A Financial Crisis in a Credit Economy

(Very incomplete and preliminary)

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Abstract

Why the aggregate productivity declined in the Great Depression and slowed down in the 1990s in Japan? Is monetary policy effective for the output losses during the financial crises? We consider these problems by analyzing a business cycle model in which credit is essential. The novel feature of our model is the assumption that agents can choose production technologies that differ in their productivities and pledgeabilities of the outputs. The agents choose the production technology ("complex" or "simple") and they borrow from banks. On one hand, the complex technology is more productive than the simple technology. On the other hand, the output of the complex technology is not pledgeable for repayment to the banks, while the output of the simple technology is pledgeable. We show that when the asset price is low or the agents are debt-ridden as a consequence of a financial crisis, they are forced to use the simple technology, leading to the decline in the aggregate productivity. In this case, monetary injections cannot restore the productivity after a financial crisis.

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1 Introduction

There are many issues concerning financial crises. In this paper we focus on the following two specific issues, which we believe have important policy implications:

- **Long-term decline in the aggregate productivity after a financial crisis.** While a financial crisis is characterized by the liquidity shortage in the short run, it is often observed that the level or the growth rate of the productivity declines over a long period after a financial crisis. The Great Depression is an example. Ohanian (2001) shows that 13 percentage points in the 18 percent decline of the detrended TFP during the 1929–1933 period cannot be explained by the ordinary cyclical factors. Ohanian argues that the destruction of the "organization capital" could be the cause of the TFP decline during the Great Depression. We also observed the long-term slowdown of the TFP growth in Japan during the 1990s after the collapse of the land price in 1991 (see Hayashi and Prescott 2002). Boyd, Kwak and Smith (2005) show that many banking crises were followed by prolonged stagnations of output and that the real output losses associated with modern banking crises have been very large. The causality between the productivity declines and the financial crises is a big research topic that may lead to an important policy implication for the financial crisis management. We try to formalize a mechanism that the low asset prices and/or the balance-sheet deteriorations of households and firms cause the destruction of specific types of production, which may be interpreted as a model of Ohanian's destruction of the organization capital. The idea that the destruction of specific types of transactions might have caused the aggregate productivity declines after financial crises is explored in Kobayashi and Inaba (2004) and Kobayashi (2006, 2007).

- **Whether or not monetary injection (or liquidity provision) can mitigate the output loss associated with a financial crisis.** In standard models of financial crises, the robust policy implication is that sufficient monetary injection can mitigate the real damage of the financial crisis almost completely (e.g., Diamond and Rajan 2006, Allen and Gale 1998). The episodes of financial crises in reality indicate that the monetary policy may not be almighty as a tool of the crisis management. Is the liquidity shortage the central factor in the financial crisis that damages the economy? We show in this paper that in the economy where credit is essential a sub-optimal equilibrium may emerge as a result of the decline of asset prices and/or the balance-sheet problem due to the financial crisis, and the sub-optimal equilibrium cannot be eliminated by monetary policy.

To analyze these problem, we consider a model of the credit economy, which is a standard business cycle model with the following features that make credit essential.

1. As in Diamond (1984), the size of one production project is too large for one agent to finance by his own assets. Therefore, agents invest their assets in the banks as bank deposits and the production projects should be financed by the bank loans.

2. The entrepreneurs, who are the borrowers of the bank loans, cannot precommit to the repayment of the bank loans. They need to put up productive assets ("land" in our model) as collateral for the bank loans. They can also pledge some portion of the output for repayment to the banks, depending on the production technology they use.

3. There are two production technologies that the entrepreneurs can choose: the "simple" technology and the "complex" technology. On one hand, the complex technology is more productive than the simple technology. On the other hand, the output of the complex technology is not pledgeable for repayment to the banks, while the output of the simple technology is pledgeable.

The difference in the pledgeability is generated from a spacial friction. In the simple technology, production takes place in the same town where the lending bank is located and the banker can costlessly monitor the production by the borrower and seize the output if necessary. If the borrower uses the complex technology, the borrower needs to go very far from the bank and he produces the good in a faraway town where the lending bank cannot monitor the borrower's activity. The borrower can hide the produced goods from the bank and make an untruthful claim that the output is lost by an accident, and the bank cannot verify that the claim is a false. This spacial friction makes the output of the complex technology unpledgeable.

(To be completed)

This paper is related to the literature of banking crises (e.g., Diamond and Dybvig 1983; Allen and Gale 1998; Diamond and Rajan 2001, 2005), money and banking (e.g., Schreft and Smith 1996, 1998; Smith 2002; Paal and Smith 2000), and the models of the Great Depression (Cooper and Ejarque 1995; Cooper and Corbae 2002). While all these models except for Cooper and Ejarque (1995) are finite-horizon models with two or three periods or the overlapping generations models, our model is a version of the standard infinite-horizon model, which is utilized in the business cycle and macroeconomic policy analyses.

(To be completed)

The organization of the paper is as follows. In the next section, we show a simplistic model, in which the decline in the asset price is associated with a sub-optimal equilibrium. In Section 3, we describe a general model, in which the agents make intertemporal borrowings and the sub-optimal equilibrium emerges as a result of a surge of the debt outstanding. Section 4 concludes.

2 The Simplistic Model

The model is a deterministic variant of the standard business cycle model, in which credit is essential. We also introduce fiat money, though money is not essential in this economy.

2.1 The Environment

Time is discrete and continues forever: $t = 0, 1, 2, \ldots$. There are four agents in the economy: the representative consumer, the firm, the bank, and the government. There are two goods, consumption good and labor, and two assets, land and cash. We use the consumption good as the numeraire throughout in this paper.

Consumer: The representative consumer lives forever. At the beginning of period t, he owns k_t units of land and rents it to the firm in exchange for k_t units of equity share. The price of land (and the equity) is q_t in period t. He supplies labor l_t, receives the wage $w_t l_t$ in the form of cash, buys and consumes c_t units of the consumption good in period t, where w_t is the wage rate. At the end of period t the bank offers that if the consumer deposits m_{t+1} units of real balance the bank will return $(1+r_{t+1})m_{t+1}$ at the end of period $t+1$. If $1+r_{t+1} > \pi_{t+1}^{-1}$, where π_{t+1} is the inflation rate between periods t and $t+1$, the consumer deposits all cash into the bank. The consumer purchases c_t, land k_{t+1}, and the bank deposit m_{t+1} at the end of period t. We assume that purchase of the consumption good can be implemented with credit and is not subject to the cash-in-advance constraint. The consumer's utility is

$$\sum_{t=0}^{\infty} \beta^t U(c_t, 1-l_t), \tag{1}$$

where β is the discount factor $(0 < \beta < 1)$, $U(c, 1-l)$ is increasing and concave with respect to the first and the second arguments, and $1-l_t$ is the leisure in period t. The consumer chooses $\{c_t, k_{t+1}, m_{t+1}, l_t\}$ to maximize (1) subject to the following budget constraint:

$$c_t + q_t k_{t+1} + m_{t+1} \leq w_t l_t + (d_t + q_t)k_t + (1+r_t)m_t + \tau_t, \tag{2}$$

where d_t is the dividend from the firm and τ_t is the cash injection from the government.

Firm: The representative firm lives for one period. It is born at the beginning of period t and die at the end of period t. The firm rents land k_t from the consumer by issuing k_t units of equity. The firm works as an agent of the consumer to maximize the return on the equity. It can produce the consumption good y_t from land k_t and labor l_t in period t. It needs to pay cash to buy labor l_t from the consumer in the labor market.[1]

[1] There is the anonymity in the labor market that makes labor supply with credit unfeasible, though we assume there are a single consumer and firm for simplicity.

Therefore, the firm needs to borrow $w_t l_t$ units of real balance in the form of cash from the bank at the beginning of period t. The firm must repay $(1+r_t^L)w_t l_t$ to the bank at the end of period t, where r_t^L is the loan rate. There are two production technologies available for the firm: the "simple" technology and the "complex" technology. The output of the simple technology is

$$y_t = A(s)l_t,$$

and the output of the complex technology is

$$y_t = A(c)k_t^{\alpha} l_t^{1-\alpha},$$

where $A(c)$ is sufficiently larger than $A(s)$. The firm chooses whether "simple" or "complex" technology at the beginning of period t before it borrows the bank loan $w_t l_t$. The firm's choice of the technology is observable for the bank. The production takes place near the lending bank if the firm uses the simple technology and therefore the bank can costlessly observe the production process and seize the output if the firm repudiates the repayment of the bank loan. Therefore, the output $A(s)l_t$ is pledgeable as collateral when the firm borrows the bank loan if it chose the simple technology at the beginning of period t. The production takes place very far away from the lending bank if the firm uses the complex technology and the bank cannot observe the production process. The firm can make an untruthful claim that the output is lost by an accident, and the bank cannot verify that the claim is a false. This spacial friction makes the output of the complex technology unpledgeable when the firm borrows the bank loan if it chose the complex technology at the beginning of period t. The land k_t that the firm rents from the consumer can be put up as collateral for the bank loan $w_t l_t$. We implicitly assumed an economic institution that the bank loan has seniority to the equity. Therefore, if the firm repudiates the repayment of the bank loan, the bank can seize the collateral land and sell it in the market at the market price q_t to recover the loan repayment. The firm that rents k_t maximizes the dividend $d_t k_t = \max_{i_t \in \{s,c\}}\{d_t(i_t)k_t\}$, where $i_t = s$ represents the simple technology and $i_t = c$ represents the complex technology. $d_t(s)k_t$ is the dividend when the firm uses the simple technology and

$$d_t(s)k_t = \max_{l_t} A(s)l_t - (1 + r_t^L)w_t l_t, \tag{3}$$

$$\text{s. t.} \quad (1 + r_t^L)w_t l_t \leq q_t k_t + A(s)l_t, \tag{4}$$

and $d_t(c)k_t$ is the dividend when it uses the complex technology and

$$d_t(c)k_t = \max_{l_t} A(c)k_t^{\alpha} l_t^{1-\alpha} - (1 + r_t^L)w_t l_t, \tag{5}$$

$$\text{s. t.} \quad (1 + r_t^L)w_t l_t \leq q_t k_t. \tag{6}$$

Constraints (4) and (6) are the collateral constraints for the bank loan in the case of the simple and the complex technologies, respectively.

Bank: The bank live for two periods. It is born at the end of period t and dies at the end of period $t+1$. The bank borrows cash m_{t+1} from the consumer at the end of period t and repays $(1 + r_{t+1})m_{t+1}$ to the consumer at the end of period $t+1$. (m_{t+1} is the real balance.) At the beginning of period $t+1$ the price level changes such that the real balance that the bank holds becomes m_{t+1}/π_{t+1}. The bank lends the real balance m_{t+1}/π_{t+1} to the firm at the beginning of period $t+1$ and receives the loan repayment $(1 + r_{t+1}^L)m_{t+1}/\pi_{t+1}$ at the end of period $t+1$. Therefore the profit that the bank can get at the end of period $t+1$ is

$$\left[\frac{(1 + r_{t+1}^L)}{\pi_{t+1}} - (1 + r_{t+1}) \right] m_{t+1}. \tag{7}$$

The bank chooses m_{t+1} to maximize (7).

Government: The government decides the gross inflation rate π_{t+1} and makes the cash injection τ_t to the consumer at the end of period t.

Supply of land: The total supply of land is fixed at K:

$$k_t = K, \qquad \text{for all } t. \tag{8}$$

2.2 Essentiality of Credit

We assumed that the firm lives only for one period and that it needs to borrow the bank loan for wage payment. This setup implies that there is no possibility for the firm to accumulate the internal fund for wage payment. This setup can be regarded as a shortcut for modeling the reality that the size of the fund necessary to operate a production project usually exceeds the internal fund of the entrepreneur or the firm. In the general model in Section 3 we allow the accumulation of the internal funds but we still have a sub-optimal equilibrium where the firm (or the entrepreneur) needs to borrow from the bank, because the debt outstanding is too large for the firm to accumulate the sufficient amount of the internal fund.

2.3 Steady-State Equilibria

The consumer's optimization implies that in equilibrium

$$w_t = \frac{U_l(t)}{U_c(t)}, \tag{9}$$

$$1 + r_{t+1} = \frac{U_c(t)}{\beta U_c(t+1)}, \tag{10}$$

$$q_t = \frac{1}{1 + r_{t+1}}\{d_{t+1} + q_{t+1}\}, \tag{11}$$

where $U_c(t) = \frac{\partial}{\partial c_t}U(c_t, 1 - l_t)$ and $U_l(t) = \frac{\partial}{\partial l_t}U(c_t, 1 - l_t)$. Since the bank's profit should be finite in equilibrium, it must be the case in equilibrium that

$$1 + r_t^L = (1 + r_t)\pi_t. \tag{12}$$

In the case where the firm chooses the simple technology, the first-order condition (FOC) for the firm's optimization is the following on the premise that constraint (4) does not bind:

$$(1 + r_t^L)w_t = A(s). \tag{13}$$

Therefore, $d_t(s) = 0$. We will justify later that (4) does not bind in a steady-state equilibrium. In the case where the firm chooses the complex technology, the firm's optimization implies the following on the premise that constraint (6) does not bind:

$$(1 + r_t^L) w_t = (1 - \alpha) A(c) \left(\frac{k_t}{l_t} \right)^{\alpha}, \tag{14}$$

$$d_t(c) = \alpha A(c) \left(\frac{l_t}{k_t} \right)^{1-\alpha}. \tag{15}$$

We will justify later that (6) does not bind in a steady-state equilibrium. The equilibrium of this economy is defined as follows:

Definition 1 *Given monetary policy $\{\pi_t\}_{t=1}^{\infty}$, the competitive equilibrium is a sequence of quantities $\{c_t, l_t, d_t, k_{t+1}, m_{t+1}, \tau_t\}$, prices $\{q_t, w_t, r_t, r_t^L\}$, and technologies $\{i_t\}$, where $i_t \in \{s, c\}$, such that (i) the consumer maximizes his utility; (ii) the firm maximizes the profit; (iii) the bank maximizes the profit; and (iv) all markets clear.*

Because of the existence of technology choice, the dynamics of the equilibrium in this economy are not as easily characterized by the FOCs and the resource constraints as in the case of the standard business cycle models. There may exist an equilibrium path in which the production technology changes over time in a complicated way. Leaving the full characterization of the dynamics of the equilibria to the future research, we focus on the analysis of the steady-state equilibria in this paper. In the following steady-state analysis we assume that the inflation rate is constant:

$$\pi_t = \pi, \quad \text{for all } t, \tag{16}$$

where $\pi \geq \beta$. As we show below there are two steady-state equilibria, the good equilibrium and the bad equilibrium, which may be interpreted as a normal time and a financial crisis, respectively.

Good Equilibrium: We assume and justify later that the prevailing technology is the complex technology in the good equilibrium. The FOCs for the consumer and the bank imply that

$$1 + r = \beta^{-1}, \tag{17}$$

$$1 + r^L = \pi / \beta. \tag{18}$$

Assuming that (6) is not binding, the variables $\{l, c, q\}$ are determined by

$$(1-\alpha)A(c)\left(\frac{K}{l}\right)^{\alpha} = (1+r^L)\frac{U_l}{U_c}, \tag{19}$$

$$c = A(c)K^{\alpha}l^{1-\alpha}, \tag{20}$$

$$q = \frac{\beta\alpha c/K}{1-\beta}. \tag{21}$$

It is easily shown that (6) is not binding for a reasonable set of parameter values that induce a large value of q, e.g., $\beta = .95$ and $\alpha = .33$. Also since $d(c) > 0 = d(s)$ under these prices, the profit-maximizing firm never chooses the simple technology. Therefore, if the agents share the optimistic expectations that the asset price q is going to be high, the economy stays in the good equilibrium.

Bad Equilibrium: We assume and justify that the simple technology prevails in the bad equilibrium. $1 + r = 1/\beta$ and $1 + r^L = \pi/\beta$. Assuming that (4) is not binding, the variables $\{l, c, q\}$ are determined by

$$A(s) = (1+r^L)\frac{U_l(t)}{U_c(t)}, \tag{22}$$

$$c = A(s)l, \tag{23}$$

$$q = 0. \tag{24}$$

It is easily shown that (4) is not binding and holds with equality. Also since $q = 0$, constraint (6) implies that the production by the complex technology is infeasible under the prices of the bad equilibrium. Therefore, the firm cannot choose the complex technology. Therefore, if the agents share the pessimistic expectations about the asset price that $q_t = 0$ for all t, the economy stays in the bad equilibrium.

On Monetary Friction: Equations (19) and (22) say that the marginal product of labor on the left-hand side equals $(1 + r^L)$ times the marginal rate of substitution between the consumption and the leisure on the right-hand side. The loan rate r^L works as the labor wedge (Chari, Kehoe, and McGrattan 2007; Shimer 2009) that distorts the efficiency in the labor input in the same way as the labor-income tax does. The labor wedge is eliminated if the government adopts the monetary policy such that $\pi = \beta$, that is, the Friedman rule. Note that the monetary friction appears only as the labor wedge in this model, which can be completely eliminated by the Friedman rule. But the Friedman rule cannot eliminate the multiplicity of the equilibria and the bad equilibrium may emerge even under the Friedman rule if the pessimism that $q = 0$ is strongly shared by the agents.

3 The General Model with Debt

In the simplistic model in the previous section, the "simple" technology is oversimplified for realistic analysis of the modern financial crisis or business cycles. In the general model in this section, we assume that the simple technology also utilizes the capital as input.

3.1 The Environment

Time is discrete and continues forever: $t = 0, 1, 2, \ldots$. There are four agents in the economy: the representative consumer, the entrepreneur, the bank, and the government. There are two goods, consumption good and labor, and three assets, land, cash, and bonds. The entrepreneur who lives forever produces the consumption good.

Consumer: The representative consumer lives forever. At the beginning of period t, he owns b_t units of real bonds that are issued by the entrepreneur as his asset. He supplies labor l_t, receives the wage $w_t l_t$ in the form of cash, buys and consumes c_t units of the consumption good in period t, where w_t is the wage rate. At the end of period t the bank offers that if the consumer deposits m_{t+1} units of real balance the bank will return $(1 + r_{t+1})m_{t+1}$ at the end of period $t + 1$. If $1 + r_{t+1} > \pi_{t+1}^{-1}$, where π_{t+1} is the inflation rate between periods t and $t + 1$, the consumer deposits all cash into the bank. At the end of period t, the consumer receives the gross return on the bonds, $(1 + r_t)b_t$, from the entrepreneur. He purchases c_t, bonds b_{t+1}, and the bank deposit m_{t+1} at the end of period t. We assume that purchase of the consumption good can be implemented with credit and is not subject to the cash-in-advance constraint. The consumer's utility is the same as in the model of the previous section, which is given by (1). The consumer chooses $\{c_t, b_{t+1}, m_{t+1}, l_t\}$ to maximize (1) subject to the following budget constraint:

$$c_t + b_{t+1} + m_{t+1} \leq w_t l_t + (1 + r_t)(m_t + b_t) + \tau_t. \tag{25}$$

Entrepreneur: The representative entrepreneur lives forever. The entrepreneur owns land k_t, the market price of which is q_t, and issues real bonds b_t. The bonds must be secured by the collateral k_t and the pledgeable output because the entrepreneur does not have the ability to precommit to redeem the bond. He can also hold cash m'_t. The entrepreneur can produce the consumption good y_t from land k_t and labor l_t in period t. He needs to pay cash to buy labor l_t from the consumer in the labor market. Therefore, the entrepreneur needs to finance $w_t l_t$ using his cash m'_t and borrowing from the bank if necessary. The entrepreneur must repay $(1 + r_t^L)(w_t l_t - m'_t)$ to the bank at the end of period t. There are two production technologies available for the firm: the "simple" technology and the "complex" technology. The output of the simple technology is

$$y_t = A(s) k_t^{\alpha} l_{t-1}^{1-\alpha},$$

and the output of the complex technology is

$$y_t = A(c) k_t^{\alpha} l_{t-1}^{1-\alpha},$$

where $A(c) > A(s)$. Unlike the simplistic model in Section 2, we assumed in this model that there is one-period lag in the production process so that the labor input l_t in period t generates the output in period $t + 1$. We made this assumption in order to have all choice variables in (28) determined in period t. The entrepreneur chooses whether the simple or the complex technology at the beginning of period t before he borrows the bank loan $w_t l_t$. The choice of the technology is observable for the bank. The production takes place near the lending bank if the entrepreneur uses the simple technology and therefore the bank can costlessly observe the production process and seize the output if the entrepreneur repudiates the repayment of the bank loan. Therefore, the output

$A(s)k_t^{\alpha}l_{t-1}^{1-\alpha}$ is pledgeable as collateral when the entrepreneur borrows the bank loan if he chose the simple technology at the beginning of period t. The production takes place very far away from the lending bank if the entrepreneur uses the complex technology and the bank cannot observe the production process. The entrepreneur can hide the output from the bank and make an untruthful claim that the output is lost by an accident, and the bank cannot verify that the claim is a false. This spacial friction makes the output of the complex technology unpledgeable when the entrepreneur borrows the bank loan if he chose the complex technology at the beginning of period t. The land k_t and the pledgeable output can be put up as collateral for the bank loan $w_t l_t$. If the entrepreneur repudiates the repayment of the bank loan, the bank can seize the collateral land and sell it in the market at the market price q_t to recover the loan repayment. The entrepreneur's utility is given by

$$\sum_{t=0}^{\infty} \beta^t c_t', \tag{26}$$

where c_t' is the consumption in period t. The entrepreneur chooses $\{i_{t+1}, c_t', l_t, b_{t+1}, m_{t+1}', k_{t+1}\}_{t=0}^{\infty}$, where $i_t \in \{s, c\}$ and s (c) stands for the simple (complex) technology, to maximize (26) subject to

$$\begin{aligned} & c_t' + (1+r_t)b_t + q_t k_{t+1} + \max\{(1+r_t^L)[w_t l_t - m_t'/\pi_t], w_t l_t - m_t'/\pi_t\} \\ & \qquad \le A(i_t)k_t^{\alpha}l_{t-1}^{1-\alpha} + q_t k_t + b_{t+1} - m_{t+1}', \end{aligned} \tag{27}$$

$$(1+r_{t+1})b_{t+1} \le q_{t+1}k_{t+1} + \sigma(i_{t+1})A(i_{t+1})k_{t+1}^{\alpha}l_t^{1-\alpha}, \tag{28}$$

$$\max\{(1+r_t^L)[w_t l_t - m_t'/\pi_t], 0\} \le q_t k_t + \sigma(i_t)A(i_t)k_t^{\alpha}l_{t-1}^{1-\alpha} - (1+r_t)b_t, \tag{29}$$

$$c_t' \ge 0, \tag{30}$$

where $\sigma(i_t)$ is the ratio of pledgeable output: $\sigma(s) = 1$ and $\sigma(c) = 0$, Constraints (28) and (29) are the collateral constraints for the intertemporal bonds and the intratemporal borrowing for the wage payment. We assume that the bonds are senior debt, and therefore the collateralizable assets and the pledgeable goods are put up for the redemption of the bonds and the remaining can be used as collateral for the bank loan.

The bank and the government behave in the same way as in the model of the previous section. The total supply of land is also fixed at K in this model.

3.2 Steady-State Equilibria

The equilibrium of this economy is defined as follows:

Definition 2 *Given monetary policy $\{\pi_t\}_{t=1}^{\infty}$, the competitive equilibrium is a sequence of quantities $\{c_t, l_t, k_{t+1}, m_{t+1}, m'_{t+1}, \tau_t\}$, prices $\{q_t, w_t, r_t, r_t^L\}$, and technologies $\{i_t\}$, where $i_t \in \{s, c\}$, such that (i) the consumer maximizes his utility; (ii) the entrepreneur maximizes his utility; (iii) the bank maximizes the profit; and (iv) all markets clear.*

In the case where the choice of the production technology is invariant over time, the equilibrium is characterized by the following set of the FOCs and resource constraints. We use μ_t and ψ_t to denote the Lagrange multipliers for (29) and (30), respectively. Note that constraint (28) does not bind in equilibrium. Therefore the equilibrium, given $\{i_t\}$, is characterized by

$$w_t = \frac{U_l(t)}{U_c(t)}, \tag{31}$$

$$1 + r_{t+1} = \frac{U_c(t)}{\beta U_c(t+1)}, \tag{32}$$

$$1 + \psi_t = (1 + \mu_{t+1} + \psi_{t+1})(1 + r_{t+1})\beta, \tag{33}$$

$$(1 + \psi_t)q_t = \beta \left\{ (1 + \psi_{t+1} + \sigma(i_{t+1})\mu_{t+1})\alpha A(i_{t+1}) \left(\frac{l_t}{k_{t+1}} \right)^{1-\alpha} + (1 + \psi_{t+1} + \mu_{t+1})q_{t+1} \right\}, \tag{34}$$

$$(1 + \psi_{t+1} + \mu_{t+1})(1 + r_t^L)w_t = \beta(1 + \psi_{t+1} + \sigma(i_{t+1})\mu_{t+1})(1 - \alpha)A(i_{t+1}) \left(\frac{k_{t+1}}{l_t} \right)^{\alpha}. \tag{35}$$

Leaving the analysis of the dynamics to the future research, we consider two steady-state equilibria, the good and the bad, in which the production technology is time-invariant.

Good Equilibrium: We assume and justify later that the prevailing technology is the complex technology and that m'/π is sufficiently large or b is sufficiently small such that collateral constraints (28) and (29) and the nonnegativity constraint (30) are all nonbinding. Under these assumptions, the good equilibrium is characterized by (17)–(18), (20), and

$$\beta(1-\alpha)A(c)\left(\frac{K}{l}\right)^{\alpha} = (1+r^L)\frac{U_l}{U_c}, \tag{36}$$

$$q = \frac{\beta[1-(1-\alpha)\beta]A(c)(l/K)^{1-\alpha}}{1-\beta}. \tag{37}$$

Given these prices and quantities, in turn, it is easily shown that (28), (29), and (30) are nonbinding for sufficiently small b or sufficiently large m'/π ($\leq wl$). When these constraints are nonbinding, the utility-maximizing entrepreneur chooses the complex technology under these prices and quantities, since $A(c) > A(s)$.

Bad Equilibrium: We assume and justify that the prevailing technology is the simple technology and that $c' = m' = 0$ and b is sufficiently large such that the entrepreneur's profit is all spent as the interest payment for b. We also assume that μ and ψ are zero. In equilibrium, $1 + r = 1/\beta$ and $1 + r^L = \pi/\beta$. The variables $\{c, l, q\}$ are determined by

$$\beta(1-\alpha)A(s)\left(\frac{K}{l}\right)^{\alpha} = (1+r^L)\frac{U_l}{U_c}, \tag{38}$$

$$c = A(s)K^{\alpha}l^{1-\alpha}, \tag{39}$$

$$q = \frac{\beta[1-(1-\alpha)\beta]A(s)(l/K)^{1-\alpha}}{1-\beta}. \tag{40}$$

The above assumption implies that b should be

$$b = \frac{\beta}{1-\beta}[1-(1-\alpha)\beta]A(s)K^{\alpha}l^{1-\alpha} = qK. \tag{41}$$

If b is determined by (41), it is shown that $m' = c' = 0$ and $rb = A(s)K^{\alpha}l^{1-\alpha} - (1+r^L)wl$, that is, all profits of the entrepreneur are spent as the interest payment for b and the entrepreneur cannot reduce b_{t+1} nor accumulate cash m'. It is also easily shown as follows that the entrepreneur has no other choice than to choose the simple technology:

Consider a representative period t; since the entrepreneur can pay at most rb, he must set $b_{t+1} = b$; since $1 + r = 1/\beta$, constraint (28) becomes

$$b/\beta \leq qK, \tag{42}$$

if the entrepreneur chooses the complex technology for period $t + 1$; the constraint (42) does not hold, however, because the left-hand side is $b/\beta = qK/\beta$, which is larger than the right-hand side; therefore, the entrepreneur must choose the simple technology.

Therefore, if the outstanding amount of the bonds is given by (41) and there prevails the pessimism that q is determined by (40) forever, then the economy stays in the bad equilibrium.

Debt and productivity: In this general model, the technology in the steady-state equilibrium corresponds to the debt level, b. If b is small, the good equilibrium is realized and the aggregate productivity is high ($A(c)$), since the complex technology prevails. If b is large and given by (41) and if the pessimism that the asset price is given by (40) prevails, then the simple technology is used and the aggregate productivity is low ($A(s)$). Note that even if b is given by (41), it is possible that the complex technology prevails if $A(c)$ is sufficiently larger than $A(s)$ and the agents share the optimism that q is given by (37). Therefore, if $A(c)$ is sufficiently larger than $A(s)$, the multiplicity of equilibria emerges for a large b and the good or the bad equilibrium is realized depending on the expectations on the asset price q.

3.3 On dynamics of the model (Incomplete)

Because we assumed the linear utility for the entrepreneur, we have a clear result on the dynamics of the model.

Lemma 1 *There cannot exist a steady-state equilibrium in which the entrepreneur chooses the simple technology and b is strictly less than the value defined in (41) .*

(Proof) Suppose that there exists a steady-state equilibirum in which the entrepreneur chooses the simple technology and $b < qK$, where q is determined by (38) and (40). In this steady state, the profit for the entrepreneur in each period is strictly larger than rb and $c' > 0$. In this case the entrepreneur can reduce b_{t+1} by setting $c'_t = 0$ and can make the debt burden b_T for $\exists T$ such that the collateral constraints (28) and (29) are nonbinding if the entrepreneur chooses the complex technology for $T+1$. Since the entrepreneur can obtain a strictly positive gain by this deviation he will choose the complex technology within finite periods. This contradicts the assumption for the steady state that the entrepreneur chooses the simple technology forever. Therefore, the steady-state equilibria with the simple technology and b less than qK does not exist. (End of Proof)

It would be shown that if the initial value of b is close to but strictly less than the value defined in (41) then in the equilibrium path there exists a finite T such that the entrepreneur chooses the simple technology for $t = 0, 1, 2, \cdots, T$ and the complex technology for $t \geq T+1$; and he keeps reducing b_{t+1} by setting $c'_t = 0$ and spending all profits for the redemption of the bonds for $t = 0, 1, 2, \cdots, T$ and both (28) and (29) become nonbinding for the complex technology for the first time at $t = T$. (Incomplete.)

Concave utility for the entrepreneur (Incomplete): The above arguments show that since the entrepreneur's utility is linear in c'_t in our model, the bad equilibrium is realized only for the unique value of b, which is defined in (41). If we assume that the entrepreneur's utility $u(c'_t)$ is concave in c'_t such that $\lim_{c \to +0} u'(c) = +\infty$, it would be shown that for a range of large values of b, there exists a steady-state equilibrium where the entrepreneur optimally chooses the simple technology; and that the deviation from the steady state is welfare reducing for the entrepreneur. (We can consider a possible deviation for the entrepreneur in which he reduces c'_t by a small amount and repays the outstanding debt b_t for some finite periods, and shifts the production technology to the complex at a certain period T. If the entrepreneur has the concave utility, it would be shown that for a deviation with any T the loss of the current utility due to the reduction of consumption exceeds the gain of the future productivity by shifting from the simple to the complex technology.)

4 Conclusion

The simplistic model in Section 2 demonstrates that when the value of the collateralizable asset (k_t) is lowered, the agents may choose a less productive but more easily pledgeable technology because they need to finance the production projects by the external funds, and the aggregate productivity of the economy declines as a result of the lower asset price. The model with debt in Section 3 demonstrates that when the debt burden is very heavy for the entrepreneurs they may choose a less productive and more easily pledgeable technology, and the aggregate productivity declines as a result of a surge of the debt burden. Both of these models may give explanations of the productivity declines or the output losses during and after the financial crises. Since the mechanisms of the productivity declines in our models are not monetary but real, a policy implication from these models is that the problem cannot be resolved by money injection. The problem is associated with the pessimistic expectations on the current and future asset prices and/or the balance-sheet deteriorations of the economic agents. If these models describe the major mechanism of the productivity declines after the financial crises, it can be said that in addition to the monetary injections we may need other policy measures that may entail fiscal outlays for financial crisis management, such as reduction of excessive debts of the debt-ridden borrowers through subsidies and bankruptcy procedures.

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