Liquidity Constraints in a Monetary Economy

Leo Ferraris    Makoto Watanabe

Departamento de Economía, Universidad Carlos III de Madrid

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Objective

- Money: the medium to transfer resources on the spot

- Liquidity: the availability of a medium to transfer resources over time

Explore a (monetary) model to study the issue of liquidity.
Key ingredients

(i) Use of money in spot exchange (Kiyotaki and Wright (1989)):

- Anonymity;
- Absence of double coincidence of wants.

⇒ Pledgeability of returns: the fundamental impediments arising in spot trade may seep into the credit market.
(ii) Liquidity (Holmstrom and Tirole (1998)):

- liquid project;

  \[ \text{pledgeable returns} = \text{expected returns} \]

- illiquid project;

  \[ \text{pledgeable returns} < \text{expected returns} \].
Preview of main results:

- The same frictions generating an essential role for money may also make firms liquidity constrained;

- Money can perform two roles - as a provider of liquidity service and exchange service;

- The binding liquidity constraint constitutes a channel through which under-investment occurs.
Literature

- Kiyotaki and Wright (1989)

- Kiyotaki and Moore (2001)
Model

A divisible/competitive version of monetary model, Lagos and Wright (2005), with a consumption and an investment market

- Time: discrete, infinite w./ three sub-periods (morning, afternoon and evening)

- Agents: entrepreneurs, investors; homogeneous, unit mass, infinitely lived

- Goods: consumption goods, investment goods; all production costs are normalized to one.
- Morning market (investment market):

  • Investors produce an investment good;

  • Entrepreneurs and investors meet randomly and bilaterally;

  • An investment good $q_1$ generates returns, *early returns* and *late returns*, to entrepreneurs with technology $g(q_1)$;

  • $g(\cdot)$ is continuously differentiable, strictly increasing and concave with $g(0) = 0$, $g'(0) = \infty$, $g'(\infty) = 0$;

  • The investment is a one-period event.
- Afternoon market (consumption market):

- Anonymous trading;

- Uncertainty in production and consumption opportunities; a buyer with prob $\delta$; a seller with prob $1 - \sigma$;

- A consumption good $q_2$ yields utility $u(q_2)$ to buyers. $u(\cdot)$ is differentiable and strictly increasing and concave with $u'(0) = \infty$, $u'(\infty) = 0$;

- Sellers have production technologies.
- Evening market (Walrasian market):

  Agents can produce and trade an output whose market price is normalized to one.

  Fiat money can also be traded at a market price, denoted by $\phi$. 
Timing

Morning

Investors
q1 → early returns g(q1)
Entrepreneurs

Afternoon

-Sellers: consume late returns g(q1)
produce consumption goods q2
-Buyers: consume q2 → u(q2)

Evening

Walrasian market
(output, money)
Efficiency

The planner solves

\[
\max_{q_1, q_2 \geq 0} \left[ g(q_1) - q_1 \right] + \left[ (1 - \sigma) g(q_1) + \sigma (u(q_2) - q_2) \right].
\]

The optimal solution \( q_1^*, q_2^* > 0 \) satisfies

\[
(2 - \sigma) g'(q_1^*) = 1, \\
u'(q_2^*) = 1.
\]
Contract with investors

- Long term contracts are not available;

- Only early returns of investment are pledgeable;

A contract between an entrepreneur and an investor specifies the amount $q_1$ of investment goods, generating output with technology $g(q_1)$, and its payment $z, \theta$ that satisfies

$$z + \theta \phi m = q_1 \quad (1)$$
$$z \leq g(q_1) \quad (2)$$
$$\theta \in [0, 1]. \quad (3)$$
Berman equations

[Evening market]:

\[
W(\hat{m}) = \max_{x,e,m+1 \geq 0} \left[ x - e + \beta V(m+1) \right]
\]

s.t. \( x - e = \phi(\hat{m} - m+1) + \tau \)

where \( \hat{m} = (1 - \theta)m - pq_2 \) or \( \hat{m} = (1 - \theta)m + pq_2^s \).
[Afternoon market]:

\[
Z(q_1, (1 - \theta)m) = \sigma \left\{ \max_{q_2 \geq 0} \left[ u(q_2) + W((1 - \theta)m - pq_2) \right] \right\}
\text{s.t. } pq_2 \leq (1 - \theta)m
\]

\[
+ (1 - \sigma) \left\{ \max_{q_2^s \geq 0} \left[ g(q_1) - q_2^s + W((1 - \theta)m + pq_2^s) \right] \right\}
\]
[Morning market]:

\[
V(m) = \max_{q_1, z, \theta \geq 0} [g(q_1) - z + Z(q_1, (1 - \theta)m)]
\]

s.t. (1)-(3)

or by \( z = q_1 - \theta \phi m \),

\[
V(m) = \max_{q_1, \theta \geq 0} [g(q_1) - (q_1 - \theta \phi m) + Z(q_1, (1 - \theta)m)]
\]

s.t. \( q_1 - \theta \phi m \leq g(q_1) \)

\( \theta \in [0, 1] \)
First order conditions

\[(2 - \delta)g'(q_1) = 1 + \mu(1 - g'(q_1))\]

\[\mu + \frac{\gamma}{\phi m} = \delta(u'(q_2) - 1)\]
Complementary slackness condition

\[ \mu [g(q_1) - q_1 + \theta \phi m] = 0 \]

\[ \gamma \theta = 0 \]

Two situations are possible:

1. Binding liquidity constraint.

2. Non-binding liquidity constraint.
Euler equation

\[ \phi = \beta \phi_+ \left[ (1 - \theta)(\delta u'(q_2) + 1 - \delta) + \theta(\mu + 1) \right] \]
Euler Equation

\[ 1 \to \beta \frac{\varphi + 1}{\varphi} \left[ \theta(1 + \mu) + (1 - \theta)(\delta u'(q_2) + 1 - \delta) \right] \]
Stationary monetary equilibrium

\[ \mu + \frac{\gamma}{q_2} = \delta(u'(q_2) - 1) = \frac{\pi}{\beta} - 1 \]

Three possible cases for \( \pi > \beta \):

[1] liquidity constraint is not binding \( \mu = 0 \) and no money is pledged \( \theta = 0 \);

[2] liquidity constraint is binding \( \mu > 0 \) and no money is pledged \( \theta = 0 \);

[3] liquidity constraint is binding \( \mu > 0 \) and a positive amount of money is pledged \( \theta > 0 \).
Proposition 1 Suppose $g(q_1^*)/q_1^* \geq 1$. Then, a unique equilibrium exists for all $\pi > \beta$ in which the liquidity constraint is not binding, $\mu = 0$, and no money is pledged, $\theta = 0$. Further, it satisfies: $q_1 = q_1^*$ for all $\pi > \beta$; $q_2 \in (0, q_2^*)$ is strictly decreasing in $\pi \in (\beta, \infty)$; $q_1 \to q_1^*$, $q_2 \to q_2^*$ as $\pi \to \beta$. 


The case $g(q^*1)/q^*1 \geq 1$: unconstrained
**Proposition 2** Suppose $g(q^*_1)/q^*_1 < 1$. Then, a unique equilibrium exists for all $\pi > \beta$ in which the liquidity constraint is binding, $\mu > 0$. It satisfies: $q_2 \in (0, q^*_2)$ is strictly decreasing in $\pi \in (\beta, \infty)$; $q_1 \to q^*_1$, $q_2 \to q^*_2$ as $\pi \to \beta$. Further, there exists a unique $\hat{\pi} \in (\beta, \infty)$ such that $q_1 = \hat{q}_1 \in (0, q^*_1)$ at $\pi = \hat{\pi}$ and:

1. $\theta > 0$ for $\pi \in (\beta, \hat{\pi})$ and $\theta = 0$ for $\pi \in [\hat{\pi}, \infty)$;

2. $q_1 \in (\hat{q}_1, q^*_1)$ is strictly decreasing in $\pi \in (\beta, \hat{\pi})$ and $q_1 = \hat{q}_1$ for all $\pi \in [\hat{\pi}, \infty)$. 
The case $g(q^{*1})/q^{*1} < 1$: constrained
Discussion 1: money and credit

“Evil is the root of all money” (Kiyotaki and Moore (2001))

versus

“Money is the root of all evil” (The Bible, 1 Timothy 6:10)
Discussion 2: policy and empirical implications

• Impact of inflation on investment according to the stage of country development (Gertler and Rogoff (1990))

• Negative but decreasing effect of inflation on investment (Boyd, Levine and Smith (2001))
Discussion 3: definition of liquidity

- Completeness of markets (Holmstrom and Titole (1998))
- Means of payment (Shubik (1999), Kiyotaki and Moore (2000))
- Thinnes of market (Diamond (1986), Jones and Ostroy (1984), Morris and Shin (2003))
- Agents' ability to sell contingent promises of future deliveries (Diamond and Rajan (2001), Caballero and Krishnamurthy (2001))
- Flexibility to move goods (Fostel and Geanakoplos (2008))
Conclusion

- Liquidity constraints

- Money can play two roles - as a provide of liquidity services and exchange services

- Interaction of an investment and a consumption market