t. \end{aligned} \tag{C.35}$$

The optimal price markup is given by equation [17] in GS. Rewrite this as

$$\{L(\epsilon-1)+(1-L)(\eta-1)v(p;P_B)\}\,\mu(p;P_B) = L\epsilon+(1-L)\eta v(p;P_B). \tag{C.36}$$

Log-linearization yields

$$\frac{\lambda(\epsilon-1)l_t - \lambda(\eta-1)vl_t + (1-\lambda)(\eta-1)vv_t}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v} + \mu_t = \frac{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t}{\lambda\epsilon+(1-\lambda)\eta v}$$

$$
\begin{aligned}
\mu_t &= \frac{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t}{\lambda\epsilon + (1-\lambda)\eta v} - \frac{\lambda(\epsilon-1) l_t - \lambda(\eta-1) v l_t + (1-\lambda)(\eta-1) v v_t}{\lambda(\epsilon-1) + (1-\lambda)(\eta-1) v} \\
&= \frac{\{\lambda\epsilon l_t - \lambda\eta v l_t + (1-\lambda)\eta v v_t\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} \\
&+ \frac{\{\lambda(\epsilon-1) l_t - \lambda(\eta-1) v l_t + (1-\lambda)(\eta-1) v v_t\}\{\lambda\epsilon + (1-\lambda)\eta v\}}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} \\
&= \frac{\{\epsilon - \eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\} - \{(\epsilon-1) - (\eta-1)v\}\{\lambda\epsilon + (1-\lambda)\eta v\}}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} \lambda l_t \\
&+ \frac{\eta\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\} - (\eta-1)\{\lambda\epsilon + (1-\lambda)\eta v\}}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} (1-\lambda) v v_t \\
&= \frac{\epsilon(1-\lambda)(\eta-1)v - \eta v\lambda(\epsilon-1) - (\epsilon-1)(1-\lambda)\eta v + (\eta-1)v\lambda\epsilon}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} \lambda l_t \\
&+ \frac{\lambda(1-\lambda)(\epsilon-\eta)v}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} v_t \\
&= \frac{(1-\lambda)(\eta-\epsilon)v + v\lambda(\eta-\epsilon)}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} \lambda l_t \\
&+ \frac{\lambda(1-\lambda)(\epsilon-\eta)v}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} v_t \\
&= \frac{\lambda(\eta-\epsilon)v}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} l_t \\
&- \frac{\lambda(1-\lambda)(\eta-\epsilon)v}{\{\lambda\epsilon + (1-\lambda)\eta v\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v\}} v_t. \qquad \text{(C.37)}
\end{aligned}
$$

We define

$$
\varrho_S = \frac{\lambda(1-\lambda)(\eta-\epsilon)v_S}{\{\lambda\epsilon + (1-\lambda)\eta v_S\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v_S\}} \qquad \text{(C.38)}
$$

$$
\varrho_N = \frac{\lambda(1-\lambda)(\eta-\epsilon)v_N}{\{\lambda\epsilon + (1-\lambda)\eta v_N\}\{\lambda(\epsilon-1) + (1-\lambda)(\eta-1)v_N\}}, \qquad \text{(C.39)}
$$

and equations [E.4a] and [E.4b] are transformed into

$$
\mu_{S,j,t} = -\varrho_S v_{S,j,t} + \frac{1}{1-\lambda}\varrho_S l_t, \qquad \text{(C.40)}
$$

$$
\mu_{N,j,t} = -\varrho_N v_{N,j,t} + \frac{1}{1-\lambda}\varrho_N l_t. \qquad \text{(C.41)}
$$

Overall demand,

$$
Q_{j,t} = s_{j,t} Q_{S,j,t} + (1-s_{j,t}) Q_{N,j,t},
$$

is log-linearized as

$$Qq_{j,t} = sQ_S(s_{j,t} + q_{S,j,t}) - sQ_N s_{j,t} + (1-s)Q_N q_{N,j,t}.$$

Using $\chi = Q_S/Q_N$, we obtain equation [E.5] in GS:

$$(s\chi + 1 - s)q_{j,t} = s\chi(s_{j,t} + q_{S,j,t}) - ss_{j,t} + (1-s)q_{N,j,t},$$

$$q_{j,t} = \frac{\chi - 1}{s\chi + 1 - s} ss_{j,t} + \frac{s\chi}{s\chi + 1 - s} q_{S,j,t} + \frac{(1-s)}{s\chi + 1 - s} q_{N,j,t}. \tag{C.42}$$

Note we define $s_{j,t}$ as the log deviation, while GS define it as the deviation from steady state.

Define the weighted average of variables as in equations [E.6] and [E.7] in GS:

$$s_t \equiv (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j s_{j,t},$$

$$p_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j r_{N,t-j}, \quad q_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j q_{N,j,t},$$
$$v_{N,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j v_{N,j,t}, \tag{C.43}$$
$$p_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j p_{S,j,t}, \quad q_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j q_{S,j,t},$$
$$v_{S,t} = (1-\phi_p)\sum_{j=0}^{\infty} \phi_p^j v_{S,j,t}. \tag{C.44}$$

The bargain hunters' price index $P_{B,t}$ is log-linearized as in equation [E.8] in GS:

$$p_{B,t} = \theta_B p_{S,t} + (1-\theta_B)p_{N,t} - \varphi_B s s_t, \tag{C.45}$$

where

$$\theta_B = \frac{s}{s + (1-s)\mu^{\eta-1}},$$
$$\varphi_B = \frac{1}{\eta - 1}\frac{1 - \mu^{\eta-1}}{s + (1-s)\mu^{\eta-1}}. \tag{C.46}$$

The price index for a hypothetical loyal customer $P_{L,t}$ is log-linearized as in equation

[E.10] in GS:

$$p_{L,t} = \theta_L p_{S,t} + (1-\theta_L) p_{N,t} - \varphi_L s s_t, \tag{C.47}$$

where

$$\begin{aligned}
\theta_L &= \frac{s}{s + (1-s)\mu^{\epsilon-1}}, \\
\varphi_L &= \frac{1}{\eta - 1} \frac{1 - \mu^{\epsilon-1}}{s + (1-s)\mu^{\epsilon-1}}.
\end{aligned} \tag{C.48}$$

The aggregate price level given by equation (C.17) is transformed into

$$P = \left[L P_L^{1-\epsilon} + (1-L) P_B^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{C.49}$$

In steady state, using $\overline{h} = P_B/P_L$, we have

$$\begin{aligned}
1 &= \lambda \left(\frac{P_L}{P} \right)^{1-\epsilon} + (1-\lambda) \left(\frac{P_B}{P} \right)^{1-\epsilon}, \\
&= \lambda \left(\frac{P_L}{P} \right)^{1-\epsilon} + (1-\lambda) \left(\overline{h} \frac{P_L}{P} \right)^{1-\epsilon}.
\end{aligned}$$

$$\left(\frac{P_L}{P} \right)^{1-\epsilon} = \frac{1}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}}, \tag{C.50}$$

$$\left(\frac{P_B}{P} \right)^{1-\epsilon} = \frac{\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}}. \tag{C.51}$$

The aggregate price level is log-linearized as

$$\begin{aligned}
(1-\epsilon) P^{1-\epsilon} p_t = {} & \lambda (1-\epsilon) P_L^{1-\epsilon} p_{L,t} + (1-\lambda)(1-\epsilon) P_B^{1-\epsilon} p_{B,t} \\
& + \lambda P_L^{1-\epsilon} l_t - \lambda P_B^{1-\epsilon} l_t,
\end{aligned}$$

$$
\begin{aligned}
p_t &= \lambda \left(\frac{P_L}{P}\right)^{1-\epsilon} p_{L,t} + (1-\lambda)\left(\frac{P_B}{P}\right)^{1-\epsilon} p_{B,t} \\
&+ \frac{\lambda}{(1-\epsilon)} \left\{ \left(\frac{P_L}{P}\right)^{1-\epsilon} - \left(\frac{P_B}{P}\right)^{1-\epsilon} \right\} l_t \\
&= \frac{\lambda}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} p_{L,t} + \frac{(1-\lambda)\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} p_{B,t} \\
&+ \frac{\lambda}{(1-\epsilon)} \frac{1-\overline{h}^{1-\epsilon}}{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}} l_t \\
&= \frac{\lambda \overline{h}^{\epsilon-1}}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} p_{L,t} + \frac{1-\lambda}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} p_{B,t} \\
&+ \frac{\lambda}{(1-\epsilon)} \frac{\overline{h}^{\epsilon-1} - 1}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)} l_t.
\end{aligned}
$$

Equation [E.11] in GS thus becomes

$$
p_t = (1-\varpi) p_{L,t} + \varpi p_{B,t} - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t, \tag{C.52}
$$

where

$$
\begin{aligned}
\varpi &\equiv \frac{1-\lambda}{\lambda \overline{h}^{\epsilon-1} + (1-\lambda)}, \\
\overline{h} &\equiv \frac{(s + (1-s)\mu^{\epsilon-1})^{\frac{1}{\epsilon-1}}}{(s + (1-s)\mu^{\eta-1})^{\frac{1}{\eta-1}}}.
\end{aligned} \tag{C.53}
$$

Equation [E.12] in GS becomes

$$
\begin{aligned}
p_t &= (1-\varpi)p_{L,t} + \varpi p_{B,t} - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t \\
&= (1-\varpi)(\theta_L p_{S,t} + (1-\theta_L)p_{N,t} - \varphi_L ss_t) \\
&+ \varpi(\theta_B p_{S,t} + (1-\theta_B)p_{N,t} - \varphi_B ss_t) \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t \\
&= \{(1-\varpi)\theta_L + \varpi\theta_B\} p_{S,t} \\
&+ \{(1-\varpi)(1-\theta_L) + \varpi(1-\theta_B)\} p_{N,t} \\
&- \{(1-\varpi)\varphi_L + \varpi\varphi_B\} ss_t \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t,
\end{aligned}
$$

$$
\begin{aligned}
p_t &= \theta_P p_{S,t} + (1-\theta_P)p_{N,t} - \varphi_P ss_t \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) l_t,
\end{aligned} \tag{C.54}
$$

where

$$
\begin{aligned}
\theta_P &\equiv (1-\varpi)\theta_L + \varpi\theta_B \\
\varphi_P &\equiv (1-\varpi)\varphi_L + \varpi\varphi_B.
\end{aligned} \tag{C.55}
$$

Regarding production, we have

$$
Q_{j,t} = Z_t^a \left(Z_t^h H_{j,t} \right)^\alpha, \tag{C.56}
$$

where Z_t^a represents a a stochastic shock to productivity, with its logarithm deviation denoted by ε_t^a. Production input includes the labor supply shock, Z_t^h, that is introduced in equation (C.1). Then, equations [E.13] and [E.14] in GS become

$$
q_t = \alpha h_t + \alpha \varepsilon_t^h + \varepsilon_t^a, \tag{C.57}
$$

$$x_t = \gamma q_t + w_t. \tag{C.58}$$

Each firm's profit maximizing problem yields equation [27] in GS:

$$\frac{p_{S,j,t} q_{S,j,t} - r_{N,t-j} q_{N,j,t}}{q_{S,j,t} - q_{N,j,t}} = X_{j,t}. \tag{C.59}$$

Equation [E.15] holds:

$$(\chi - 1)X_{j,t} = \mu_S \chi p_{S,j,t} - \mu_N r_{N,t-j} + (\mu_S - 1)\chi(q_{S,j,t} - q_{N,j,t}). \tag{C.60}$$

Substituting equations (C.34) and (C.35), we obtain

$$\begin{aligned}
q_{S,j,t} &= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t - \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}(\eta-\epsilon)(p_{S,j,t}-p_{B,t}) - \epsilon(p_{S,j,t}-p_t) + y_t \\
&= \frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} l_t \\
&\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S} p_{S,j,t} + (\eta-\epsilon)\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} p_{B,t} + \epsilon p_t + y_t, && \text{(C.61)} \\
q_{N,j,t} &= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}(\eta-\epsilon)(r_{N,t-j}-p_{B,t}) - \epsilon(r_{N,t-j}-p_t) + y_t \\
&= \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} l_t \\
&\quad - \frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} r_{N,t-j} + (\eta-\epsilon)\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} p_{B,t} + \epsilon p_t + y_t. && \text{(C.62)}
\end{aligned}$$

$$\begin{aligned}
(\chi - 1)X_{j,t} = \mu_S \chi p_{S,j,t} - \mu_N r_{N,t-j} + (\mu_S - 1)\chi \\
\cdot \Bigg\{ \left(\frac{\lambda(1-v_S)}{\lambda+(1-\lambda)v_S} - \frac{\lambda(1-v_N)}{\lambda+(1-\lambda)v_N} \right) l_t \\
- \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S} p_{S,j,t} \\
+ \frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} r_{N,t-j} \\
+(\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N} \right) p_{B,t} \Bigg\}.
\end{aligned}$$

$$
\begin{aligned}
(\chi-1)X_{j,t} &= \left\{\mu_S - (\mu_S-1)\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi p_{S,j,t} \\
&- \left\{\mu_N - (\mu_S-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N}\right\} r_{N,t-j} \\
&+ (\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S} - \frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi p_{B,t} \\
&+ \left(\frac{1-v_S}{\lambda+(1-\lambda)v_S} - \frac{1-v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi\lambda l_t.
\end{aligned} \tag{C.63}
$$

Note that, under flexible prices, the optimal markup for the sales price is given by equation [17] in GS:

$$
\mu(p;P_B) = \frac{L\epsilon+(1-L)\eta v(p;P_B)}{L(\epsilon-1)+(1-L)(\eta-1)v(p;P_B)}. \tag{C.64}
$$

Its steady-state value is given by

$$
\mu_S = \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}, \tag{C.65}
$$

$$
\mu_S - 1 = \frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}, \tag{C.66}
$$

so the coefficient on $p_{S,l,t}$ becomes

$$
\begin{aligned}
&\left\{\mu_S - (\mu_S-1)\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi \\
&= \left\{\mu_S - \frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda+(1-\lambda)v_S}\right\}\chi \\
&= \left\{\mu_S - \frac{\lambda\epsilon+(1-\lambda)\eta v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\right\}\chi \\
&= 0.
\end{aligned}
$$

The coefficient on $r_{N,t-l}$ similarly becomes

$$
\begin{aligned}
&\mu_N - (\mu_S-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} \\
&= \mu_N - (\mu_N-1)\chi\frac{\lambda\epsilon+(1-\lambda)\eta v_N}{\lambda+(1-\lambda)v_N} \\
&= 0.
\end{aligned}
$$

The coefficient on $p_{B,t}$ becomes

$$
\begin{aligned}
&(\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}-\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi \\
&=(\eta-\epsilon)\left(\begin{array}{c} \frac{(1-\lambda)v_S}{\lambda+(1-\lambda)v_S}\frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi \\ -\frac{(1-\lambda)v_N}{\lambda+(1-\lambda)v_N}\frac{\lambda+(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N} \end{array}\right) \\
&=(\eta-\epsilon)\left(\frac{(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi-\frac{(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\right) \\
&=\left(\frac{\lambda\epsilon+(1-\lambda)\eta v_S-\{\lambda\epsilon+(1-\lambda)\epsilon v_S\}}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi-\frac{\lambda\epsilon+(1-\lambda)\eta v_N-\{\lambda\epsilon+(1-\lambda)\epsilon v_N\}}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\right) \\
&=\mu_S\chi-(\mu_S-1)\chi\epsilon-\mu_N+(\mu_N-1)\epsilon \\
&=\mu_S\chi-\mu_N \\
&=\mu_S\frac{\mu_N-1}{\mu_S-1}-\mu_N=\frac{\mu_S(\mu_N-1)-\mu_N(\mu_S-1)}{\mu_S-1}=\frac{\mu_N-1-(\mu_S-1)}{\mu_S-1} \\
&=\chi-1.
\end{aligned}
$$

The coefficient on l_t becomes

$$
\begin{aligned}
&\left(\frac{1-v_S}{\lambda+(1-\lambda)v_S}-\frac{1-v_N}{\lambda+(1-\lambda)v_N}\right)(\mu_S-1)\chi\lambda \\
&=\frac{1-v_S}{\lambda+(1-\lambda)v_S}\frac{\lambda+(1-\lambda)v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi\lambda \\
&-\frac{1-v_N}{\lambda+(1-\lambda)v_N}\frac{\lambda+(1-\lambda)v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\lambda \\
&=\frac{1-v_S}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_S}\chi\lambda \\
&-\frac{1-v_N}{\lambda(\epsilon-1)+(1-\lambda)(\eta-1)v_N}\lambda \\
&=-\left\{\frac{\mu_S-\eta(\mu_S-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_S-\epsilon(\mu_S-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\chi\lambda \\
&+\left\{\frac{\mu_N-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_N-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&=-\left\{\frac{\mu_S\chi-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_S\chi-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&+\left\{\frac{\mu_N-\eta(\mu_N-1)}{(\eta-\epsilon)\lambda}+\frac{\mu_N-\epsilon(\mu_N-1)}{(\eta-\epsilon)(1-\lambda)}\right\}\lambda \\
&=-\frac{\mu_S\chi}{(\eta-\epsilon)\lambda(1-\lambda)}\lambda+-\frac{\mu_N}{(\eta-\epsilon)\lambda(1-\lambda)}\lambda \\
&=-\frac{\mu_S\chi-\mu_N}{(\eta-\epsilon)(1-\lambda)} \\
&=-\frac{\chi-1}{(\eta-\epsilon)(1-\lambda)}.
\end{aligned}
$$

Therefore, equation (C.63) is simplified as

$$
\begin{aligned}
(\chi-1)X_{j,t}&=(\chi-1)p_{B,t}-\frac{\chi-1}{(\eta-\epsilon)(1-\lambda)}l_t, \\
X_{j,t}&=p_{B,t}-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t. \qquad \text{(C.67)}
\end{aligned}
$$

The right hand side of the equation is independent of j, so all firms have the same marginal cost.

Equation [27] in GS suggests

$$
p_{S,j,t}=\mu_{S,j,t}+X_{j,t}.
$$

Substitution of equations (C.33) and (C.40) yields

$$\begin{aligned}
p_{S,j,t} &= -\varrho_S v_{S,j,t} + \frac{1}{1-\lambda}\varrho_S l_t + X_{j,t} \\
&= \varrho_S(\eta-\epsilon)(p_{S,j,t} - p_{B,t}) + \frac{1}{1-\lambda}\varrho_S l_t + X_{j,t} \\
&= \varrho_S(\eta-\epsilon)p_{S,j,t} - \varrho_S(\eta-\epsilon)\left\{X_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right\} + \frac{1}{1-\lambda}\varrho_S l_t + X_t,
\end{aligned}$$

and equation [E.17] in GS holds:

$$\begin{aligned}
\{1-\varrho_S(\eta-\epsilon)\}\left(p_{S,j,t} - X_t\right) &= \left\{\frac{\varrho_S}{1-\lambda} - \frac{\varrho_S(\eta-\epsilon)}{(\eta-\epsilon)(1-\lambda)}\right\} l_t \\
&= 0. \qquad \text{(C.68)}
\end{aligned}$$

We thus have

$$p_{S,j,t} = X_t. \qquad \text{(C.69)}$$

Regarding normal prices, the log-linearization of the first-order condition, equation [26] in GS, becomes

$$\sum_{j=0}^{\infty} \left(\beta\phi_p\right)^j E_t\left[r_{N,t} - \mu_{N,j,t+j} - X_{t+j}\right] = 0, \qquad \text{(C.70)}$$

which corresponds to equation [E.18] in GS. From equations (C.33) and (C.41), the optimal markup $\mu_{N,j,t+j}$ becomes

$$\begin{aligned}
\mu_{N,j,t} &= -\varrho_N v_{N,j,t} + \frac{1}{1-\lambda}\varrho_N l_t \\
&= \varrho_N(\eta-\epsilon)(r_{N,t-j} - p_{B,t}) + \frac{1}{1-\lambda}\varrho_N l_t.
\end{aligned}$$

Equation (C.67) yields

$$\begin{aligned}
\mu_{N,j,t} &= \varrho_N(\eta-\epsilon)\left(r_{N,t-j} - X_{j,t} - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right) + \frac{1}{1-\lambda}\varrho_N l_t \\
&= \varrho_N(\eta-\epsilon)\left(r_{N,t-j} - X_{j,t}\right).
\end{aligned}$$

Equation (C.70) thus becomes

$$\{1-\varrho_N(\eta-\epsilon)\}\sum_{j=0}^{\infty}(\beta\phi_p)^j E_t\left[r_{N,t}-X_{t+j}\right]=0,$$

and equation [E.19] in GS is obtained:

$$r_{N,t}=(1-\beta\phi_p)\sum_{j=0}^{\infty}(\beta\phi_p)^j E_t X_{t+j}. \tag{C.71}$$

Equation (C.54) and $p_{S,j,t}=X_t$ change equation [E.20] in GS as

$$\begin{aligned}\varphi_P ss_t &= \theta_P p_{S,t}+(1-\theta_P)p_{N,t}-p_t-\lambda\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t\\ &= \theta_P(X_t-p_t)+(1-\theta_P)(p_{N,t}-p_t)-\lambda\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t. \end{aligned}\tag{C.72}$$

Equations (C.45), (C.67), and $p_{S,j,t}=X_t$ change equation [E.21] in GS as

$$\begin{aligned}\varphi_B ss_t &= \theta_B p_{S,t}+(1-\theta_B)p_{N,t}-p_{B,t}\\ &= \theta_B X_t+(1-\theta_B)p_{N,t}-X_t-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\\ &= (1-\theta_B)(p_{N,t}-X_t)-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t. \end{aligned}\tag{C.73}$$

Using equations (C.72) and (C.73), we obtain

$$\begin{aligned}&\left\{(1-\theta_B)(p_{N,t}-X_t)-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right\}/\varphi_B\\ &=\left\{\theta_P(X_t-p_t)+(1-\theta_P)(p_{N,t}-p_t)-\lambda\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t\right\}/\varphi_P\\ &=\left\{\theta_P(X_t-p_t)+(1-\theta_P)(p_{N,t}-X_t+X_t-p_t)-\lambda\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)l_t\right\}/\varphi_P\end{aligned}$$

$$\begin{aligned}\frac{X_t-p_t}{\varphi_P} &= \left\{\frac{1-\theta_B}{\varphi_B}-\frac{1-\theta_P}{\varphi_P}\right\}(p_{N,t}-X_t)\\ &\quad -\left\{\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}-\frac{\lambda}{\varphi_P}\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)\right\}l_t\end{aligned}$$

$$X_t - p_t = \left\{ \frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B} \right\} (p_{N,t} - X_t) - Al_t.$$

where

$$A \equiv \frac{\varphi_P}{(\eta-\epsilon)(1-\lambda)\varphi_B} - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right). \tag{C.74}$$

Equation [E.22] in GS becomes

$$X_t - p_t = (1-\psi)(X_t - p_{N,t}) - Al_t, \tag{C.75}$$

where equation [E.23] is defined as

$$\begin{aligned} 1-\psi &= -\frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B} \\ \psi &= 1 + \frac{(1-\theta_B)\varphi_P - (1-\theta_P)\varphi_B}{\varphi_B} \\ &= \frac{(1-\theta_B)\varphi_P + \theta_P\varphi_B}{\varphi_B}. \end{aligned} \tag{C.76}$$

Equation (C.43) is rearranged as equation [E.24] in GS:

$$p_{N,t} = \phi_p p_{N,t-1} + (1-\phi_p) r_{N,t}, \tag{C.77}$$

and equation (C.71) is rearranged as equation [E.25] in GS:

$$r_{N,t} = \beta\phi_p E_t r_{N,t+1} + (1-\beta\phi_p) X_t. \tag{C.78}$$

Multiplying the above by $(1-\phi_p)$ and substituting it to equation (C.77) yields

$$p_{N,t} = \phi_p p_{N,t-1} + (1-\phi_p)\beta\phi_p E_t r_{N,t+1} + (1-\phi_p)(1-\beta\phi_p) X_t,$$

$$\begin{aligned} p_{N,t} - \phi_p p_{N,t-1} &= (1-\phi_p)\beta\phi_p E_t r_{N,t+1} + (1-\phi_p)(1-\beta\phi_p) X_t \\ &= \beta\phi_p \left\{ E_t p_{N,t+1} - \phi_p p_{N,t} \right\} + (1-\phi_p)(1-\beta\phi_p) X_t. \end{aligned}$$

Defining $\pi_{N,t} \equiv p_{N,t} - p_{N,t-1}$ and adding $(\phi_p - 1) p_{N,t}$ to both terms, we obtain equation

[E.26] in GS:

$$
\begin{aligned}
\phi_p(p_{N,t}-p_{N,t-1}) &= \beta\phi_p\left\{E_t p_{N,t+1}-p_{N,t}+p_{N,t}-\phi_p p_{N,t}\right\}+(1-\phi_p)(1-\beta\phi_p)X_t \\
&\quad +(\phi_p-1)p_{N,t} \\
\phi_p\pi_{N,t} &= \beta\phi_p E_t\pi_{N,t+1}+(1-\phi_p)(1-\beta\phi_p)X_t-(\phi_p-1)(1-\beta\phi_p)p_{N,t} \\
\phi_p\pi_{N,t} &= \beta\phi_p E_t\pi_{N,t+1}+(1-\phi_p)(1-\beta\phi_p)(X_t-p_{N,t}) \\
\pi_{N,t} &= \beta E_t\pi_{N,t+1}+\frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}(X_t-p_{N,t}),
\end{aligned}
\tag{C.79}
$$

where we define

$$
\kappa \equiv \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}. \tag{C.80}
$$

Taking the first difference of equation (C.73) yields an equivalent equation of [E.27] in GS:

$$
\begin{aligned}
\varphi_B s\Delta s_t &= (1-\theta_B)(\Delta p_{N,t}-\Delta X_t)-\frac{1}{(\eta-\epsilon)(1-\lambda)}\Delta l_t, \\
s\Delta s_t &= -\frac{1-\theta_B}{\varphi_B}(\Delta X_t-\pi_{N,t})-\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\Delta l_t.
\end{aligned}
\tag{C.81}
$$

The first difference of equation (C.54) becomes

$$
\begin{aligned}
\pi_t &= \theta_P(p_{S,t}-p_{S,t-1})+(1-\theta_P)\pi_{N,t}-\varphi_P s\Delta s_t \\
&\quad -\lambda\left(\frac{\frac{1-\varpi}{\lambda}-\frac{\varpi}{1-\lambda}}{\epsilon-1}\right)\Delta l_t.
\end{aligned}
$$

From $p_{S,j,t} = X_t$, it becomes

$$
\begin{aligned}
\pi_t &= \theta_P \Delta X_t + (1-\theta_P)\pi_{N,t} - \varphi_P s \Delta s_t - \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) \Delta l_t \\
&= \pi_{N,t} + \theta_P(\Delta X_t - \pi_{N,t}) \\
&+ \varphi_P \left\{ \frac{1-\theta_B}{\varphi_B}(\Delta X_t - \pi_{N,t}) + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\Delta l_t \right\} \\
&- \lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right) \Delta l_t \\
&= \pi_{N,t} + \frac{(1-\theta_B)\varphi_P + \theta_P \varphi_B}{\varphi_B}(\Delta X_t - \pi_{N,t}) + A\Delta l_t \\
&= \pi_{N,t} + \psi(\Delta X_t - \pi_{N,t}) + A\Delta l_t.
\end{aligned}
$$

Defining $x_t \equiv X_t - p_t$, we transform equation (C.75):

$$
x_t = (1-\psi)(x_t + p_t - p_{N,t}) - Al_t, \tag{C.82}
$$

$$
\begin{aligned}
\Delta x_t &= (1-\psi)(\Delta x_t + \pi_t - \pi_{N,t}) - A\Delta l_t, \\
\pi_{N,t} &= \pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t.
\end{aligned}
$$

Substituting this into equation (C.79) yields

$$
\begin{aligned}
&\pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t \\
&= \beta E_t \left\{ \pi_{t+1} - \frac{\psi}{1-\psi}\Delta x_{t+1} - \frac{A}{1-\psi}\Delta l_{t+1} \right\} + \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p}(X_t - p_{N,t}).
\end{aligned}
$$

From equation (C.75), it becomes

$$
\begin{aligned}
&\pi_t - \frac{\psi}{1-\psi}\Delta x_t - \frac{A}{1-\psi}\Delta l_t \\
&= \beta E_t \left\{ \pi_{t+1} - \frac{\psi}{1-\psi}\Delta x_{t+1} - \frac{A}{1-\psi}\Delta l_{t+1} \right\} \\
&+ \frac{(1-\phi_p)(1-\beta\phi_p)}{\phi_p} \left\{ \frac{1}{1-\psi}x_t + \frac{A}{1-\psi}l_t \right\},
\end{aligned}
$$

and equivalent equation to [32] in GS is obtained:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} \\ &+ \frac{1}{1-\psi}\left\{\kappa x_t + \psi(\Delta x_t - \beta E_t \Delta x_{t+1}) + \kappa A l_t + A(\Delta l_t - \beta E_t \Delta l_{t+1})\right\}. \end{aligned} \tag{C.83}$$

Lemma 4 in GS Using

$$\mu_t = p_{S,t} - p_{N,t}, \tag{C.84}$$

equation (C.75) suggests

$$\begin{aligned} X_t - p_t &= (1-\psi)(X_t - p_{N,t}) - A l_t, \\ X_t - p_t &= (1-\psi)(p_{S,t} - p_{N,t}) - A l_t \\ x_t &= (1-\psi)\mu_t - A l_t, \end{aligned}$$

yielding an equivalent equation to [E.37] in GS:

$$\mu_t = \frac{1}{1-\psi}\left(x_t + A l_t\right). \tag{C.85}$$

From equations (C.31) to (C.33), we have

$$q_{S,t} = \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S}\left\{-(\eta - \epsilon)(p_{S,t} - p_{B,t})\right\} - \epsilon(p_{S,t} - p_t) + y_t,$$

$$q_{N,t} = \frac{\lambda(1-v_N)}{\lambda + (1-\lambda)v_N} l_t + \frac{(1-\lambda)v_N}{\lambda + (1-\lambda)v_N}\left\{-(\eta - \epsilon)(p_{N,t} - p_{B,t})\right\} - \epsilon(p_{N,t} - p_t) + y_t.$$

Using equation (C.67) and $p_{S,t} = X_t$, we have

$$\begin{aligned} q_{S,t} &= \frac{\lambda(1-v_S)}{\lambda + (1-\lambda)v_S} l_t + \frac{(1-\lambda)v_S}{\lambda + (1-\lambda)v_S}(\eta - \epsilon)\frac{1}{(\eta - \epsilon)(1-\lambda)} l_t - \epsilon x_t + y_t \\ &= l_t - \epsilon x_t + y_t \end{aligned} \tag{C.86}$$

$$\begin{aligned}
q_{N,t} &= \frac{\lambda(1-\upsilon_N)}{\lambda+(1-\lambda)\upsilon_N} l_t - \frac{(1-\lambda)\upsilon_N}{\lambda+(1-\lambda)\upsilon_N}(\eta-\epsilon)\left(p_{N,t}-p_{S,t}-\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right) \\
&\quad - \epsilon(p_{N,t}-p_t)+y_t \\
&= l_t - \frac{(1-\lambda)\upsilon_N}{\lambda+(1-\lambda)\upsilon_N}(\eta-\epsilon)\left(p_{N,t}-p_{S,t}\right) - \epsilon(p_{N,t}-p_{S,t}+p_{S,t}-p_t)+y_t \\
&= l_t - \frac{(\eta-\epsilon)(1-\lambda)\upsilon_N+\epsilon\{\lambda+(1-\lambda)\upsilon_N\}}{\lambda+(1-\lambda)\upsilon_N}\left(p_{N,t}-p_{S,t}\right) - \epsilon(X_t-p_t)+y_t \\
&= l_t + \frac{\epsilon\lambda+\eta(1-\lambda)\upsilon_N}{\lambda+(1-\lambda)\upsilon_N}\mu_t - \epsilon x_t + y_t. \qquad \text{(C.87)}
\end{aligned}$$

Then, the quantity ratio becomes

$$\begin{aligned}
\chi_t &= q_{S,t} - q_{N,t} \\
&= -\varsigma_N \mu_t, \qquad \text{(C.88)}
\end{aligned}$$

where

$$\varsigma_N = \frac{\epsilon\lambda+\eta(1-\lambda)\upsilon_N}{\lambda+(1-\lambda)\upsilon_N}. \qquad \text{(C.89)}$$

From equation (C.85), the quantity ratio becomes

$$\chi_t = -\frac{\varsigma_N}{1-\psi}\left(x_t + Al_t\right). \qquad \text{(C.90)}$$

Equation (C.73) is transformed into an equivalent equation to [E.39] in GS:

$$\begin{aligned}
ss_t &= \frac{1-\theta_B}{\varphi_B}(p_{N,t}-X_t) - \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t \\
&= \frac{1-\theta_B}{\varphi_B}(p_{N,t}-p_{S,t}) - \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t \\
&= -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}\left(x_t+Al_t\right) - \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}l_t \\
&= -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t - \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right)l_t. \qquad \text{(C.91)}
\end{aligned}$$

Let

$$\Delta_t \equiv y_t - q_t. \qquad \text{(C.92)}$$

Using equation (C.42), total output becomes

$$q_t = \frac{\chi-1}{s\chi+1-s}ss_t + \frac{s\chi}{s\chi+1-s}q_{S,t} + \frac{(1-s)}{s\chi+1-s}q_{N,t}.$$

Equations (C.86) and (C.87) yield

$$
\begin{aligned}
q_t &= \frac{\chi-1}{s\chi+1-s} ss_t + \frac{s\chi}{s\chi+1-s}(l_t - \epsilon x_t + y_t) \\
&+ \frac{(1-s)}{s\chi+1-s}\left\{ l_t + \frac{\epsilon\lambda + \eta(1-\lambda)v_N}{\lambda + (1-\lambda)v_N}\mu_t - \epsilon x_t + y_t \right\} \\
&= \frac{\chi-1}{s\chi+1-s} ss_t + l_t - \epsilon x_t + y_t \\
&+ \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}\left(x_t + Al_t\right).
\end{aligned}
$$

From equation (C.91), it becomes

$$
\begin{aligned}
q_t &= \frac{\chi-1}{s\chi+1-s}\left\{ -\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi}x_t - \left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) l_t \right\} \\
&+ l_t - \epsilon x_t + y_t + \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}\left(x_t + Al_t\right) \\
&= y_t - \delta x_t - Bl_t,
\end{aligned}
$$

where equation [E.40] in GS becomes

$$
\begin{aligned}
\delta &\equiv \epsilon + \frac{\chi-1}{s\chi+1-s}\frac{1-\theta_B}{\varphi_B}\frac{1}{1-\psi} - \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi} \\
&= \epsilon + \frac{1}{s\chi+1-s}\frac{1}{1-\psi}\left(\frac{(\chi-1)(1-\theta_B)}{\varphi_B} - (1-s)\varsigma_N\right), \qquad \text{(C.93)}
\end{aligned}
$$

$$
\begin{aligned}
B &\equiv -1 + \frac{\chi-1}{s\chi+1-s}\left(\frac{1-\theta_B}{\varphi_B}\frac{A}{1-\psi} + \frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}\right) \\
&- \frac{1-s}{s\chi+1-s}\frac{\varsigma_N}{1-\psi}A \\
&= -1 + \frac{1}{s\chi+1-s}\frac{1}{1-\psi}\left(\frac{(\chi-1)(1-\theta_B)}{\varphi_B} - (1-s)\varsigma_N\right) A \\
&+ \frac{\chi-1}{s\chi+1-s}\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B} \\
&= -1 + (\delta-\epsilon)A + \frac{\chi-1}{s\chi+1-s}\frac{1}{(\eta-\epsilon)(1-\lambda)\varphi_B}. \qquad \text{(C.94)}
\end{aligned}
$$

Thus, we have

$$\begin{aligned}\Delta_t &\equiv y_t - q_t \\ &= \delta x_t + Bl_t. \end{aligned} \tag{C.95}$$

From equation (C.58), the real marginal cost becomes

$$\begin{aligned} x_t &= \gamma q_t + w_t \\ &= \gamma(y_t - \Delta_t) + w_t \\ &= \gamma(y_t - \delta x_t - Bl_t) + w_t, \end{aligned} \tag{C.96}$$

and then equation [A.9b] in GS becomes

$$x_t = \frac{1}{1+\gamma\delta} w_t + \frac{\gamma}{1+\gamma\delta}\left(y_t - Bl_t\right). \tag{C.97}$$

Quantity becomes

$$\begin{aligned} q_t &= y_t - \Delta_t \\ &= y_t - \delta x_t - Bl_t \\ &= y_t - \delta\left(\frac{1}{1+\gamma\delta} w_t + \frac{\gamma}{1+\gamma\delta}\left(y_t - Bl_t\right)\right) - Bl_t \\ &= \frac{1}{1+\gamma\delta} y_t - \frac{\delta}{1+\gamma\delta} w_t - \frac{1}{1+\gamma\delta} Bl_t. \end{aligned} \tag{C.98}$$

Log-linearization of households' part From equations (C.12) and (C.13), the consumption wedge F_t becomes

$$\begin{aligned} f_t &= \frac{\eta}{\eta - 1} \frac{\lambda P_{SN}(l_t + p_{SN,t}) - \lambda l_t}{\lambda P_{SN} + (1-\lambda)} \\ &= \frac{\eta}{\eta - 1} \lambda \frac{P_{SN} p_{SN,t} - (1 - P_{SN}) l_t}{\lambda P_{SN} + (1-\lambda)}, \end{aligned} \tag{C.99}$$

where

$$
\begin{aligned}
p_{SN,t} &= \frac{s\mu^{\epsilon\frac{1-\eta}{\eta}}(s_t + \epsilon\frac{1-\eta}{\eta}\mu_t) - ss_t}{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)} - \frac{\epsilon}{\eta}\frac{s\mu^{1-\eta}(s_t + (1-\eta)\mu_t) - ss_t}{s\mu^{1-\eta} + (1-s)} \\
&= \frac{s(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)s_t + s\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\mu_t}{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)} - \frac{\epsilon}{\eta}\frac{s(\mu^{1-\eta} - 1)s_t + s\mu^{1-\eta}(1-\eta)\mu_t}{s\mu^{1-\eta} + (1-s)} \\
&= \frac{(\mu^{\epsilon\frac{1-\eta}{\eta}} - 1)\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}(\mu^{1-\eta} - 1)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} ss_t \\
&+ \frac{\mu^{\epsilon\frac{1-\eta}{\eta}}\epsilon\frac{1-\eta}{\eta}\left\{s\mu^{1-\eta} + (1-s)\right\} - \frac{\epsilon}{\eta}\mu^{1-\eta}(1-\eta)\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}}{\left\{s\mu^{\epsilon\frac{1-\eta}{\eta}} + (1-s)\right\}\left\{s\mu^{1-\eta} + (1-s)\right\}} s\mu_t \qquad \text{(C.100)}
\end{aligned}
$$

Using equation (C.26), we transform equation (C.20) into

$$
\begin{aligned}
0 &= \frac{v_{CC}}{v_C}C(E_t c_{t+1} - c_t) - (i_t - E_t\pi_{t+1}) + E_t(f_{t+1} - f_t) - \epsilon E_t\left[(p_{B,t+1} - p_{t+1}) - (p_{B,t} - p_t)\right] \\
&= -\theta_c^{-1}\left\{E_t(y_{t+1} - \varepsilon^g_{t+1} + f_{t+1} - \epsilon\,(p_{B,t+1} - p_{t+1})) - (y_t - \varepsilon^g_t + f_t - \epsilon\,(p_{B,t} - p_t))\right\} \\
&- (i_t - E_t\pi_{t+1}) + E_t(f_{t+1} - f_t) - \epsilon E_t\left[(p_{B,t+1} - p_{t+1}) - (p_{B,t} - p_t)\right],
\end{aligned}
$$

where ε^g_t represents a stochastic government shock. From equation (C.67), it becomes

$$
\begin{aligned}
0 = \theta_c^{-1} &\left\{ \begin{array}{c} E_t(y_{t+1} - \varepsilon^g_{t+1} + f_{t+1} - \epsilon\left[x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_{t+1}\right]) \\ -(y_t - \varepsilon^g_t + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right]) \end{array} \right\} \\
&- (i_t - E_t\pi_{t+1}) - E_t(f_{t+1} - f_t) \\
&+ \epsilon E_t\left[x_{t+1}\frac{1}{(\eta-\epsilon)(1-\lambda)}l_{t+1} - x_t - \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right],
\end{aligned}
$$

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \theta_c(i_t - E_t\pi_{t+1}) + \varepsilon^g_t - \varepsilon^g_{t+1} \\
&+ (1-\theta_c)\left\{\Delta f_{t+1} - \epsilon\left(\Delta x_{t+1} + \frac{1}{(\eta-\epsilon)(1-\lambda)}\Delta l_{t+1}\right)\right\}. \qquad \text{(C.101)}
\end{aligned}
$$

Equation (C.21) becomes

$$\begin{aligned}
&\frac{\upsilon_{HH}}{\upsilon_H}\left\{Hh_t - \theta_L\phi_L\frac{(1-\lambda)^{\theta_L-1}}{(1-\lambda)^{\theta_L}}\lambda l_t\right\} \\
&+\varepsilon_t^h - \frac{\upsilon_{CC}}{\upsilon_C}Cc_t \\
&= w_t + f_t - \epsilon(p_{B,t} - p_t), \qquad \text{(C.102)}
\end{aligned}$$

where ε_t^h represents a stochastic shock to labor supply. The first term on the right-hand side of the equation implies that h_t are positively correlated with l_t if all other things equal. A decline in hours worked involves a decrease in loyal customers. From equations (C.26) and (C.67), this equation becomes

$$\begin{aligned}
&\theta_h^{-1}\left(h_t - \theta_L\phi_L(1-\lambda)^{-1}\frac{\lambda}{H}l_t\right) + \varepsilon_t^h \\
&+ \theta_c^{-1}(y_t - \varepsilon_t^g + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right]) \\
&= w_t + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right]. \qquad \text{(C.103)}
\end{aligned}$$

From equations (C.57) and (C.98), hours worked are given by

$$\begin{aligned}
h_t &= \frac{q_t - \varepsilon_t^a}{\alpha} - \varepsilon_t^h \\
&= \frac{\frac{1}{1+\gamma\delta}y_t - \frac{\delta}{1+\gamma\delta}w_t - \frac{1}{1+\gamma\delta}Bl_t - \varepsilon_t^a}{\alpha} - \varepsilon_t^h \\
&= \frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - Bl_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a - \varepsilon_t^h. \qquad \text{(C.104)}
\end{aligned}$$

Shifting ε_t^h to the left-hand side yields production input:

$$\varepsilon_t^h + h_t = \frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - Bl_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a. \qquad \text{(C.105)}$$

Substituting equation (C.104) yields

$$\begin{aligned}
&\theta_h^{-1}\left(\frac{1}{1+\gamma\delta}\frac{y_t-\delta w_t-Bl_t}{\alpha}-\frac{1}{\alpha}\varepsilon_t^a-\varepsilon_t^h\right)+\varepsilon_t^h \\
&-\theta_h^{-1}\theta_L\phi_L(1-\lambda)^{-1}\frac{\lambda}{H}l_t \\
&+\theta_c^{-1}(y_t-\varepsilon_t^g+f_t-\epsilon\left[x_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right]) \\
&=w_t+f_t-\epsilon\left[x_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right],
\end{aligned}$$

$$\begin{aligned}
0=&\left(\theta_c^{-1}+\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)y_t-\left(1+\frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)w_t \\
&-\frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a-(\theta_h^{-1}-1)\varepsilon_t^h-\theta_c^{-1}\varepsilon_t^g \\
&-\left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B+\theta_h^{-1}\theta_L\phi_L(1-\lambda)^{-1}\frac{\lambda}{H}\right)l_t \\
&-(1-\theta_c^{-1})\left\{f_t-\epsilon\left(x_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\}.
\end{aligned} \tag{C.106}$$

In the presence of wage stickiness, the right-hand side of the equation deviates from zero:

$$\begin{aligned}
\pi_{W,t}=&\beta\pi_{W,t+1} \\
&+\frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w}\frac{1}{1+\varsigma\theta_h^{-1}}[\\
&\left(\theta_c^{-1}+\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)y_t-\left(1+\frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right)w_t \\
&-\frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a-(\theta_h^{-1}-1)\varepsilon_t^h-\theta_c^{-1}\varepsilon_t^g \\
&-\left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}B+\theta_h^{-1}\theta_L\phi_L\frac{\lambda}{(1-\lambda)H}\right)l_t \\
&-(1-\theta_c^{-1})\left\{f_t-\epsilon\left(x_t+\frac{1}{(\eta-\epsilon)(1-\lambda)}l_t\right)\right\}\Bigg].
\end{aligned} \tag{C.107}$$

Equation (C.23) becomes

$$
\begin{aligned}
&-(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
&+ \frac{\upsilon_{HH}}{\upsilon_H}(Hh_t - \theta_L \phi_L (1-\lambda)^{-1}\lambda l_t) - \frac{\upsilon_{CC}}{\upsilon_C} C c_t + \varepsilon_t^h \\
&= c_t - \frac{P_{SN}}{1-P_{SN}} p_{SN,t} - \frac{\lambda P_{SN}(l_t + p_{SN,t}) - \lambda l_t}{\lambda P_{SN} + (1-\lambda)},
\end{aligned}
$$

$$
\begin{aligned}
&-(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
&+ \theta_h^{-1} h_t - \theta_h^{-1}\phi_L (1-\lambda)^{-1}\frac{\lambda}{H} l_t + \theta_c^{-1} c_t + \varepsilon_t^h \\
&= c_t - \frac{P_{SN}}{1-P_{SN}} p_{SN,t} - \frac{\eta - 1}{\eta} f_t,
\end{aligned}
$$

$$
\begin{aligned}
&-(\theta_L - 1)\frac{\lambda}{1-\lambda} l_t \\
&+ \theta_h^{-1}\left(\frac{1}{1+\gamma\delta}\frac{y_t - \delta w_t - B l_t}{\alpha} - \frac{1}{\alpha}\varepsilon_t^a - \varepsilon_t^h\right) + \varepsilon_t^h \\
&- \theta_h^{-1}\phi_L (1-\lambda)^{-1}\frac{\lambda}{H} l_t \\
&+ (\theta_c^{-1} - 1)\left(y_t - \varepsilon_t^g + f_t - \epsilon\left[x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right]\right) \\
&= -\frac{P_{SN}}{1-P_{SN}} p_{SN,t} - \frac{\eta - 1}{\eta} f_t.
\end{aligned}
$$

$$
\begin{aligned}
0 = &\left(\theta_c^{-1} - 1 + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
&- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a - (\theta_h^{-1} - 1)\varepsilon_t^h - (\theta_c^{-1} - 1)\varepsilon_t^g \\
&- \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B + (\theta_L - 1)\frac{\lambda}{1-\lambda} + \theta_h^{-1}\phi_L \frac{\lambda}{(1-\lambda)H}\right) l_t \\
&+ (\theta_c^{-1} - 1)\left\{f_t - \epsilon\left(x_t + \frac{1}{(\eta-\epsilon)(1-\lambda)} l_t\right)\right\} \\
&+ \frac{P_{SN}}{1-P_{SN}} p_{SN,t} + \frac{\eta - 1}{\eta} f_t. \qquad \text{(C.108)}
\end{aligned}
$$

Steady state conditions To calibrate parameters associated with the fraction of loyal customers ϕ_L and θ_L, we consider steady state conditions.

From equations (C.21) and (C.23),

$$\frac{\upsilon_H(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}})}{\upsilon_C(C_t)} = \frac{W_t}{P_t} F_t \left(\frac{P_{B,t}}{P_t}\right)^{-\epsilon},$$

$$\theta_L \phi_L (1-L_t)^{-1} \frac{\upsilon_H(H_t + \phi_L \frac{(1-L_t)^{\theta_L}}{(1-\lambda)^{\theta_L}})}{\upsilon_C(C_t)}$$
$$= \frac{\eta}{\eta - 1} C_t \frac{1 - P_{SN,t}}{L_t P_{SN,t} + (1-L_t)},$$

we obtain the following steady state condition:

$$\theta_L \phi_L (1-\lambda)^{-1} \frac{W}{P} F \left(\frac{P_B}{P}\right)^{-\epsilon}$$
$$= \frac{\eta}{\eta - 1} C \frac{1 - P_{SN}}{\lambda P_{SN} + (1-\lambda)}.$$

Substituting equation (C.25)

$$Y = \frac{C}{F \cdot \left(\frac{P_B}{P}\right)^{-\epsilon}},$$

it becomes

$$\theta_L \phi_L (1-\lambda)^{-1} \frac{W}{PY}$$
$$= \frac{\eta}{\eta - 1} \frac{1 - P_{SN}}{\lambda P_{SN} + (1-\lambda)}. \tag{C.109}$$

As for the right-hand side of the equation, P_{SN} is given by equation (C.13):

$$P_{SN} \equiv \frac{s\mu^{\epsilon \frac{1-\eta}{\eta}} + (1-s)}{(s\mu^{1-\eta} + (1-s))^{\frac{\epsilon}{\eta}}}. \tag{C.110}$$

As for the left-side of the equation, we calculate W/PY. Firms' optimal normal price satisfies

$$\frac{P_N}{P} = \frac{\epsilon}{\epsilon - 1} \frac{X}{P},$$

from equation [26] in GS. The nominal marginal cost X is given by

$$X = \frac{\partial(Q(H)^{-1})}{\partial Q} = \frac{WH}{\alpha Q},$$

where α represents the elasticity of output with respect to hours worked. Therefore, we have

$$\begin{aligned}\frac{W}{PY} &= \frac{W}{PQ}\frac{Q}{Y} \\ &= \frac{P_N}{P}\frac{\epsilon - 1}{\epsilon}\frac{\alpha}{H}\frac{Q}{Y}.\end{aligned} \tag{C.111}$$

Here, from the definition of price index, we have

$$\begin{aligned}\frac{P_N}{P} &= \frac{P_N}{\left\{\lambda P_L^{1-\epsilon} + (1-\lambda)P_B^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \\ &= \frac{P_N/P_L}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \\ &= \frac{1}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \frac{P_N}{\left\{sP_S^{1-\epsilon} + (1-s)P_N^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \\ &= \frac{1}{\left\{\lambda + (1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}} \frac{1}{\left\{s\mu^{1-\epsilon} + 1 - s\right\}^{\frac{1}{1-\epsilon}}}.\end{aligned} \tag{C.112}$$

The relationship between Q and Y is given by

$$\begin{aligned}Q &= sQ_S + (1-s)Q_N \\ &= s(\lambda + (1-\lambda)v_S)\left(\frac{P_S}{P}\right)^{-\epsilon} Y \\ &+ (1-s)(\lambda + (1-\lambda)v_N)\left(\frac{P_N}{P}\right)^{-\epsilon} Y,\end{aligned}$$

where

$$\begin{aligned}v_S &= \left(\frac{P_S}{P_B}\right)^{-(\eta-\epsilon)} = \left(\frac{P_S}{\overline{h}P_L}\right)^{-(\eta-\epsilon)} \\ &= \left(\frac{1}{\overline{h}}\frac{\mu}{\left\{s\mu^{1-\epsilon} + 1 - s\right\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}\end{aligned}$$

$$
\begin{aligned}
v_N &= \left(\frac{P_N}{P_B}\right)^{-(\eta-\epsilon)} = \left(\frac{P_N}{\overline{h}P_L}\right)^{-(\eta-\epsilon)} \\
&= \left(\frac{1}{\overline{h}}\frac{1}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}.
\end{aligned}
$$

Therefore, we obtain

$$
\begin{aligned}
\frac{Q}{Y} &= s(\lambda+(1-\lambda)v_S)\left(\frac{P_S}{P}\right)^{-\epsilon} \\
&+ (1-s)(\lambda+(1-\lambda)v_N)\left(\frac{P_N}{P}\right)^{-\epsilon} \\
&= s\left\{\lambda+(1-\lambda)\left(\frac{1}{\overline{h}}\frac{\mu}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}\right\} \\
&\cdot\left\{\frac{1}{\left\{\lambda+(1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}}\frac{\mu}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right\}^{-\epsilon} \\
&+ (1-s)\left\{\lambda+(1-\lambda)\left(\frac{1}{\overline{h}}\frac{1}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right)^{-(\eta-\epsilon)}\right\} \\
&\cdot\left\{\frac{1}{\left\{\lambda+(1-\lambda)\overline{h}^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}}}\frac{1}{\{s\mu^{1-\epsilon}+1-s\}^{\frac{1}{1-\epsilon}}}\right\}^{-\epsilon}.
\end{aligned}
\tag{C.113}
$$

Equation (C.109) with equations (C.110), (C.111), (C.112), and (C.113) give the condition for the parameters ϕ_L and θ_L.

A case where firms do not observe l_t Equation (C.67) becomes independent of l_t :

$$
X_{j,t} = p_{B,t} \tag{C.114}
$$

Equation (C.73) becomes

$$
\varphi_B ss_t = (1-\theta_B)(p_{N,t}-X_t), \tag{C.115}
$$

which is the same as equation [E.21] in GS. Equation (C.74) becomes

$$A \equiv -\lambda \left(\frac{\frac{1-\varpi}{\lambda} - \frac{\varpi}{1-\lambda}}{\epsilon - 1} \right). \tag{C.116}$$

Equation (C.91) becomes

$$ss_t = -\frac{1-\theta_B}{\varphi_B} \frac{1}{1-\psi} \left(x_t + A l_t \right). \tag{C.117}$$

Equation (C.94) becomes

$$B \equiv -1 + (\delta - \epsilon) A. \tag{C.118}$$

Equation (C.101) becomes

$$\begin{aligned} y_t = {} & E_t y_{t+1} - \theta_c (i_t - E_t \pi_{t+1}) \\ & + (1-\theta_c) \left\{ \Delta f_{t+1} - \epsilon \Delta x_{t+1} \right\}. \end{aligned} \tag{C.119}$$

Equation (C.107) becomes

$$\begin{aligned} \pi_{W,t} = {} & \beta \pi_{W,t+1} \\ & + \frac{(1-\phi_w)(1-\beta\phi_w)}{\phi_w} \frac{1}{1+\varsigma\theta_h^{-1}} \Big[\\ & \left(\theta_c^{-1} + \frac{1}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} \right) y_t - \left(1 + \frac{\delta}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} \right) w_t \\ & - \frac{\theta_h^{-1}}{\alpha} \varepsilon_t^a + \varepsilon_t^h + \left(\frac{1}{1+\gamma\delta} \frac{\theta_h^{-1}}{\alpha} B - \theta_h^{-1} \phi \frac{\lambda}{H} \right) l_t \\ & - (1-\theta_c^{-1}) \left\{ f_t - \epsilon x_t \right\} \Big]. \end{aligned} \tag{C.120}$$

Equation (C.108) becomes

$$
\begin{aligned}
0 = &\left(\theta_c^{-1} - 1 + \frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha}\right) y_t - \frac{\delta}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} w_t \\
&- \frac{\theta_h^{-1}}{\alpha}\varepsilon_t^a + \varepsilon_t^h \\
&+ \left(\frac{1}{1+\gamma\delta}\frac{\theta_h^{-1}}{\alpha} B - \theta_h^{-1}\phi\frac{\lambda}{H}\right) l_t \qquad \text{(C.121)} \\
&+ \left(\theta_c^{-1} - 1\right)\{f_t - \epsilon x_t\} \\
&+ \frac{P_{SN}}{1-P_{SN}} p_{SN,t} + \frac{\eta - 1}{\eta} f_t. \qquad \text{(C.122)}
\end{aligned}
$$